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Multicriteria Analysis

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and 50 Tables

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Prof. João Clímaco
University of Coimbra
Faculty of Economics
Av. Dias da Silva, 165
3000 Coimbra
Portugal

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General Introduction

J. Clímaco and C. H. Antunes

After the pleasure which has been to host the community of researchers and practitioners in the area of multicriteria analysis (MA) in Coimbra in August 1994, this volume of proceedings based on the papers presented at the conference is the last step of that venture. Even though this may not be the appropriate place we cannot resist, however, the temptation to express herein some brief feelings about the conference. Almost everything concerning the conference organisation has been "handcrafted" by a small number of people, with the advantages and disadvantages that this approach generates. Our first word of acknowledgement is of course due to those who have had a permanent and active role in the multiple aspects which make the success of a conference: Maria João Alves, Carlos Henggeler Antunes (who is a co-author of this introduction since he has closely collaborated with me in the scientific programme), João Paulo Costa, Luís Dias (who greatly contributed to the organisation of this volume) and Paulo Melo, as well as Leonor Dias, from the Faculty of Economics, who has shown an outstanding dedication. To those who collaborated with the organisers in the framework of their professional activity, special thanks due to Adelina whose dedication greatly exceeded her duties. As you probably know from your own experience every small detail of the conference organisation required a lot of "sweating", but the atmosphere of joy and friendship then generated has been a generous "pay-off". We all remember little episodes such as the arrival to Coimbra of Prof. Tzeng (organiser of the previous conference in Taiwan) some days before the schedule, lost in the train station, and who ended having dinner in the home of a unknown family. We finally found Prof. Tzeng at his hotel around midnight after we had received several phone calls about a supposedly "Japanese" man lost in Coimbra... The social programme required an almost obsessive periodic revision to ensure an enjoyable and smooth running! When we thought it would be "almost perfect", a reception happened in which the announced group of *Fados de Coimbra* made a no-show, with the participants constantly asking for the students in black, playing guitars and singing, that someone had promised them.... Fortunately we were able to appeal to our improvisation ability to correct this in real-time at the conference dinner at S. Marcos Palace!

We sincerely believe that participants and accompanying persons actually have enjoyed those days in the special atmosphere of Coimbra. For some of our more daring colleagues those days even permitted to enter the famous Coimbra *bohemia*, since they have known Severo and spent nights at his home breathing the special perfume which still exists

in Coimbra nightlife for those who persist in forgetting for some hours his/her space- and time-connections and giving free ride to his/her imagination in front of a glass of wine! But we must acknowledge that, even for those engaged in the nightlife social activities, the plenary sessions each morning were very participated!

The outstanding scientific level of the conference, we could now appreciate in building up this proceedings volume, is due to all participants who presented communications. Please let us mention with a special word the invited plenary speakers Anna Ostanello, Jaap Spronk and E. Vermeulen, V. Ozernoy, and Jared Cohon, as well as the organisers of the workshop on Behavioural Issues in Multicriteria Decision Analysis, P. Korhonen and J. Wallenius. This important initiative has had its start kick in Coimbra (a new one has been held in Hagen the following year) as an open discussion forum to encourage and foster discussion on those topics.

When the conference was over, the difficult task of organising a proceedings volume began. As Theo Stewart writes on his paper also the state of the art of MA involves a "Babel-like confusion of tongues". This situation was clear in a conference where the participation has been very diversified as the schools of thought which are recognised in the area are concerned. We believe that this diversity is a richness which we need to make the most of it in order to potentiate the existing synergy among the different perspectives. Under this circumstances we soon understood that we had in hands a good opportunity to put together a book which, being essentially based on the communications presented at the conference, could be more than a mere collection of papers to be a structured selection of those papers in a framework of invited leading papers in specific topics. We then decided to include still a chapter of invited short papers about the future trends of MA, in such a way to help the reader perceiving the plurality of challenges that lie ahead. This has been a long and time consuming job which occupied part of the last two years. In this general introduction we would like to thank all the authors of the invited papers: the leading papers at the beginning of each chapter, the short papers of the Future Trends chapter, and the paper related to the workshop on Behavioural Issues in Multicriteria Decision Analysis. Special thanks are also due to the authors of the contributed papers which made their best in the revisions suggested by the referees. In deciding to organise this book according to thematic areas, we attempted, up to the possible extent, to obtain a certain balance. As it is obvious papers with merit may have been rejected by absolute lack of space in their thematic area having in mind the acceptable maximum page length for a proceedings volume. Thanks are also due to the referees who helped us in the difficult task of selecting the contributed papers to be included in this volume, and specially for the understanding of all our colleagues

who have seen their works rejected in function of both the referees' "value judgement" and the available space according to the structure decided by the editor. The "first page" of each chapter is to be considered as part of this general introduction, in which a brief introduction to the papers included in that chapter is made.

Within the above mentioned "Babel-like confusion of tongues" several branches of MA can be recognised such as multiobjective mathematical programming methods, utility/value function approaches, outranking methods, interactive procedures, etc., besides theoretical and methodological issues. Clear preoccupations presently exist not just with the methods, but also with the structuring and modelling of problems, as well as model validation and robustness analysis. Even though these issues are not exclusive of MA, they are specially felt by us since the development of MA has, in our opinion, a motivation on the difficulties of classical Operations Research/Decision Analysis (OR/DA) approaches. The cognitive paradigms the multicriteria approaches are based on are also far from being a peaceful issue. The need arises namely to clarify the theoretical foundations of many procedures, to unify concepts as well as the use of a certain terminology and to take into account empirical studies about the DMs' behaviour, in order that discussion could be based on the essential issues. It is our conviction that all this apparent confusion, if appropriately understood, can potentiate complementary and synergetic ways to face the problems. Finally it cannot be forgotten the relationships between MA and other areas of knowledge such as group decision, cognitive psychology, economic sciences and social sciences in general, artificial intelligence, information systems, decision support systems (DSS), new technologies generally linked to computer sciences, etc. Besides their importance DSSs are also a ground for some confusion, regarding not just the understanding of what they should be but also concerning the role of the analyst/mediator.

This melting pot appears as natural if we have in mind the complexity of the decisions to be made in a complex and turbulent real-world context. In our opinion the multicriteria approaches are essentially the way certain nonconformist minds attempt to face the challenges of OR/DA. The issue is not the systematic replacement of OR/DA methodologies, but, whenever appropriate, to use nonconformist multicriteria approaches endeavouring alternative understandings of reality looking for a better intervention on it. We will feel deeply rewarded if this book contributes to a clearer understanding of the different "branches" of MA.

This book begins with a chapter on theory and methodology in which Anna Ostanello raises some key issues from the point of view of both cognitive complexity and plurality of situations (including the existence of multiple decision makers). She points out the potentialities of the

constructivist attitude and presents the papers selected for this chapter, referring to very important subjects such as translating qualitative preference information into numerical values required by the aggregating procedures, the complexity of the aggregation problem, problems of the interdependence among the objectives or criteria, a link between the "English School of soft OR/DA" and the "MCDA European School", and an extension of a modelling/validation process. Besides the scientific developments herein presented, this chapter raises the issues of MA in a way that clarifies the reading of the whole book.

The following chapters are of a more technical nature. In Chapter Two J. Martel writes an interesting leading paper on the important issue of "aggregating preferences - utility function and outranking approaches". This leading paper is followed by four contributed papers, two dealing with outranking methods, one related to the approaches which use utility functions and one paper which discusses some key questions dealing with the selection of the most appropriate method to tackle a given problem.

Chapter Three deals with multiobjective mathematical programming. The chapter begins with a short paper by H. Nakayama who interprets the decision problem associated with a multiobjective mathematical programme as a "value judgement" problem. The chapter also includes a deep and extensive tutorial paper on the development and future aspects of vector optimisation, as well as four other technical contributed papers.

Chapters Four and Five are devoted to more specific branches of multiobjective mathematical programming. Chapter Four is devoted to uncertainty in multiobjective programming. Roman Slowinski writes an interesting leading paper on interactive fuzzy multiobjective programming (which does not cover the broader scope of the chapter). Two contributed papers also deal with the fuzzy approach, whereas Ben Abdelaziz et al. present a paper dealing with multiobjective stochastic programming (although this topic is not covered by the leading paper, the reference list of this contributed paper permits to the reader to become aware of the state-of-the-art on this topic).

The leading paper in Chapter Five makes a state-of-the-art review on multiobjective integer programming. The three contributed papers which follow present specific advances in particular cases of multiobjective integer programming.

In Chapter Six attention is paid to interactivity, which is a key issue from the methodological point of view. The interesting leading paper by L. Gardiner and D. Vanderpooten presents the historical evolution of the concept of interactivity in MA, makes a categorisation of the methods according to the underlying cognitive paradigm and points out the current

challenges concerning interactive approaches, for instance regarding the robustness analysis of the proposed solutions. This chapter includes three contributed papers on different topics of interactive mathematical programming.

Chapter Seven begins with a complete leading paper authored by G. Kersten. The relationships between multicriteria analysis and group decision and negotiation are discussed. This is an important area in which the DMs' cognitive and behavioural considerations are essential. The chapter includes four technical contributed papers on specific topics.

Chapter Eight deals with new computer science developments and multicriteria analysis. The chapter begins with a provocative paper by C. H. Antunes and A. Tsoukiàs in which the authors confess the difficulty to approach this topic, thus opting for a discussion based on some selected keywords (such as multimedia, expert systems, neural networks, etc.) and their potential relationships with MA. Two contributed papers follow, one devoted to the important topic of Human-computer interaction and the other using neural networks in multicriteria decision support.

Chapter Nine is completely devoted to the invited paper related to the workshop on Behavioural Issues on Multicriteria Decision Analysis. Its most important claim is that "to improve the success of decision tools in practice, MCDM researchers should pay more attention to the behavioural realities of decision making". Special attention is paid to the discussion on the relationships between interactivity and the DM's behavioural issues.

Chapter Ten aims at presenting multicriteria approaches in practice - applications and methodological issues. Naturally this long chapter has a lower level of integration than the others. The application and experimentation of MA approaches to provide decision support in real-world problems and/or gaining insights from laboratory case studies is an essential keystone for the development of the MA area. A foreword initiates the chapter, briefly describing the contributed papers. From the material in this chapter it is clear that MA is a key component of Multicriteria Systems for Decision Support, but not the whole system. The complexity of real-world situations implies taking into account holistic and intuitive DM's point of view.

The last chapter of this book is entirely devoted to invited papers and it aims at making the reader aware of a plurality of perspectives on the future trends of MA. It includes interesting contributions which complement the ideas expressed in the leading papers and help the reader to make clearer ideas about the melting pot we have been referring to from the very beginning.

I

Theory and Methodology

An invited leading paper by Ostanello presents an introduction to the subject of theory and methodology. This paper aims to outline a framework within which all the included papers find a common conceptual thread while some terminological differences remain. A need of new tools which might favorize a more mature and integrated development of the analyst's activities, within cultural contexts other than those hypothesized by the classical assumptions of OR/DA, is also underlined.

Bana e Costa and Vansnick present a theoretical framework for the MACBETH approach. Taking as a starting point the measurement rules used for numerical representing the qualitative (categorical) preference information assessed by the questioning procedure of MACBETH, they study the existence and unicity of a solution for the particular representation problem.

Carlsson and Fuller develop and build on measures of interdependence between the objectives (which are generally useful to be exploited when dealing with real world decision problems). The authors also develop some methods for finding effective and more realistic solutions to MCDM problems, using both crisp and fuzzy formulations.

The paper by Tsoukiàs and Vincke advances a basic distinction between uncertainty reasons, that is between lack of information and contradictory information. A semantical investigation of this approach is conducted and the PC preference structure is adopted to represent the different preference situations that may occur. The potentialities of this type of preference structure are discussed, and it is claimed that besides offering an enhanced granularity in preference modelling, it is strongly axiomatized thus enabling a precise calculus.

Daellenbach studies Checkland's Soft Systems Methodology (SSM) as a potential multi-criteria decision making methodology. The role of the *Weltanschauung* in the MCDM context and the process of finding a compromise solution involving conflicting *Weltanschauungen* among multiple-decision makers or stakeholders are explored. The author concludes that Checkland's SSM can, without any changes from its usual framework, be used to bring about compromise solutions.

Ostanello summarises some methodological considerations concerning the development/validation process of a "tool" to be used as a support in a complex situation of decision making. A "bipolar" and multistage model is proposed for such a process, within a Multicriteria Decision Aid framework. This model integrates and extends two important models of modelling/validation process literature, and it is conceived as a support tool for the analyst's work during a decision aiding intervention.

Complexity Issues and New Trends in Multiple Criteria Decision Aid

Anna Ostanello
Politecnico di Torino, Dipartimento Sistemi di Produzione ed Economia dell' Azienda
Corso Duca degli Abruzzi 24
10129 Torino, ITALY

Abstract. This paper is an introduction to the present chapter and aims to outline a framework within which all the included papers find a common conceptual thread while some terminological differences exist. These reflect the complexity that is inherent to any decision aiding process in a real organisational context, a complexity which does not tolerate, for various validity reasons, a mutilating simplification that is often imposed by classical mathematical approaches. A need of new tools which might favorize a more mature and integrated development of the analyst's activities, within cultural contexts other than those hypothesized by the classical assumptions of OR/DA, is also underlined. This permits one to recall some new trends of MCDA research which are, at the moment, found in literature.

1 MCDA tools

The studies on multiple criteria decisions in 'real' or organisational contexts have been stimulated over the last ten-year period, after a distinction between the situations of Decision Making (MCDM) and Decision Aiding (MCDA) became clear to MCD researchers (cf [28] [29] [30]). Such an 'impulse' originated from a long standing tradition of empirical research on OR/DA processes within real organisational contexts, conducted by various groups since the beginning of the '70s (cf [1] [2] [3] [6] [7] [8] [15] [16] [20] [21] [22] [24] [31]). The suggestions and proposals for possible new lines of MCDA research deriving from such studies and other important results from other fields (sometimes overlapping with MCDM/A), such as Cognitive Psychology (cf [17]), Information Systems research (cf [32]), Decision Support Systems (cf [18]), Management Sciences (cf [12]) [14]) and Process Analysis (cf [16] [23]), were rich and stimulating. These did not, however, relevantly influence MCDA/MCDM literature, which has been principally focused on 'the mythical decision moment' of an isolated individual, a decision-maker 'freed' from any context or process constraint, until very recently (cf [10]).

New tools have been developed and new ways of using the existing methods have been proposed along with a 'new stream' of MCDA studies, with the aim of assisting both the analyst/modeller (*A*), i.e. the subject who has to 'take care' of the modelling activities (a decision-maker) during his OR/DA intervention, and the client/decision-maker in a double process: a decisional process which

develops within a multiactorial organisational context and concerns a 'real object' (cf [14] [15] [24]), and a cognitive/operational process with the aim of reaching an understanding of a real problem situation and of developing problem representation(s) or potential action(s) (cf [28]). As a great deal of research has shown, this last process is not always addressed to choosing or selecting the 'best potential action(s)' as a *decision* (cf [4]), independently of any conception of a possible subsequent process by which such an 'action' is implemented in a real context (*undertaken action*). The variety of inquiry, analysis, identification, structuring, development or evaluation steps recurrently conducted by the analyst and the involved actors to produce valid elements of answer to the client's or their own questions, make the analyst's endowment of traditional methods practically unusable in real cases (cf [25]). This is especially verified if external validity criteria are activated to test and control the activities of such an 'OR/DA worker' (generally called 'analyst' even though his role changes with the different stages of the intervention process) and their results.

By *new tools* one intends the important contributions brought by what is called the *English School of soft OR/DA*, on the one hand, (cf [7] [8] [11] [13] [27], [31]) (these refer, principally, to complex pluralistic situations [13] in which ill-structured or ill-formulated problems have to be dealt with) and by the *European School of MCDA* [30], on the other hand, especially with regard to some aspects of cognitive complexity which appear with 'well formulated' or semi-structured MC problems "in the presence of ambiguity, uncertainty and an abundance of bifurcations" [30]. These tools are not restricted to well known types of methods 'procedurally converging' to some structured numerical representation(s) (the most applied, within the contexts of basic mathematical culture, but not necessarily the most 'valid' ones to supply answers for real problems). The various categories of such tools include different kinds of models, methodologies, approaches, and techniques which may have different aims from the methods that have only focused on the treatment of quantitative data, and also in relation to the different operative contexts of the actual process stage (cf [26]). They therefore have to be seen in a different perspective, which might enable the analyst "to enhance the degree of conformity and coherence between the evolution of a decision-making process and the value systems and objective of those involved in this process" [30], and tend to "reason out the change prepared by a decision in such a way as to make it more consistent with goals and system values of the one for whom decision-aid is to be performed".

2 A need for an integrated approach of MCDA

If one accepts that decision aid is within a *double process framework*, then when one says 'multiple criteria decision aid' one is referring to a "multiple criteria approach for decision aiding" (cf [5] [6] [22]). The characterising features of such an approach include a process involving participation (participative paradigm) and a 'constructivist' attitude.

A *participative paradigm* implies, among other features of the multiactorial DA process, the accepting of a hypothesis on the behaviour of the two main

actors involved, i.e. client/decision-maker (*C*) and OR/DA worker (*A*). This behaviour requires participation and commitment during the intervention, and implies that both actors are engaged in a *modelling process* that is not separable from an understanding of several issues, of different nature, produced by the process development. The modelling process therefore has to be considered not as isolated or regarding a unique subject, but as being strongly interrelated with the *organisational context* (cf [32]). Other subjects involved are the actors who are capable of influencing the process by their systems of values, information and relationships, and not just a part of the *A* or *C*'s information system. This implies that the MCDA process activities are not restricted to those considered by the cognitive paradigm, but also include organisational and political ('social') actions, which may be essential for the validity of the cognitive process and for its formal and informal (sometimes invisible) results. On the other hand, formal modelling may have important organisational and political functions, if applied as useful tool to support client's learning (cf [14]) and to help actors' communication, cooperation, compromising, negotiation and thus organisational change (cf [8]). This implies, in particular, the analyst's endowment of tools to be considered in a wider sense, from a cultural and operational point of view.

A *constructivist approach* (cf [3] [9]) helps the analyst to structure 'action' representations and build models by seeking (or simply testing) some working hypotheses to "make recommendations" [5]. This implies the construction, along with the actors involved, of a "set of keys" [30] which will enable them "to go forward, progressing according to their own objectives and their systems of values".

3 Complexity issues and methodological challenges

Some typical concerns of the OR/DA worker constitute the 'heart' of MCDA research studies, within such a framework. These regard, in particular, the following points, among other issues pointed out in paper [30]: 1) meaning and role of axiomatic results; 2) attention paid to all sources of uncertainty, imprecision and ill-determination; 3) use of a wide range of preference model structures; 4) the study of interactive procedures.

Some other important issues have to be taken into consideration, within a perspective of an integrated (multicriteria) approach of MCDA. These have been generally ignored by the mathematical 'heart' of MCDM and MCDA, and concern, for instance, the elaboration of a methodology oriented towards a multiactorial and multidimensional process conception of decision aiding (cf [8] [10] [22] [24] [32]), and the elaboration of a multistage and multicriteria framework for MCDA process validation, which might favourize a more conscious development of the analyst's activities and selection of appropriate tools to achieve valid operational results under different perspectives (internal and external) (cf [19] [26]).

Some of these concerns constitute the focus of the papers included in the present chapter.

The contribution by Alexis Tsoukiàs and Philippe Vincke falls within the first three above mentioned points. The paper underlines some important issues related to the *complexity of the aggregation problem,* and proposes a few *Extended Preference Structures in MCDA* with particular attention being paid to a 'double process' framework. The aggregation step is not seen as an application of a 'decision rule', made by the analyst/modeller to an available well structured problem, but is kept within a validity framework of the DA process and, therefore, with all its interpretive difficulties. The analyst (or a participating decision-maker) is thus confronted with a "dilemma of either loosing information", to reduce such a complexity within a classical framework of preference axioms, or "to keep as much as possible of the preference information" which is achieved through DA interactions, so as to preserve some conceptual and operational validity [19] of the preference model. The Authors, according to this second way of approaching the problem, are conscious that there might be a 'cost,' in terms of "efficiency and clarity" of the applied aggregation procedure, but also that such an approach might favourize a better understanding and representation of the client's actual preferences, congruently with the achieved information system and the value system of this actor.

The paper of Carlos Bana e Costa and Jean-Claude Vansnick deals with a critical problem (very common in real situations), that is of translating value judgements - as these may be more or less easily expressed by a decision-maker - into numerical values that are, usually, required by an aggregation procedure. A direct numerical statement of such values, normally, meets some resistance by the decision-makers in the organisational contexts, because of the ambiguity and uncertainty of possible consequences that derive from such a formulation and of the use made of it by the analyst to deal with an already structured information base. The Authors develop a theoretical framework of the MACBETH approach, that they introduced in 1994. This approach permits one to interactively construct with the decision-maker an interval scale "for which the idea of difference of attractiveness" of a potential action over another of a given set "is meaningful". Such an approach seems to overcome some methodological problems that affect other 'numerical scaling' procedures proposed in MCDA literature.

Christer Carlsson and Robert Fuller point out a methodological problem, which is often avoided by the analyst during DA (or by a decision-maker in the case in which such an actor is concerned with the mathematical foundations of a solution approach he may use). This problem regards the possible interdependences among the objectives (or criteria) introduced in a multiple-objective model. The assumption "that all criteria are independent was initially introduced as a safeguard to obtain feasible solutions to multicriteria problems, as there were no means available to deal with interdependence. MCDM problems were complex and difficult to understand even without mathematical complications". By not allowing interdependence, the MC models "are simplified beyond recognition, and the solution reached by the traditional algorithms have only marginal interest". The Authors therefore propose a method for using criteria interdependence explicitly. This method is built around a family of application functions, with the aim of favourizing a decision-maker to express his own preferences, when any chosen preference is either conflicting or supportive in

relation to any other. They also show how fuzzy MOLP problems can be interpreted as multiple fuzzy reasoning schemes, and propose a method to find a compromise solution which seems to have "promising possibilities for effectiveness and usefulness in real life decision making".

The paper by Hans Daellenbach gives an interesting methodological contribution, by which an explicit link between the *English School of soft OR/DA* and the *MCDA European School* is given. The Checkland's Soft Systems Methodology (SSM) [11] is analysed, in particular, from a multicriteria decision aid perspective: " The aim of Checkland's SSM is to bring about changes in operating procedures, structures, and/or attitudes that are systematically desirable and culturally feasible. Both these aspects are essential for any change to be accepted and implemented. Cultural feasibility usually implies that the solution derived is a compromise over conflicting world views and hence conflicting objectives. For this point of view SSM is an MCDM" (or *MCDA*) "methodology, highly effective for bringing about change involving multiple stakeholders."

The last paper proposed by Anna Ostanello extends a conception of a modelling/validation process (MVP), as introduced in a few works of literature, to a 'bipolar' framework of decision aiding. The validation process is conceived, therefore, from a multiactorial and multicriteria perspective. This permits an integrated representation of various stages of the DA process with different functions of the OR/DA worker to be given. Such a model is based on six basic elements, which include: Client's problem situation, Analyst's problem situation and data validity. It is pictorially represented by an octahedron, whose 'geometrical' parts, i.e. tetrahedrons, facets and edges, represent many validation sub-processes linked through non-linear and dynamic relations during the intervention. This model underlines the complexity of modelling/validation in real contexts, points out a need for the OR/DA worker to constantly manage and evaluate his work, and may be used as a tool that may facilitate interpretive and organisational working problems.

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A Theoretical Framework for Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH)

Carlos A. Bana e Costa[1] and Jean-Claude Vansnick[2]

[1] Technical University of Lisbon, IST, Dpt. Civil/CESUR, Av. Rovisco Pais, 1000, Lisbon, PORTUGAL

[2] University of Mons-Hainaut, F.S.E.S., Place du Parc, 20, 7000, Mons, BELGIUM

Abstract. This paper presents a theoretical framework for the macbeth approach. Taking as a starting point the measurement rules used for numerically representing the qualitative (categorical) preference information assessed by the questioning procedure of macbeth, we study the existence of a solution for our particular representation problem and we briefly discuss its unicity.

Introduction

Based on judgments of an evaluator D about the attractiveness of the elements of a finite set A of alternatives, MACBETH (**M**easuring **A**ttractiveness by a **C**ategorical **B**ased **E**valuation **Tech**nique) is an approach to guide the construction of an interval scale for which the idea of difference of attractiveness is meaningful, that is for the construction of a numerical scale $v: A \rightarrow \Re : a \rightarrow v(a)$ which not only satisfies the

Ordinal condition:

- $\forall a, b \in A,\ v(a) > v(b)$ if and only if D judges a more attractive than b,

but also satisfies the

Cardinal condition:

- with a more attractive than b and c more attractive than d, the ratio

$$\frac{v(a) - v(b)}{v(c) - v(d)}$$

expresses the relative strength for D of the difference of attractiveness between a and b taking as unit of reference the difference of attractiveness between c and d.

To derive a scale satisfying these conditions the basic idea of MACBETH is:

1º) in a first stage, to use a very simple questioning procedure which involves only two actions in each question and to assign, to each element a of A, a real number $\mu(a)$ on the basis of straightforward rules for quantifying the preference information given by D;

2º) in a second stage, to discuss with D about the cardinality of the scale μ constructed in the first stage: does this scale satisfy the cardinal condition?

MACBETH was first proposed in [Bana e Costa and Vansnick, 1994] and two applications in decision-aiding are described in [Bana e Costa and Vansnick, 1996]. This new paper presents a theoretical framework for our approach. It includes three paragraphs: in the first, we review the questioning procedure and indicate the measurement rules used in MACBETH, while the second and third are respectively devoted to the discussion of the existence and of the unicity of a solution for the representation problem we are addressing.

1. Questioning procedure and measurement rules

1.1. Questioning procedure

In the questioning procedure of MACBETH, an evaluator D is asked to make semantic judgments about the (subjective) difference of attractiveness between actions of A:

$\forall (a,b) \in A \times A$ with $a \neq b$

	yes	no
Is a more attractive than b?	☐	☐

and, if the answer is yes:

Is the difference of attractiveness between a and b

very weak ?	☐
weak ?	☐
moderate?	☐
strong ?	☐
very strong ?	☐
extreme ?	☐

Concerning the responses of D, the following terminology and notations are adopted:

- We note aPb when D judges a more attractive than b (P is a binary relation on A).
- When aPb, we say that:

  (a,b) belongs to the category C_1 when D judges that the difference of attractiveness between a and b is **very weak** $(a,b) \in C_1$

  (a,b) belongs to the category C_2 when D judges that the difference of attractiveness between a and b is **weak** $(a,b) \in C_2$

  (a,b) belongs to the category C_3 when D judges that the difference of attractiveness between a and b is **moderate** $(a,b) \in C_3$

  (a,b) belongs to the category C_4 when D judges that the difference of attractiveness between a and b is **strong** $(a,b) \in C_4$

  (a,b) belongs to the category C_5 when D judges that the difference of attractiveness between a and b is **very strong** $(a,b) \in C_5$

  (a,b) belongs to the category C_6 when D judges that the difference of attractiveness between a and b is **extreme** $(a,b) \in C_6$.

Remarks

① The categories C_1, C_2, C_3, C_4, C_5 and C_6 can be seen as six binary relations on A and we have:

$$P = C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6 . \qquad (1)$$

② We will always suppose hereafter that the binary relations P, C_1, C_2, C_3, C_4, C_5 and C_6 are asymmetric and that, $\forall i,j \in \{1,2,3,4,5,6\}$ with $i \neq j$, $C_i \cap C_j = \varnothing$. Such conditions are supposed to be verified during the interaction with D.

③ The initial responses of D can be presented in a "matrix of judgments"

$$\mathcal{M} = [m_{ab} \mid (a,b) \in A \times A] \quad \text{with} \quad \begin{cases} m_{ab} = 0 & \text{if and only if } (\textit{not } a P b) \\ m_{ab} = k & \text{if and only if } (a,b) \in C_k . \end{cases}$$

Note that k is just a code for category C_k.

1.2. Measurement rules

In order to represent numerically the qualitative information given by D, some measurement rules are of course necessary. Two basic rules are used in MACBETH for assigning, to each element a of A, a real number $\mu(a)$:

Rule I $\forall a,b \in A$: $aPb \Leftrightarrow \mu(a) > \mu(b)$

that is, the number assigned to action a is strictly greater than the number assigned to action b if and only if, for D, a is more attractive than b.

Rule II $\forall k,k' \in \{1,2,3,4,5,6\}$ with $k \neq k'$ and
$\forall a,b,c,d \in A$ with $(a,b) \in C_k$ and $(c,d) \in C_{k'}$:
$k > k' \Leftrightarrow \mu(a) - \mu(b) > \mu(c) - \mu(d)$

that is, when the pairs of alternatives (a,b) and (c,d) do not belong to the same category, the difference between the numbers associated to a and b is strictly greater than the difference between the numbers associated to c and d if and only if the difference of attractiveness between a and b is judged greater than the difference of attractiveness between c and d. (Note that this information follows indirectly from the matrix $\mathcal{M}$ of judgments.)

The justification for including the constraint $k \neq k'$ in rule II is that, when (a,b) and (c,d) are assigned to the same category, there is no information about the relation between the numerical differences $\mu(a) - \mu(b)$ and $\mu(c) - \mu(d)$. In particular, there is no reason to assume that $\mu(a) - \mu(b) = \mu(c) - \mu(d)$ as is it the case in the approaches that "abusively" associate each category with a single real number. It is worthwhile to point out that, in MACBETH, each category of difference of attractiveness is numerically represented by an interval of positive real numbers (see [Bana e Costa and Vansnick, 1994]) as it intuitively follows from the imprecise nature ("vague", see [von Winterfeldt and Edwards, 1986, page 98]) of verbal expressions like "weak", "moderate", "strong", ...

2. Existence of a solution

In this paragraph, we address the following questions:

1) what are the necessary and sufficient conditions for the existence of a scale $\mu : A \rightarrow \Re$ satisfying the measurement rules I and II,

2) how to test if these conditions are satisfied?

A main part of the answer to the first question derives from:

Proposition 1 $\exists\, \mu : A \rightarrow \Re$ satisfying rules I and II
if and only if
$\exists\, \mu : A \rightarrow \Re$ and six real numbers t_1, t_2, t_3, t_4, t_5 and t_6
satisfying the following conditions:

$$0 = t_1 < t_2 < t_3 < t_4 < t_5 < t_6$$

and $\forall a,b \in A$ and $\forall k \in \{1,2,3,4,5,6\}$, $\mu(a) > \mu(b) + t_k \Leftrightarrow a\, P_k\, b$

where $\forall a,b \in A$ and $\forall k \in \{1,2,3,4,5,6\}$:

$$a P_k b \Leftrightarrow (a,b) \in C_k \cup C_{k+1} \cup ... \cup C_6 \qquad (2)$$

Proposition 1 results from the property of "continuity" of the set of real numbers and the fact that A is a finite set. It shows that we are facing a problem theoretically similar to the "constant threshold representation of a m-tuple of binary relations". This problem was solved by Doignon [1987].

Let:

- $\mathcal{R} = (R_1, R_2, ..., R_m)$ be an m-tuple ($m \geq 2$) of binary relations on a given finite set X
- R_j^{dual} the dual relation of R_j^{dual} defined by $x R_j^{dual} y \Leftrightarrow (not\ y R_j x)$
- a "cycle" from $\mathcal{R}$ be any sequence of the form

  $$(x_1, x_2)(x_2, x_3)...(x_{j-1}, x_j)(x_j, x_1)$$

  with all the pairs taken in $R_1 \cup R_1^{dual} \cup R_2 \cup R_2^{dual} \cup ... \cup R_m \cup R_m^{dual}$.
- a "p-cyclone" from $\mathcal{R}$ is any non-empty union of at most p cycles from $\mathcal{R}$; when, for each $j = 1,2,...,m$, the same number (possibly zero) of pairs is taken in R_j and R_j^{dual}, the cyclone is called "balanced".

The main result of [Doignon, 1987] is:

Theorem 1

There are a real-valued mapping f on X and real numbers $\sigma_1, \sigma_2, ..., \sigma_m$ such that, for all $j \in \{1,2,...,m\}$ and $x,y \in X$: $x R_j y \Leftrightarrow f(x) > f(y) + \sigma_j$

if and only if

no m-cyclone from $\mathcal{R}$ is balanced.

With respect to this general case, our problem presents the following particularities:

1) the thresholds t_1, t_2, t_3, t_4, t_5 and t_6 have to be non negative and $\forall k, k' \in \{1,2,3,4,5,6\}$ with $k < k' : t_k < t_{k'}$ that is, the relations P_k have to be irreflexive and $P_1 \supset P_2 \supset P_3 \supset P_4 \supset P_5 \supset P_6$. (Note that, given the questioning mode of MACBETH – cf. § 1.1 – and the definition of the relations P_k – cf. (2) – these conditions are always satisfied.)

2) $t_1 = 0$ that is

(2.1) $P_1 (\equiv P)$ – cf. (1) and (2) – has to be a strict weak order (an asymmetric and negatively transitive relation)

and

(2.2) $\forall a,b,c,d \in A$ with (*not* aPb), (*not* bPa), aPc and dPa and $\forall k,k' \in \{1,2,3,4,5,6\}$, we must have

$$\left[(a,c) \in C_k \Leftrightarrow (b,c) \in C_k\right] \text{and} \left[(d,a) \in C_{k'} \Leftrightarrow (d,b) \in C_{k'}\right].$$

When the relations P_k are irreflexive and $P_1 \supset P_2 \supset P_3 \supset P_4 \supset P_5 \supset P_6$ (which results from remark ② above and will be always be assumed hereafter), it can be proven that the cyclone condition of Doignon (cf. Theorem 1) and the conditions (2.1) and (2.2) are necessary and sufficient conditions for the existence of a scale $\mu : A \to \Re$ satisfying the measurement rules I and II. It is interesting to observe that, if we adopt the following definitions:

- a "generalised cycle" from $\mathcal{P} = (P_1, P_2, P_3, P_4, P_5, P_6)$ is any sequence of the form $(a,x_2)(x_2,x_3)\ldots(x_{j-1},x_j)(x_j,b)$ with (*not* aP_1b) and (*not* bP_1a) and where all pairs are taken in

$$P_1 \cup P_1^{dual} \cup P_2 \cup P_2^{dual} \cup \ldots \cup P_6 \cup P_6^{dual} \ ;$$

- a "generalised p-cyclone" from $\mathcal{P}$ is any non-empty union of at most p generalised cycles from $\mathcal{P}$; when, for each $j = 1,2,\ldots,m$, the same number (possibly zero) of pairs is taken in P_j and P_j^{dual}, the generalised cyclone is called "balanced",

then the following result can be stated:

Theorem 2 $\quad \exists\, \mu : A \to \Re$ satisfying rules I and II
if and only if
no generalised n-cyclone from $\mathcal{P}$ is balanced
where n is the number of non-empty relations among $C_1, C_2, C_3, C_4, C_5, C_6$.

From a practical point of view, it is of course difficult to test the "cyclone condition". However, it is possible to verify the existence of a solution for our problem by solving the following linear program (LP1), the variables of which are $\mu(a)$ $(a \in A)$, $s_1, s_2, s_3, s_4, s_5, s_6$ and $\mathbf{c}$:

Program LP1

Min $\mathbf{c}$

s.t. each variable ≥ 0

$\forall k \in \{1,2,3,4,5\}$: $s_{k+1} \geq s_k + 2$

$\forall k \in \{1,2,3,4,5,6\}$ and $\forall (a,b) \in C_k$: $\mu(a) - \mu(b) \geq s_k + 1 - \mathbf{c}$

$\forall k \in \{1,2,3,4,5\}$ and $\forall (a,b) \in C_k$: $\mu(a) - \mu(b) \leq s_{k+1} - 1 + \mathbf{c}$

$\forall (a,b) \notin P$: $\mu(a) \leq \mu(b)$.

Indeed, it is easy to prove that:

Proposition 2 $\exists \mu : A \to \Re$ satisfying rules I and II
if and only if
the optimal solution $\mathbf{c}_{\min}$ of LP1 is equal to 0.

From proposition 2 and theorem 2, it follows that [$\mathbf{c}_{\min} > 0$] implies that there exists (at least) one generalised cyclone from $\mathcal{P}$ which is balanced. (Note that program LP1 is always feasible.)

Such a cyclone can be found from the optimal solution of the following linear program (LP2), the variables of which are $\mu(a)$ $(a \in A)$, $s_1, s_2, s_3, s_4, s_5, s_6$, $\alpha(a,b)$ with $(a,b) \in C_1 \cup C_2 \cup ... \cup C_6$, $\beta(a,b)$ with $(a,b) \in C_1 \cup C_2 \cup ... \cup C_5$ and λ:

Program LP2

$$\text{Min} \left[M.\lambda + \sum_{(a,b) \in C_1 \cup C_2 \cup ... \cup C_6} \alpha(a,b) + \sum_{(a,b) \in C_1 \cup C_2 \cup ... \cup C_5} \beta(a,b) \right]$$

s.t. each variable ≥ 0

$\forall k \in \{1,2,3,4,5\}$: $s_{k+1} \geq s_k + 2$

$\forall k \in \{1,2,3,4,5,6\}$ and $\forall (a,b) \in C_k$: $\mu(a) - \mu(b) \geq s_k + 1 - \alpha(a,b)$

$\forall k \in \{1,2,3,4,5\}$ and $\forall (a,b) \in C_k$: $\mu(a) - \mu(b) \leq s_{k+1} - 1 + \beta(a,b)$

$\forall k \in \{1,2,3,4,5,6\}$ and $\forall (a,b) \in C_k$: $\alpha(a,b) \leq \lambda$

$\forall k \in \{1,2,3,4,5\}$ and $\forall (a,b) \in C_k$: $\beta(a,b) \leq \lambda$

$\forall (a,b) \notin P$: $\mu(a) \leq \mu(b)$

where M is a big positive real number, arbitrarily fixed.

Let us point out that any $\alpha(a,b) \neq 0$ or $\beta(a,b) \neq 0$ indicates that the measurement rules I and II can not be satisfied; $\alpha(a,b) \neq 0$ means that (a,b) can belong to a balanced cyclone with $(a,b) \in P_1 \cup P_2 \cup ... \cup P_6$ and $\beta(a,b) \neq 0$ means that (b,a) can belong to a balanced cyclone with $(b,a) \in P_1^{dual} \cup P_2^{dual} \cup ... \cup P_6^{dual}$. (Note that program LP2 does not necessarily allow the identification of all balanced cyclones.)

Example

Let us consider the following matrix of judgments:

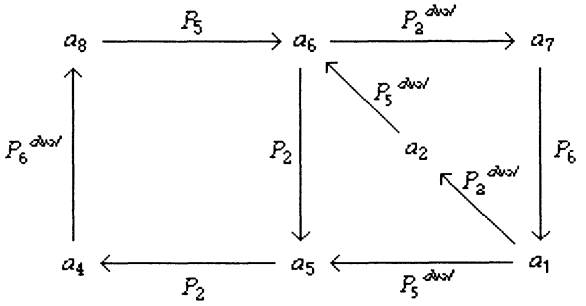
	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1
a_8	0	4	5	5	5	6	6	6
a_7	0	0	1	2	3	4	5	6
a_6	0	0	0	2	2	4	4	5
a_5	0	0	0	0	2	2	4	4
a_4	0	0	0	0	0	2	3	4
a_3	0	0	0	0	0	0	2	2
a_2	0	0	0	0	0	0	0	1
a_1	0	0	0	0	0	0	0	0

In the following tables the "×" indicate the variables $\alpha(a,b) \neq 0$ and $\beta(a,b) \neq 0$ in the optimal solution of program LP2:

α	a_1	a_4	a_5	a_6
a_5		×		
a_6			×	
a_7	×			
a_8				×

β	a_1	a_2	a_4	a_6
a_2	×			
a_5	×			
a_6		×		
a_7				×
a_8			×	

These tables and the matrix of judgments allow us to draw the graph:



which allows us to identify the two following balanced cyclones from $\mathcal{P}$:

$$(a_8,a_6)(a_6,a_7)(a_7,a_1)(a_1,a_5)(a_5,a_4)(a_4,a_8)$$

and

$$(a_8,a_6)(a_6,a_5)(a_5,a_4)(a_4,a_8) \cup (a_6,a_7)(a_7,a_1)(a_1,a_2)(a_2,a_6)\,.$$

It is interesting to note that once identified a cyclone, it is easy to understand why there is no numerical scale μ satisfying the measurement rules I and II (this is what we called a situation of theoretical incoherence in [Bana e Costa and Vansnick, 1994] or cardinal inconsistency in [Bana e Costa and Vansnick, 1995 and 1996]. For example, in

$$(a_8,a_6)\,(a_6,a_7)\,(a_7,a_1)\,(a_1,a_5)\,(a_5,a_4)\,(a_4,a_8)$$

one has (see the figure above):

$$\begin{cases} a_8\, P_5\, a_6 \text{ and } a_1\, P_5^{dual}\, a_5 \\ a_7\, P_6\, a_1 \text{ and } a_4\, P_6^{dual}\, a_8 \\ a_5\, P_2\, a_4 \text{ and } a_6\, P_2^{dual}\, a_7 \end{cases} \text{ implying } \begin{cases} \mu(a_8)-\mu(a_6) > \mu(a_5)-\mu(a_1) \\ \mu(a_7)-\mu(a_1) > \mu(a_8)-\mu(a_4) \\ \mu(a_5)-\mu(a_4) > \mu(a_7)-\mu(a_6) \end{cases}$$

which is obviously impossible, because the sum of the terms in the left members is the same as the sum of the terms in the right members.

3. Unicity of the solution and further developments

When there exists a scale $\mu : A \to \Re$ satisfying rules I and II, such a scale is not unique. These rules are satisfied not only by $\mu^* = \theta_1.\mu + \theta_2$ for each $\theta_1, \theta_2 \in \Re$ with $\theta_1 > 0$, but also by other scales. Given the non–unicity of the solution, a relevant question arises in practice: what scale "should" be proposed to the evaluator, from among all the possible ones?

To answer this question, it is clear that more conditions are needed than simply those imposed by our measurement rules. This is a subject out of the theoretical scope of this paper. Nevertheless, let us recall that in our previous papers, we gave technical answers based on the idea of minimising, under appropriate constraints, the sum of the distances between each difference $\mu(a)-\mu(b)$ and the centre $(s_k + s_{k+1})/2$ of the interval corresponding to the category C_k $(k = 1,2,3,4,5)$ to which the pair (a,b) belongs (see [Bana e Costa and Vansnick, 1994 and 1995]). In a forthcoming paper, we will show how it is possible to improve this idea in order to increase the substantive significance of the proposed solution.

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INTERDEPENDENCE IN MULTIPLE CRITERIA DECISION MAKING

Christer Carlsson
Institute for Advanced Management Systems Research
Åbo Akademi University
DataCity A 3210, 20520 Åbo, Finland

Robert Fuller
Department of Computer Science
Eötvös Loránd University
Muzeum krt. 6-8, H-1088 Budapest, Hungary

Abstract: Modeling and optimization methods for handling multiple criteria decision problems have over the last 20 years been developed in both crisp and fuzzy environments. The overwhelming majority of approaches for finding best optimal or optimal compromise solutions to MCDM problems do not make use of the interdependences among the objectives. However, when dealing with real world decision problems we often encounter problems for which it is useful to exploit interdependences, even if it is excluded from most MCDM problem formulations. In this paper we develop and build upon measures of interdependence between the objectives, in both crisp and fuzzy formulations, and develop some methods for finding effective and more realistic solutions to MCDM problems.

1. Introduction

Part of the habitual domain [18] of multiple criteria decision making is the intuitive assumption that all criteria are independent; this was initially introduced as a safeguard to get a feasible solution to a multiple criteria problem (which was complex and difficult to understand also without mathematical complications) as there were no means available to deal with interdependence. Then, gradually, conflicts were introduced as we came to realize that multiple goals or objectives almost by necessity represent conflicting interests [2, 17, 19]. Here we will leave out the assumption of interdependent criteria. By not allowing interdependence multiple criteria problems are simplified beyond recognition and the solutions reached by the traditional algorithms have only marginal interest, which is contrary to our expectations [19].

Zeleny also points to other circumstances [20] which have reduced the visibility and usefulness of MCDM: (i) time pressure reduces the number of criteria to be considered; (ii) the more precise and complete the problem definition, the less criteria are needed; (iii) autonomous decision makers are bound to use more criteria than those being controlled by a strict hierarchical decision system; (iv) isolation from the perturbations of a changing environment reduces the need for multiple criteria; (v) the more complete, comprehensive and integrated knowledge of the problem the more criteria will be used - but partial, limited and non-integrated knowledge will

significantly reduce the number of criteria; and (vi) cultures and organisations focused on central planning and collective decision making rely on aggregation and the reduction of criteria in order to reach consensus.

We believe the theory of MCDM can be more realistic and more complete, but above all more useful for the decision makers of the business world. We believe that this can be done both by introducing interdependence and by developing MCDM-based decision support systems; here we will pursue the first task.

Typically, the DM has a *dilemma*: he is expected to be able to formulate his preferences regarding the objectives, but he cannot have much more than an intuitive grasp of the trade-offs he is probably doing among them. This is taken care of with a convenient assumption: the DM is assumed to be a fully informed, rational decision maker who relies on some underlying utility function as a basis for his preferences.

It is well known that there does not exists any concept of *optimal solution* universally accepted and valid for any multiobjective problem [19]. Delgado et al [8] provided a unified framework to use fuzzy sets and possibility theory in multicriteria decision models and multiobjective programming. Felix [9] presented a novel theory for multiattribute decision making based on fuzzy relations between objectives, in which the interactive structure of objectives is inferred and represented explicitly. Carlsson [4, 5] used the fuzzy Pareto optimal set of nondominated alternatives as a basis for using an OWA-type operator [17] to find a best compromise solution to MCDM problems with interdependent criteria.

The aim of this paper is to provide a new method for finding a compromise solution to MCDM problems by using explicitly the interdependences among the objectives and combining the results [1, 2, 3, 5, 6, 8, 9, 21, 22]. Let us briefly summarize some standard definitions.

An $x \in X$ is said to be efficient for a set of objective functions f_i iff there exists no $y \in X$ such that $f_i(y) \geq f_i(x)$ for all i, with strict inequality for at least one i.

Recall that the mapping $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is a triangular norm [14] iff it is symmetric, associative, non-decreasing in each argument and $T(a, 1) = a$, for all $a \in [0, 1]$. A t-norm T is called strict if T is strictly increasing in each argument.

The basic t-norms are $min\ \{a, b\}$, $LAND\ (a, b) = \max\ \{a + b - 1, 0\}$ (the Lukasiewicz t-norm), and $PAND\ (a, b) = ab$. All t-norms may be extended, through associativity, to n > 2 arguments. The t-norm *min* is automatically extended, and

$$PAND\ (a_1, \ldots, a_n) = a_1\, a_2 \ldots a_n$$
$$LAND\ (a_1, \ldots, a_n) = \max\ \{a_1 + \ldots + a_n - n + 1, 0\}.$$

2. Interdependences in MCDM problems

In this section we will define *interdependences* between objectives of a decision problem defined in terms of multiple objectives.

Consider the following problem,

$$\max_{x \in X} \{ f_1(x), \ldots f_k(x) \} \qquad (1)$$

where $f_i : \mathbf{R}^n \to \mathbf{R}$ are objective functions, $x \in \mathbf{R}^n$ is the decision variable, and X is a subset of $\mathbf{R}^n$ without any additional conditions for the moment.

Definition 2.1 Let f_i and f_j be two objective functions of (1). We say that,

(i) f_i supports f_j on X (denoted by $f_i \uparrow f_j$) if $f_i(x') \geq f_i(x)$ entails $f_j(x') \geq f_j(x)$, for all $x', x \in X$;

(ii) f_i is in conflict with f_j on X (denoted by $f_i \downarrow f_j$) if $f_i(x') \geq f_i(x)$ entails $f_j(x') \leq f_j(x)$, for all $x', x \in X$;

(iii) f_i and f_j are independent on X otherwise;

If $X = \mathbf{R}^n$ then we say that f_i supports f_j (or is in conflict with) *globally*; it is easy to see that the relations $\uparrow$ and $\downarrow$ have the following properties:

(i) symmetry: if $f_i \uparrow f_j$ then $f_j \uparrow f_i$ (if $f_i \downarrow f_j$ then $f_j \downarrow f_i$),

(ii) associativity: if $f_i \uparrow f_j$ and $f_j \uparrow f_l$ then $f_i \uparrow f_l$ (if $f_i \downarrow f_j$ and $f_j \downarrow f_l$ then $f_i \downarrow f_l$),

(iii) reflexivity of $\uparrow$: $f_i \uparrow f_i$,

(iv) if $f_i \uparrow f_j$ and $f_j \downarrow f_l$ then $f_i \downarrow f_l$ (if $f_i \downarrow f_j$ and $f_j \uparrow f_l$ then $f_i \uparrow f_l$),

If the objective functions are differentiable on X, then we have,

(i) $f_i \uparrow f_j$ on X iff $\partial_{\mathbf{e}} f_i(x)\, \partial_{\mathbf{e}} f_j(x) \geq 0$ for all $\mathbf{e} \in \mathbf{R}^n$ and $x \in X$,

(ii) $f_i \downarrow f_j$ on X iff $\partial_{\mathbf{e}} f_i(x)\, \partial_{\mathbf{e}} f_j(x) \leq 0$ for all $\mathbf{e} \in \mathbf{R}^n$ and $x \in X$,

where $\partial_{\mathbf{e}} f_i(x)$ denotes the derivative of f_i with respect to the direction $\mathbf{e} \in \mathbf{R}^n$ at $x \in \mathbf{R}^n$.

If for a given direction $\mathbf{e} \in \mathbf{R}^n$,

$$\partial_{\mathbf{e}} f_i(x)\ \partial_{\mathbf{e}} f_j(x) \geq 0 \quad [\, \partial_{\mathbf{e}} f_i(x)\ \partial_{\mathbf{e}} f_j(x) \leq 0 \,]$$

holds for all $x \in X$ then we say that f_i supports f_j [f_i is in conflict with f_j] with respect to the direction $\mathbf{e}$ on X.

Let f_i be an objective function of (1); then we define the *grade of interdependency*, denoted by $\Delta(f_i)$, of f_i as,

$$\Delta(f_i) = \sum_{f_i \uparrow f_j,\, i \neq j} 1 \quad - \sum_{f_i \downarrow f_j} 1, \quad i = 1, \ldots, k$$

If $\Delta(f_i)$ is positive and large, then f_i supports a *majority* of the objectives; if $\Delta(f_i)$ is negative and large then f_i is *in conflict with* a majority of the objectives; if $\Delta(f_i)$ is positive and small then f_i *supports* more objectives than it hinders, and if $\Delta(f_i)$ is negative and small, then f_i *hinders* more objectives than it supports. Finally if $\Delta(f_i) = 0$, then f_i is *independent* from the others or supports the same number of objectives as it hinders.

Example 2.1: In corporate takeover negotiations the Buyer and the Seller have two conflicting objectives: the Buyer wants the takeover price to be *as low as possible* (c_1), but the Seller wants it to be *as high as possible* (c_2). There is, however, much more behind corporate takeovers. In a real case, in which two Finnish companies were involved and finally merged, there were a number of more objectives which could be identified and gradually formulated.

The Seller's objectives (c_4), (c_6), and (c_8) all support his objective of getting a high aquisition price; nevertheless, the objectives (c_4, c_6), (c_6, c_8) and (c_8, c_4) are all pairwise conflicting. The Buyer's objective (c_1) supports his objectives (c_3), (c_5) and (c_7). There is no conflict among his objectives, but the objectives (c_3) and (c_7) support each others. There is also some interaction among the Seller's and the Buyer's objectives, which partly explains why they are negotiating: (c_3) and (c_4) are supporting each others, like (c_6) and (c_3), but (c_5) and (c_8) are conflicting:

Buyer	*Seller*
c_1 aquisition price *low*	c_2 aquistion price *high*
c_3 overall profits *high*	c_4 cash inflow *high*
c_5 investments *medium*	c_6 *max* corporate ROC
c_7 total loans *low*	c_8 RD investments *high*

With the notation we introduced above, the takeover has the following objective structure:

Buyer
$c_2 \uparrow c_4$ $c_2 \uparrow c_6$ $c_2 \uparrow c_8$ $c_4 \downarrow c_6$ $c_6 \downarrow c_8$ $c_8 \downarrow c_4$

Seller
$c_1 \uparrow c_3$ $c_1 \uparrow c_5$ $c_1 \uparrow c_7$ $c_3 \uparrow c_7$

Buyer / Seller
$c_3 \uparrow c_4$ $c_6 \uparrow c_3$ $c_5 \downarrow c_8$

It seems clear that it would be rather difficult to find a negotiated solution which would be simultaneously optimal for all the objectives, as the conflicts seem to eliminate this possibility. It should, however, be noted that the conflicts are fuzzy, as most of the objectives are given in a fuzzy form (*high, medium, low*), which indicates that some other solution than a simultaneous optimum for all the objectives should be attempted. There are two possibilities: (i) a negotiated compromise, based on trade offs among the conflicting objectives (this was carried out in an intuitive fashion in the real case), or (ii) alternate optima for combinations of subsets of the objectives during a negotiated interval (this was also attempted by representatives of the Seller, but without any success).

2.1 The linear case

If the objective functions are linear then their derivatives are constant. Then it follows that if two objectives are parallel and grow in the same direction, they will *support* each others; in all other cases we have varying degrees of conflict between them.

Consider the following problem with multiple objectives,

$$\max_{x \in X} \{ f_1(x), \ldots, f_k(x) \} \qquad (2)$$

where $f_i(x) = \langle c_i, x \rangle = c_{i1} x_1 + \ldots + c_{in} x_n$.

Definition 2. Let $f_j(x) = \langle c_i, x \rangle$ and $f_i(x) = \langle c_i, x \rangle$ be two objective functions of (2). Then the measure of conflict between f_i and f_j, denoted by $\kappa(f_i, f_j)$, is defined by,

$$\kappa(f_i, f_j) = (1 - \langle c_i, c_j \rangle) / 2$$

Let us in the following illustrate the meaning of the measure of conflict by a biobjective two-dimensional decision problem:

$$\max_{x \in X} \{ \alpha(x), \beta(x) \}$$

where $\alpha(x) = \langle \mathbf{n}, x \rangle$ and $\beta(x) = \langle \mathbf{m}, x \rangle$.

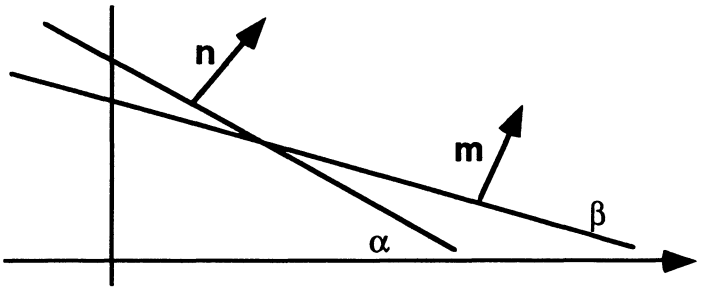

Fig. 1 The measure of conflict between α and β is $|\mathbf{n}|\,|\mathbf{m}|\cos(\mathbf{n}, \mathbf{m})$.

The larger the angle between the lines and the larger the degree of conflict between them.

If $\kappa(\alpha, \beta) = 1/2$ and the set of feasible solutions is a convex polyhedron in $\mathbf{R}^n$ then and reach their independent maxima at neighbouring vertexes of X.

If $\kappa(\alpha, \beta) = 0$ and the set of feasible solutions is a convex polyhedron in $\mathbf{R}^n$ then and reach their independent maxima at the same vertex of X.

Definition 3. The complexity of the problem (2) is defined by,

$$\Omega = \sum_{i,j}^{k} [\kappa(f_i, f_j)/2]$$

It is clear that $\Omega = 0$ iff all the objectives are parallel, i.e. we have a single objective problem.

2.2 Constructing the scalarizing function

Following [8, 21] we introduce an application function with the following properties,

$$h_i : \mathbf{R} \to [0, 1]$$

such that $h_i(t)$ measures the degree of fulfillment of the decision maker's requirements abouth the i-th objective by the value t. In other words, with the notation of $H_i(x) = h_i(f(x))$, $H_i(x)$ may be considered as the degree of membership of x in the fuzzy set "good solutions" for the i-th objective. Then a "good compromise solution" to (1) may be defined as an $x \in X$ being "as good as possible" for the whole set of objectives. Taking into consideration the nature of $H_i(.)$, $i = 1, ..., k$ it is quite reasonable to look for such a kind of solution by means of

the following auxiliary problem,

$$\max \{H_1(x), ..., H_k(x)\} \quad x \in X \tag{3}$$

As $\max \{H_1(x), ..., H_k(x)\}$ may be interpreted as a synthetical notation of a conjunction statement (maximize jointly all objectives) and $H_i(x) \in [0, 1]$, it is reasonable to use a t-norm T [14] to represent the connective AND. In this way (3) turns into the single objective problem,

$$\max T\{H_1(x), ..., H_k(x)\} \quad x \in X \tag{4}$$

There exists several ways to introduce application functions. Usually, the authors have used increasing membership functions (growing membership represents better solutions) of the following form,

$$h_i(t) = \begin{cases} 1 & \text{if } t \geq M_i \\ v_i(t) & \text{if } m_i < t < M_i \\ 0 & \text{if } t \leq m_i \end{cases} \tag{5}$$

where $m_i := \min_{x \in X} f_i(x)$ is the independent minimum and
$M_i := \max_{x \in X} f_i(x)$ is the independent maximum of the i-th criterion.

As we have stated above, our idea is to use the interdependences explicitly in the multiple criteria problem solving method.. To do so, let us first define H_i by,

$$H_i(x) = \begin{cases} 1 & \text{if } f_i(x) \geq M_i \\ 1 - [(M_i - f_i(x)) / (M_i - m_i)] & \text{if } m_i < f_i(x) < M_i \\ 0 & \text{if } f_i(x) \leq m_i \end{cases} \tag{6}$$

i.e. all membership functions are defined to be linear.

Now we can compute $\kappa(f_i)$ for $i = 1, ..., k$, and we change the shape of H_i according to the value of (f_i) as follows,

i. if $\kappa(f_i) = 0$, then we do not change the shape;
ii. if $\kappa(f_i) > 0$, then instead of the linear membership function we use,

$$H_i(x, \Delta(f_i)) = \begin{cases} 1 & \text{if } f_i(x) \geq M_i \\ 1 - [(M_i - f_i(x)) / (M_i - m_i)] / [1 / ((f_i) + 1)] & \text{if } m_i < f_i(x) < M_i \\ 0 & \text{if } f_i(x) \leq m_i \end{cases}$$

iii. if $(f_i) < 0$, then instead of the linear membership function we use,

$$H_i(x, \Delta(f_i)) = \begin{cases} 1 & \text{if } f_i(x) \geq M_i \\ 1 - [(M_i - f_i(x)) / (M_i - m_i)] / [|(f_i)| + 1)] & \text{if } m_i < f_i(x) < M_i \\ 0 & \text{if } f_i(x) \leq m_i \end{cases}$$

Then we solve the following auxiliary problem,

$$\max_{x \in X} T\{H_1(x, \Delta(f_1)), ..., H_k(x, \Delta(f_k))\} \tag{7}$$

Let us now assume that we have a decision problem with many ($k \geq 7$) objective functions. It is clear (due to the interdependences between the objectives), that we find optimal compromise solutions rather closer to the values of independent minima than maxima.

The basic idea of introducing the shape functions can be explained as follows: if we manage to increase the value of the i-th objective function, which has a large $\Delta(f_i)$, then from this follows a growth in a majority of the criteria (because the i-th objective supports a majority of the criteria), which takes us closer to the optimum of the scalarizing function (as the losses on the conflicting objectives are smaller, according to the definition).

One of the most important characteristics is the efficiency of the obtained compromise solutions. Delgado et al [8] obtained the following results:

Theorem 2.1 Let x^* be an optimal solution to,

$$\max_{x \in X} T\{H_1(x), ..., H_k(x)\} \tag{8}$$

where T is a t-norm, $H_i(x) = h_i(f_i(x))$ and h_i is an application function of the form (5), $i = 1, ..., k$. If h_i is strictly increasing on $[m_i, M_i]$, $i = 1, ..., k$ then x^* is efficient for the problem:

$$\max_{x \in X} T\{f_1(x), ..., f_k(x)\} \tag{9}$$

if either (i) x^* is unique, or (ii) T is strict and $H_i(x^*) \neq 0$, $i = 1, ..., k$.

It is easy to see that our application functions are strictly increasing on $[m_i, M_i]$, and therefore any optimal solution to the auxiliary problem (7) is an efficient solution to (1), if either (i) x^* is unique, or (ii) T is strict and $H_i(x^*) \neq 0$, $i = 1, ..., k$.

The choice of the particular t-norm depends upon several factors, such as the charcteristics of the problem, the context of the decision problem and the decision maker's knowledge representation model. Minimum and product t-norms are primarily used in literature to solve (9) through (8). The associated problems are, respectively,

$$\max_{x \in X} \min_{i=1,k} \{H_i(x, \Delta(f_i))\}$$

$$\max_{x \in X} \prod_{i=1,k} \{H_i(x, \Delta(f_i))\}$$

We prefer to use the Lukasiewicz t-norm (*LAND*) in (7), because it contains the sum of the particular application functions. This sum will increase rapidly if we manage to improve the value of an objective function which supports a majority of the objectives. With the *LAND* we get an auxiliary problem of the following form:

$$\max_{x \in X} \{\sum_{i=1,k} (H_i(x, \Delta(f_i)) - k + 1, 0\} \qquad (10)$$

The Lukasiewicz t-norm is not strict, so an optimal solution x^* to (10) is efficient for (1) iff x^* is the only optimal solution to (10).

Example 2.2: Let us illustrate the method with a 5-objective one dimensional decision problem. Consider the following problem formulation:

$$\max_{x \in X} \{f_1(x), ..., f_5(x)\}$$

with objective functions $f_1(x) = x$, $f_2(x) = (x + 1)^2 - 1$, $f_3(x) = 2x + 1$, $f_4(x) = x^4 - 1$, $f_5(x) = -3x + 1$ and $X = [0, 2]$.

It is easy to verify that we have the following interdependences,

$$f_1 \uparrow f_2 \quad f_2 \uparrow f_3 \quad f_3 \uparrow f_4 \quad f_4 \downarrow f_5$$

Then the grades of interdependences are,

$$\Delta(f_1) = \Delta(f_2) = \Delta(f_3) = \Delta(f_4) = 3, \ \Delta(f_5) = -4,$$

and we get

$$H_1(x, \Delta(f_1)) = H_3(x, \Delta(f_3)) = (x/2)^{1/4}, H_2(x, \Delta(f_2)) = [(x+1)/4]^{1/4},$$

$$H_4(x, \Delta(f_4)) = x/2, H_5(x, \Delta(f_5)) = (1 - x/2)^5$$

If we now select the *PAND* operator to represent the decision maker's preferences, we get the following single objective MP,

$$\max (x/2)^{3/2} [(x + 1)/3]^{1/2} (1 - x/2)^5$$

It turns out that this problem has a unique solution $x^* \approx 0.53$, which is a nondominated solution with the following values of the objective functions (0.53, 1.34, 2.06, -0.92, -0.59).

3. Conclusions

There has been a development over time in multiple criteria optimisation methods (cf [13]). The *first* stage, the multiple atribute utility theory and the utility theory based multiple objective programming methods, aimed at aggregating preferences to a unique function, which then should be optimized. Thus the main focus was on exploring the mathematical conditions for a consistent aggregation, the forms of the aggregating functions and the methods for constructing mathematically useful preference structures with some fairly strong assumptions on the rationality of the decision maker.

The *second* stage, which is overlapping with the first stage, was developed around the outranking relation, which represents the decision maker's preferences in a studied context when he has a certain set of information available. This outranking relation is assumed neither complete nor transitive, and is thus much more realistic than the multiple attribute utility theory; it has also turned out to be quite useful for actual problem solving [13].

The *third* stage has evolved around interactive methods, which combine successive optimisation steps to find optimal compromise solutions with a dialogue with the decision maker to extract more information on his preferences. At its best an interactive method will serve as a searching and/or learning instrument in teh problem space, which will help the decision maker to both formulate his preferences and to find optimal solutions.

Fuzzy set theory has proved to offer several distinct advantages as a basis for multiple criteria optimisation. It has been the basis for new and rather non-conventional solution concepts, which are based on searching for some optimal option, which "best satisfies most of the important objectives", and differ significantly from the traditional notion to try to find an optimal option which best satisfies "all the objectives". It has turned out that the problem of conflicting objectives can be resolved, as a fuzzy conflict in a multiple objective programming model is not absolute and thus can be used as a basis for compromise solutions. There are also some practical advantages: when a solution derived from a well-formulated MOLP-model is applied to an actual problem there are some major problems to consider: (i) the set of feasible decision alternatives is fuzzy, and this set

changes during the problem solving process; (ii) the decision maker does not always exist as an active entity, and the preferences consist of badly formulated beliefs, which are riddled with conflicts and contradictions; (iii) data on preferences are imprecise, and (iv) a decision should be good or bad not only in relation to some model, but in relation to the actual context.

In this paper we have developed a method for explicitly using interdependences, both conflict and support, among the criteria of a multiple criteria problem. The method is built around a family of application functions, which will help a decision maker to express his preferences, when any chosen preference is either conflicting or supportive in relation to any other preference. We have shown that the method will produce a well-defined solution and we have illustrated the technique with examples.

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Extended Preference Structures in MultiCriteria Decision Aid

Alexis Tsoukiàs(+), Philippe Vincke(*)
(+) LAMSADE, Université Paris Dauphine 75775 Paris Cédex 16, France
e-mail: tsoukias@lamsade.dauphine.fr
(*) Université Libre de Bruxelles, CP 210/01 Bd. du Triomphe,
1050 Bruxelles, Belgium, e-mail: pvincke@ulb.ac.be

Abstract: Uncertainty and ambiguity are common situations in decision aid and therefore in preference modelling and aggregation. A basic distinction between uncertainty reasons is advanced in the paper, that is between lack of information and contradictory information. A semantical investigation of this approach is conducted and the PC preference structure is adopted in order to represent the different preference situations that may occur. This type of preference structure, besides offering an enhanced granularity in preference modelling, is strongly axiomatized, enabling a precise calculus. Some of its potentialities are discussed using a preference aggregation problem.

Introduction

It is a well known problem that when preferences have to be aggregated one necessarily faces a dilemma. Either you accept to lose information, reducing the complexity of the aggregation problem, in order to gain in efficiency and clarity of the results or you keep as much as possible of the original information and then you lose in efficiency and effectiveness (see Vincke 1982a and 1982b).

Suppose that you do not always accept a reductionist approach because of the critical nature of some problem situations. You need to distinguish different situations of preferences that may arise, mainly in order to model the hesitation of the decision maker. In these cases the conventional preference structure appears to be unsuitable even from a probabilistic point of view (for a discussion see Roy, 1977; Bell et al., 1988). The decision maker's hesitation may have different origins (see Roy, 1988; Bouyssou, 1989). In this paper we propose a basic distinction between lack of information and excess of information. Even when the result is the same (confusion and/or hesitation) it seems important to be able to explain to the decision maker why such situations occur.

The basic idea introduced in this paper is that each time two alternatives x and y have to be compared, then it is quite natural to try to find the "positive reasons" supporting one of the two alternatives and the "negative reasons" that go against it. Under this reasoning (comparable with the concept of concordance and discordance introduced by Roy, 1985) it is possible to have either situations

where such reasons are absent (ignorance situations) or situations where all are present (contradictory situations). Using such concepts it is possible to identify new "extended preference structures" in the sense of a higher "granularity" of the usual relations.

The paper is therefore organized as follows. In section 1, a small example is given in order to present the kind of problems that appear in preference aggregation. In section 2, a new approach is presented and the PC preference structure is introduced. In section 3 the example of section 1 is exploited by the new approach in order to present its advantages and disadvantages.

1 An example

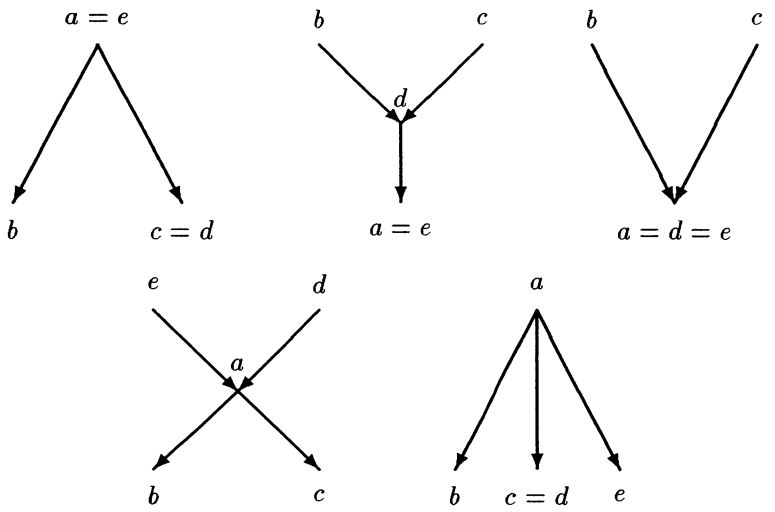


Figure 1: five quasi orders which must be aggregated
(transitive arcs are omitted)

Suppose there are five examinators and a set A of five candidates ($A = \{a, b, c, d, e\}$). Each examinator used a multicriteria model to evaluate the candidates. The results are presented in figure 1 where each arc should be read as "strict preference", the sign of equality as "indifference" and the absence of arcs as "incomparability". These are well known quasi orders that can be easily obtained when preferences are aggregated on the basis of several criteria.

We are interested in obtaining a final ranking. The aggregation rule is the following: "for all $x, y \in A$, x is globally at least as good as y **iff** there exists

a majority in this sense and there is no strong opposition against it". More formally:

$$\forall x, y \;\; s(x,y) \Leftrightarrow \;\mid s_j(x,y) \mid \geq \alpha \text{ and } \mid p_j^{-1}(x,y) \mid \leq \beta$$

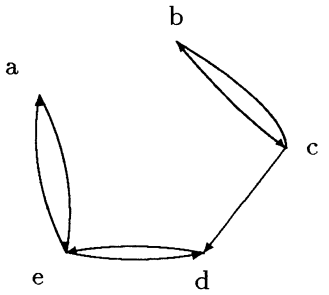
where
$s(x,y)$ means that *"x is globally at least as good as y"*,
$\mid s_j(x,y) \mid$ is the number of preorders in which x is at least as good as y (preferred or indifferent),
$\mid p_j^{-1}(x,y) \mid$ is the number of preorders in which y is strictly preferred to x.

It is well known that:
- "x is globally better than y" **iff** "x is globally at least as good as y" ($s(x,y)$ and "y is not globally at least as good as x" ($\neg s^{-1}(x,y)$);
- "x and y are globally equivalent" **iff** "x is globally at least as good as y" ($s(x,y)$ and "y is globally at least as good as x" ($s^{-1}(x,y)$);
- "x and y are globally incomparable" **iff** "x is not globally at least as good as y" ($\neg s(x,y)$ and "y is not globally at least as good as x" ($\neg s^{-1}(x,y)$);

Let us consider for example $\alpha = 3$ (a strict majority is required) and $\beta = 1$ (no more than one opposition is required). There are different ways to obtain a result. The first one is to reduce these partial orders to one of their underlying complete orders and then aggregate the obtained rankings. This can be done using a simple exploiting procedure (reduction of circuits and then dominance; see Vincke 1992). The result is shown in Table 1. We obtain $c, b \leq a, d, e$. This is a reasonable result from a technical point of view, but it can be strongly argued by any decision maker as the couple b, c which appears as the best is the worst for three of the five examinators. However this is the usual solution in such cases. In any case, it presents the inconvenience of eliminating useful information as the incomparabilities and to impose technical transformations that may be obscure for the decision maker.

	a	b	c	d	e
a	-	0	0	0	1
b	0	-	1	0	0
c	0	1	-	1	0
d	0	0	0	-	1
e	1	0	0	1	-

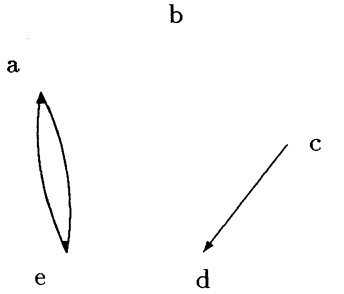
Table 1: the graph obtained after aggregating the five complete orders



Another way is to try to directly aggregate the five partial orders (using the rule previously defined). The result is shown in Table 2. The information contained in this result is rather poor. No reasonable conclusion can be obtained here.

	a	b	c	d	e
a	-	0	0	0	1
b	0	-	0	0	0
c	0	0	-	1	0
d	0	0	0	-	0
e	1	0	0	0	-

Table 2: the graph obtained aggregating the five partial orders



What can we do now? Where is the problem? In fact there exist several problems. It is obvious that the preference structure we are using forces different situations to be represented by the same preference relation, creating confusion. For instance in Table 1, b and c are considered indifferent while they are incomparable. Moreover, the incomparability holding between a and c does not have the same meaning as the one between c and e. The first case presents some reasons (not sufficient for the decision rule) to conclude $s(a, c)$ while it is certain that it will never be $s(c, a)$. On the other hand there exists a symmetrically conflicting situation between c and e. These are differences that appear clearly in Table 2. Here, all the incomparabilities are detected, but they are all considered in the same way. For the decision maker(s) it may be interesting to know that b and c are incomparable because of lack of information while e and d are in a strong conflicting position.

Concequently exists an expresssiveness problem of the conventional preference structure $< P, I, R >$ towards particular decision problems, such as the ones that naturally occur in multicriteria decision aid and social choice. We claim that we need preference structures presenting a higher "granularity". It might be thought that we could just add the particular preference relations we need in order to take in account our specific situations. There are two objections to such a direction.

1. The new relations cannot be completely arbitrary, because in the extreme case one may end up defining a preference relation for each ordered couple in $A \times A$. It is necessary to follow some general rules or axioms which define the set of possible preference relations.

2. The new set of preference relations should have an axiomatization, that

is should fulfill some basic properties considered relevant in order to have sound and reliable models.

We therefore claim that a preference structure used for decision aid purposes should fulfill the following axioms (this may not be always valid; it is not a normative axiom set):

A1 any preference structure should be a f.r.s.p. (fundamental relational system of preferences), that is should define a partition on $A \times A$ for any given A; in other words the preference relations included in the preference structure should be exhaustive for all possible situations and not redundant;

A2 the preference structure should obey the axiom of "independence from irrelevant alternatives"; in a more general version the evaluation, if a specific ordered couple belongs (and in which way) to a specific relation, should depend on information concerning only this ordered couple;

A3 the preference structure should be "well founded" in the sense that any binary relation in it should be univoquely defined by its properties.

Under these axioms, the problem of extending the conventional preference structure becomes more complex. We consider that the main difficulty in this direction is the language adopted as a basic formalism for preference modeling. Not surprisingly the language generally used is formal logic. The conventional preference structure $< P, I, R >$ is based on classic two valued logic which also supports the usual mathematical notation adopted in this case. In Tsoukiàs and Vincke, 1995 we proved that, using a language with a finite number of truth values, a limited number of possible preference relations can be defined, at least under the three axioms previously defined. Using classic logic, only three preference relations can be defined, exactly the $< P, I, R >$ preference structure. In fact it is easy to prove that using classic logic the $< P, I, R >$ preference structure is a maximal, well founded f.r.s.p.

So, if we want to extend a preference structure and retain the three axioms we have to change the language. The best known alternative is to use fuzzy sets and logics or the so called "valued binary relations". In this case, it is possible to imagine the existence of an infinity of preference relations each one defined for a particular value of the membership function. This is the direction chosen by the majority of researchers working in non conventional preference modeling (see Tsoukiàs and Vincke, 1992), but is not the one followed here. The reason for this choice is the following: in extending the set of preference relations we want to have a clear distinction between situations where the information is missing, not satisfactory, very poor and situations in which the information is too rich, contradictory, conflictual, ambiguous. We consider these situations as completely different so they should be treated in different ways. This is

not the case when using the valued extensions of the conventional preference structure $< P, I, R >$. The use of the concept of credibility (or membership of each ordered couple in a binary relation) does not make any difference in the above mentioned situations. Low credibility can occur either because of strong evidence in the negation or because of ignorance. The fuzzy approach formally treats these cases in the same way.

Under these considerations we definitely chose to adopt a four valued paraconsistent logic, named DDT, as the basic formalism under which preference modeling could be represented (see Doherty et al., 1993; Tsoukiàs and Vincke, 1995). This logic enables an explicit representation for situations under lack of information and situations under conflicting information. The first order extension of this logic is immediate (necessary for the preference modeling case). The use of this formalism will be explained in the next section.

2 A new approach

Suppose you must compare two actions x and y. Naturally you may compare the consequences of adopting one of the two different actions. Now if you have very clear and sufficient information you can conclude the following situations:

1 x is definitely better than y as the consequences of x are judged definitely better than those of y;

2 x and y are equivalent because their consequences are equivalent from any point of view;

3 x and y are in a conflicting position as you have clear different and conflicting consequences between them.

However these conclusions are possible because we made the assumption of complete and sure information. They do not cover all possible situations a decision maker can be faced with. Consider now the situations in which you have symmetrically lack of information and excess of information in the sense that you have contradictory inputs. You may introduce two new situations:

4 x and y could be either equivalent or conflicting, but you hesitate because you have no relevant information in any of the two directions;

5 x and y could be either equivalent or conflicting, but you hesitate because you have contradictory informations in both directions.

Another possibility may occur when it is easy to positively compare the consequences of x with these of y, but the inverse is more difficult again either because you do not know or because you know too much. We have two more situations:

6 x could be better than y, but you hesitate to conclude if it is preferred or equivalent, because you do not have all the relevant information;

7 x could be better than y, but you hesitate between preference and equivalence due to some conflicting information.

Another possibility occurs when it is easy to conclude something in a negative way, but the inverse is more difficult. So you may have two more situations:

8 y cannot be better than x, but you hesitate whether they are conflicting or if x is better than y, because you do not have sufficient information;

9 y cannot be better than x, but you hesitate wheter they are conflicting or if x is better than y, because you have contradictory information.

Finally you find yourself in the most ambiguous situation, that is having simultaneously both some lack of information and some contradictory information so that:

10 x could be better than y, but you hesitate before concluding anything, because you have only bad information.

By these definitions all the situations of hesitation are represented. The question is now wheter it is possible to formalize these situations in a way which constitutes a sound preference modeling theory satisfying the three axioms we introduced above.

In order to build a new extended preference structure we will start with a preference relation that we call "outranking", which we will denote by $s(x, y)$ and read as *"x is at least as good as y"*. In order to satisfy axioms **A2, A3** the only logical evaluations to perform will concern $s(x, y)$ and $s^{-1}(x, y)$ where

$$s^{-1}(x, y) \equiv s(y, x)$$

We generally accept that there may exist a set of "positive reasons" which support the sentence $s(x, y)$ (or $s^{-1}(x, y)$) and a set of negative reasons against this sentence or that supports the sentence $\neg s(x, y)$ ($\neg s^{-1}(x, y)$ respectively). This kind of reasoning can be generally accepted in many human problem situations. In any case, it seems reasonable when decision aid must be provided. The balance between reasons supporting a certain assertion and reasons against it is a natural "common sense" situation. Moreover, when a prescription has to be presented and explained to a decision maker a clear presentation of the positive and negative reasons behind it eases the discussion, the acceptance and the reflection about the problem situation and the action to be undertaken. There are therefore four possible states:

- there exist sufficient positive reasons to establish $s(x, y)$ and there are not enough negative reasons to establish $\neg s(x, y)$; $s(x, y)$ is "true";

- there exist sufficient positive reasons to establish $s(x,y)$ and sufficient negative reasons to establish $\neg s(x,y)$; $s(x,y)$ is "contradictory";
- there do not exist sufficient positive reasons to establish $s(x,y)$ and there are not enough negative reasons to establish $\neg s(x,y)$; $s(x,y)$ is "unknown";
- there do not exist sufficient positive reasons to establish $s(x,y)$ and there exist enough negative reasons to establish $\neg s(x,y)$; $s(x,y)$ is "false";

More formally, we accept that $s(x,y)$ and $\neg s(x,y)$ are not complementary or that the extensions of the two predicates may overlap and that they do not cover the whole set of possible situations. We can express this idea by introducing (using the DDT language) the sentence $\triangle s(x,y)$ to be read as "there is presence of truth in saying $s(x,y)$". The definitions of the sentences $\triangle\neg s(x,y)$, $\neg\triangle s(x,y)$, $\neg\triangle\neg s(x,y)$ are straightforward, with the corresponding meanings. Using these sentences we can define also the following (the formal machinery of this language is described in Doherty at al., 1993 and Tsoukiàs and Vincke, 1995):

- $\mathbf{T}s(x,y) \equiv \triangle s(x,y) \wedge \neg\triangle\neg s(x,y)$: the "true" extension of the predicate $s(x,y)$;
- $\mathbf{K}s(x,y) \equiv \triangle s(x,y) \wedge \triangle\neg s(x,y)$: the "contradictory" extension of the predicate $s(x,y)$;
- $\mathbf{U}s(x,y) \equiv \neg\triangle s(x,y) \wedge \neg\triangle\neg s(x,y)$: the "unknown" extension of the predicate $s(x,y)$;
- $\mathbf{F}s(x,y) \equiv \neg\triangle s(x,y) \wedge \triangle\neg s(x,y)$: the "false" extension of the predicate $s(x,y)$.

Let us now give names to the ten situations we previously introduced. We will have: case 1: relation P; case 2: relation I; case 3: relation R; case 4: relation U; case 5: relation J; case 6: relation K; case 7: relation H; case 8: relation V; case 9: relation Q; case 10: relation L;

It is easy to see that semantically the true extension of the relation S will be the union of the true extensions of the relations P, I, K and H; the ones where in fact we are at least sure that "x is at least as good as y". Remember that in P "x is definitely better than y", that in I "x and y are equivalent" and that in K and H "x could be better than y", but you hesitate for the inverse relation. By the same reasoning we have that the false extension of the relation S will be the union of the true extensions of the relations P^{-1}, Q^{-1}, V^{-1} and R. The unknown extension of S will be the union of the true extensions of the relations V, U, K^{-1} and L^{-1}. The contradictory extension of S will be the union of the true extensions of the relations Q, L, J and H^{-1}. We can give a formal representation of these concepts with the following DDT formulas:

- $\forall x,y \in A \;\; \mathbf{T}s(x,y) \equiv \mathbf{T}p(x,y) \vee \mathbf{T}i(x,y) \vee \mathbf{T}k(x,y) \vee \mathbf{T}h(x,y)$.
- $\forall x,y \in A \;\; \mathbf{K}s(x,y) \equiv \mathbf{T}q(x,y) \vee \mathbf{T}l(x,y) \vee \mathbf{T}j(x,y) \vee \mathbf{T}h^{-1}(x,y)$.
- $\forall x,y \in A \;\; \mathbf{U}s(x,y) \equiv \mathbf{T}v(x,y) \vee \mathbf{T}u(x,y) \vee \mathbf{T}k^{-1}(x,y) \vee \mathbf{T}l^{-1}(x,y)$.
- $\forall x,y \in A \;\; \mathbf{F}s(x,y) \equiv \mathbf{T}r(x,y) \vee \mathbf{T}p^{-1}(x,y) \vee \mathbf{T}q^{-1}(x,y) \vee \mathbf{T}v^{-1}(x,y)$.

We summarize the formulas given above in Table 3.

$A \times A$	$\mathbf{T}s^{-1}(x,y)$	$\mathbf{F}s^{-1}(x,y)$	$\mathbf{U}s^{-1}(x,y)$	$\mathbf{K}s^{-1}(x,y)$
$\mathbf{T}s(x,y)$	$\mathbf{T}i(x,y)$	$\mathbf{T}p(x,y)$	$\mathbf{T}k(x,y)$	$\mathbf{T}h(x,y)$
$\mathbf{F}s(x,y)$	$\mathbf{T}p^{-1}(x,y)$	$\mathbf{T}r(x,y)$	$\mathbf{T}v^{-1}(x,y)$	$\mathbf{T}q^{-1}(x,y)$
$\mathbf{U}s(x,y)$	$\mathbf{T}k^{-1}(x,y)$	$\mathbf{T}v(x,y)$	$\mathbf{T}u(x,y)$	$\mathbf{T}l^{-1}(x,y)$
$\mathbf{K}s(x,y)$	$\mathbf{T}h^{-1}(x,y)$	$\mathbf{T}q(x,y)$	$\mathbf{T}l(x,y)$	$\mathbf{T}j(x,y)$

Table 3: the PC preference structure

We call these ten different basic preference relations the PC preference structure $< P, Q, K, L, I, J, H, R, U, V >$. In Tsoukiàs and Vincke (1995) we proved that the PC preference structure is a maximal well founded f.r.s.p.. What are the semantics of the ten preference relations defined if we consider the relation S as the basic information to ask to the decision maker or to compute? We give a brief discussion hereunder.

1. $p(x,y)$ (strict preference):

   we can definitely establish that "x is at least as good as y" as there exist sufficient positive reasons supporting it and there are not enough negative reasons against it, while we can surely establish that "y is not as least as good as x" for the opposite reasons; therefore "x is strictly better than y".

2. $q(x,y)$ (weak preference):

   we have doubts concerning "x is at least as good as y" as there exist both sufficient positive reasons supporting it and sufficient negative reasons against it, while we can surely establish that "y is not as least as good as x" because we do not have sufficient positive reasons to assume the opposite and there exist sufficient negative reasons against it; therefore "x could be better than y, but we have some doubts due to the presence of strong negative reasons about $s(x,y)$".

3. $k(x,y)$ (semi preference):

   we can surely establish that "x is at least as good as y" as there exist sufficient positive reasons supporting it and there are not sufficient negative reasons against it, while we cannot establish that "y is not as least as good as x" or the opposite because we have neither sufficient positive nor negative reasons; therefore "x could be better than y, but we have some doubts because we do not know what happens with $s^{-1}(x,y)$".

4. $l(x,y)$ (semi weak preference):

   we have doubts concerning "x is at least as good as y" as there exist both sufficient positive reasons supporting it and sufficient negative reasons

against it, while we cannot establish that "y is not as least as good as x" or the opposite because we have neither sufficient positive nor negative reasons; therefore "x could be better than y, but we have some doubts both because we have evidence indicating that may hold $\neg s(x, y)$ and we do not know anything about $s^{-1}(x, y)$".

5. $i(x, y)$ (strict indifference):

   we can surely establish that "x is at least as good as y" as there exist sufficient positive reasons supporting it and there are not sufficient negative reasons against it, while we can also surely establish that "y is as least as good as x" for the same reasons; therefore "x and y are strictly indifferent as they could be considered equivalent".

6. $j(x, y)$ (semi indifference):

   we have doubts concerning "x is at least as good as y" as there exist both sufficient positive reasons supporting it and sufficient negative reasons against it, while we have also doubts to establish that "y is at least as good as x" for the same reasons; therefore "x and y could be indifferent", but we doubt due to the presence of strong evidence against it in both directions.

7. $h(x, y)$ (weak indifference):

   we can surely establish that "x is at least as good as y" as there exist sufficient positive reasons supporting it and there are not sufficient negative reasons against it, while we have doubts about "y is as least as good as x" because of the presence of both positive and negative reasons; therefore "x could be indifferent to y (notice in this case that "indifference" is is not symmetric), but we have some doubts to about $s^{-1}(x, y)$".

8. $r(x, y)$ (incomparability):

   we can surely establish that "x is not at least as good as y" as there are not sufficient positive reasons supporting the opposite and there are sufficient negative reasons against it, while we can also surely establish that "y is not as least as good as x" for the same reasons; therefore "x and y are in conflicting position due to strong contrasting information".

9. $u(x, y)$ (semi incomparability):

   we cannot establish that "x is at least as good as y" as there exist neither sufficient positive reasons supporting it nor sufficient negative reasons against it, while we cannot establish that "y is as least as good as x" for the same reasons; therefore "we cannot establish what holds between x and y because of the absence of relevant information".

10. $v(x, y)$ (weak incomparability):

we can surely establish that "x is not at least as good as y" as there are not sufficient positive reasons supporting the opposite and there are sufficient negative reasons against it, while we cannot establish that "y is as least as good as x" because of the absence of either positive or negative reasons; therefore "x could be in opposition to y, but we have some doubts due to the absence of all the necessary information".

To summarize, comparing x to y and y to x in order to establish a preference between them we have the ten following possible situations:
- x is strictly better than y (strict preference $p(x,y)$);
- y is not better than x, but we are not sure that x is better than y because of some evidence against it (weak preference $q(x,y)$);
- x could be better than y, but we are not sure due to lack of all the necessary information (semi preference $k(x,y)$);
- x is possibly better than y, but we have both contradictory information and lack of all the necessary information (semi weak preference $l(x,y)$);
- x and y are strictly equivalent (strict indifference $i(x,y)$);
- x and y could be indifferent, but there exist contradictions in both directions (semi indifference $j(x,y)$);
- x could be indifferent to y, but there is some contradictory information if y is also indifferent to x (weak indifference $h(x,y)$);
- x and y are in strong opposition (incomparability $r(x,y)$);
we cannot establish what holds between x and y (semi incomparability $u(x,y)$);
- x could be in opposition to y, but we are not sure due to lack of all the necessary information (weak incomparability $v(x,y)$).

These ten situations cover all the possibilities that may occur when comparing x and y.

3 The example again

Let us now see what happens if we apply the previously developed theory to the preference aggregation problem presented in section 1. We shall make for the moment a basic assumption: the preferences expressed by each examinator are modelled by the conventional preference structure $< P, I, R >$. We can now translate the decision rule by the following DDT formula:

$$\begin{cases} \triangle s(x,y) \Leftrightarrow \mid s_j(x,y) \mid \geq \alpha, \\ \triangle \neg s(x,y) \Leftrightarrow \mid p_j^{-1}(x,y) \mid \geq \beta. \end{cases}$$

Table 4 gives the results for all ordered couples in $A \times A$ on the basis of the definitions of the **PC** preference structure.

Let us dicuss the result. We recall here that the result of the direct aggregation of the five partial orders in section 1 was:
$P = \{(c,d)\}$, $I = \{(a,e)\}$, $R = A \times A \setminus (P \cup I)$

Using the **PC** preference structure we have instead:
$P = K = L = \emptyset$, $Q = \{(a,b),(a,c)\}$, $R = \{(c,e),(b,e)\}$, $U = \{(c,b),(d,e)\}$, $V = \{(b,d)\}$, $I = \{(a,e)\}$, $J = \{(a,d)\}$, $H = \{(c,d)\}$

	a	b	c	d	e
a	I	Q	Q	J	I
b	Q^{-1}	I	U	V^{-1}	R
c	Q^{-1}	U	I	H	R
d	J	V	H^{-1}	I	U
e	I	R	R	U	I

Table 4: The **PC** preference structure applied to set A

We note that only two couples ((b,e) and (c,e)) are effectively incomparable because of symmetric conflicting reasons (two examinators for part). The other couples included in the incomparability relation have been assigned to other relations, more precisely:
- for both couples (a,b) and (a,c) a could be better, but the existence of "strong" opposition (as valued by the decision rule: two examinators against) leads to a hesitation between preference and incomparability;
- for both couples (b,c) and (d,e) there is a strong lack of information (in the first case no examinator compares them and in the second case only three examinators compare them, but in a conflicting way) and therefore they could be either indifferent or incomparable;
- for couple (b,d) we have some evidence that "d cannot be better than b", but we have no other relevant information, so we hesitate between incomparability and preference (but this situation is obviously different from the case of couples (a,b) and (a,c));
- for couple (a,d) we have reasons both to conclude equivalence and strong opposition (two examinators for each and one considering them equivalent) so we hesitate between equivalence and incomparability (again this is a different situation from the one concerning couples (b,c) and (d,e));

Finally it should be noted that couple (c,d) which was in the preference relation, has now been assigned to an hesitation situation. Observing the five quasi orders it is possible to see that in fact two examinators consider them equivalent, two prefer c and one prefers d. It is more reasonable to hesitate between equivalence and preference rather to express a definite preference and this is our conclusion.

At this point one must remember that until now we considered for each examinator a conventional model based on the $< P, I, R >$ preference structure. It is easy to observe that if each examinator reported his preferences in a more "rich" model (reporting his positive and negative reasons) the final evaluations may be different (see for example Mainka et al., 1993).

So what are the final prescriptions? The most important ones are to ask the committee to clarify the uncertain situations (if possible) providing the missing information or eliminating contradictions. A more thorough investigation should be pursued, but the direction in which such investigation should take place is now clear. And what if, by any mean a final ranking must be given? No time or no will for further investigation. What could be a final prescription?

A basic idea could be the following:
- build the graphs of the "presence of truth in outranking" relation and of the "presence of truth in not outranking" relation (these are classic binary relations);
- compute the two rankings, one for each graph, using any appropriate procedure and then combine them.

It is possible to give some considerations about this final problem. Assuming the idea of working on the two graphs previously introduced, there are two directions of research in order to obtain a more reliable answer to an eventual demand of a final ranking (or prescription).

1. Define "a priori" the properties that the final ranking should fulfill and then try to identify which exploiting procedure fits the requirements. In this sense the results obtained by Vincke (1992) can be used.

2. Bouyssou (1994) recently proposed some results about the existence of structural properties in outranking relations. This result applies in conventional definitions of the outranking relation based on the $< P, I, R >$ preference structure. It may be interesting to verify if the outranking relation, as defined by the PC preference structure, has any structural properties (conditioning the exploiting procedure) and under which conditions.

4 Conclusions

The problem of extending the conventional preference structure has been addressed in this paper in order to take into account situations of uncertainty, imprecision, ambiguity etc.. A basic distinction between hesitation due to ignorance and hesitation due to contradiction is introduced and it is applied to the case where a preference model must be established. Under this perspective, all the possible situations are reviewed and formalized, using the DDT language, in a new preference structure named PC. Such a preference structure fulfils some basic axioms, that is a maximal well founded f.r.s.p.. An example of aggregation of partial orders is introduced in the paper in order to present the limits of the conventional approach and the eventual advantages of the new one. Some future research directions conclude the paper.

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Multiple Criteria Decision Making Within Checkland's Soft Systems Methodology

Hans G. Daellenbach
Department of Management, University of Canterbury
Private Bag 4800, Christchurch, New Zealand
Fax 64 3 364 2020, e-mail: h.daellenbach@mang.canterbury,ac.nz

Abstract: This paper studies Checkland's Soft Systems Methodology (SSM) as a potential multi-criteria decision making methodology. The paper first gives a brief outline of Checkland's SSM, highlighting its salient features. It then explores in what sense it is an MCDM methodology (in contrast to an MCDM technique) for dealing with multiple-decision maker situations. In particular, we explore the role of the Weltanschauung in the MCDM context and the process of finding a compromise solution involving conflicting Weltanschauungen among multiple-decision makers or stakeholders. We conclude that Checkland's SSM can, without any changes from its usual framework, be used to bring about compromise solutions. By its very nature, it is particularly suitable for problem situations that are ill-defined and largely qualitative. For problems that lend themselves to quantification, MCDM techniques for exploring the solution space may be useful aids. The final section then explores its suitability for single-decision maker situations. The tentative conclusion reached is that, in principle, this is the case. However, the lack of a proper debate among advocates with opposing views may call for the use of MCDM techniques to get better insights into the solution space.

1. INTRODUCTION

Soft systems methodologies have been around for more than 20 years. Most of them are intended to deal with problem situations that exhibit many of the aspects MCDM methodologies would consider to be within their domain of concern, such as multiple-decision makers with different preference structures, multiple conflicting objectives, and lack of or only partially externalized preference structures. One particular soft systems methodology, namely Checkland's SSM [1981, 1990], has a proven track record of successful real-life applications in industry, business, and government (in contrast to potential example applications without any real implementations, so commonly found in the MCDM literature).

This paper briefly looks at how Checkland's SSM naturally deals with many MCDM issues, in particular those involving conflicting preference structures with multiple-decision makers. It then analyzes how it proposes to deal with multiple criteria problems involving a single decision maker and the possible difficulties associated with this.

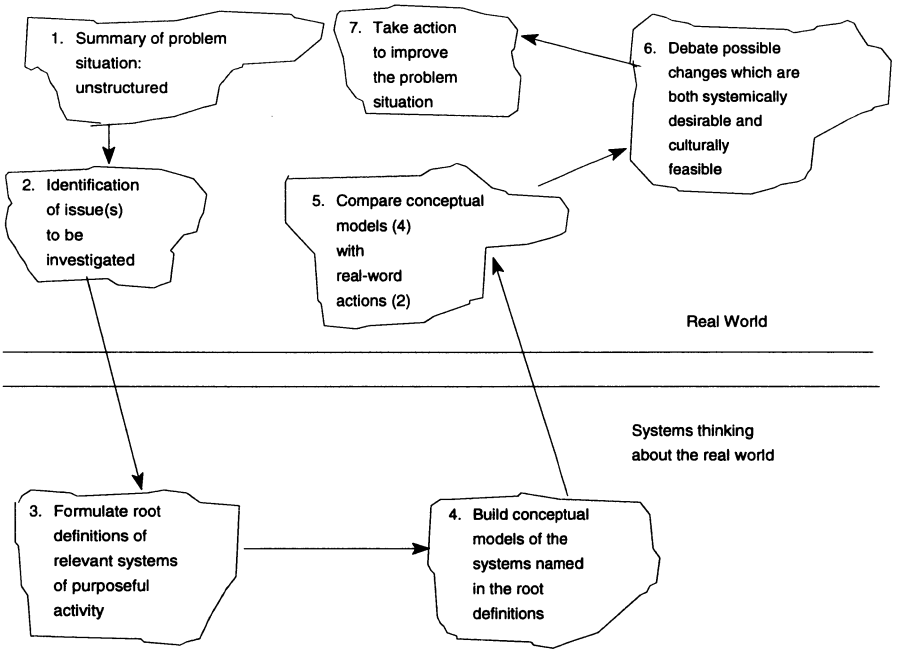
It should be stressed at the outset that Checkland's approach is not a technique (in the sense of most MCDM techniques), but a methodology — a general decision aiding approach, nor is it restricted to situations that lend themselves to be

quantified. In fact, the vast majority of its uses has been for problem situations that would be characterizes as `messes' in Russel Ackoff's terminology.

2. CHECKLAND'S SOFT SYSTEMS METHODOLOGY

Peter Checkland's soft systems methodology or SSM is without doubt the most researched, the most rigorously based in terms of its theoretical premises and philosophical reasoning, and the most widely applied, with well over a thousand known real-life applications, many of them documented in the literature. It is a fair assessment that its track record is reasonably good. Based on an action research programme at the University of Lancaster, started in 1969, the methodology slowly evolved, using itself as a learning system to grow and develop. Although in their latest book, Checkland and Scholes [1990] profess a somewhat more flexible use, its best known version is the one presented in Checkland's 1981 book *Systems Thinking, Systems Practice*, a text that any serious student of management science should study carefully. It is a seven-stage process, as shown in Figure 1. We shall now briefly review these seven stages. The intention is not to provide a full understanding of the methodology, but rather a flavour of the ideas underlying it.

Figure 1: A flow diagram of Checkland's SSM


1. Summary of problem situation: unstructured
7. Take action to improve the problem situation
6. Debate possible changes which are both systemically desirable and culturally feasible
2. Identification of issue(s) to be investigated
5. Compare conceptual models (4) with real-word actions (2)
Real World
Systems thinking about the real world
3. Formulate root definitions of relevant systems of purposeful activity
4. Build conceptual models of the systems named in the root definitions

Checkland sees problem solving within a management or social systems context as a never-ending learning process. It begins with one or more people viewing a

situation as problematical, with its own history and various stakeholders, all having possibly different perceptions of the situation and different perspectives in terms of their aims. The first stage is to assemble a **situation summary**, or **rich picture**, which summarizes everything that is known about the problem situation, hard factual information as well as soft information, such as differing perceptions, opinions, ideas and views of the organizational climate. Note, this is not the problem, but things about the problem and its context. An effective means for capturing this rich picture is via a cartoon-like drawing, showing the various aspects that make up the problem situation in an unstructured way. It deliberately avoids imposing an organized framework or systems view on the problem situation. The idea is to keep a completely open mind about what the `problem' to be analyzed is deemed to be.

Stage 2 leads to the identification of various possible candidate issues and the ultimate choice of one of them for further investigation.

At stage 3, several different systems views are developed for the issue selected. Note that Checkland views systems definitions as intellectual constructs. There is no implication that the various components and relationships necessarily exist, or should exist, in the real world. Each relevant systems view is expressed as a succinct unambiguous statement, a so-called **root-definition**, specifying the owners of the problem, the transformation process to be achieved and by whom, the world view or **Weltanschauung** (see the next section) that makes that transformation process a meaningful activity, the customers (victims and beneficiaries) of the system, and the environment of the system. The reason for formulating several root-definitions, and hence corresponding systems views, is to gain deeper and more varied and contrasting insights into the problem, each possibly reflecting a different Weltanschauung. Experience indicates that this raises the level of the constructive debate at stage 6 which hopefully leads to some changes.

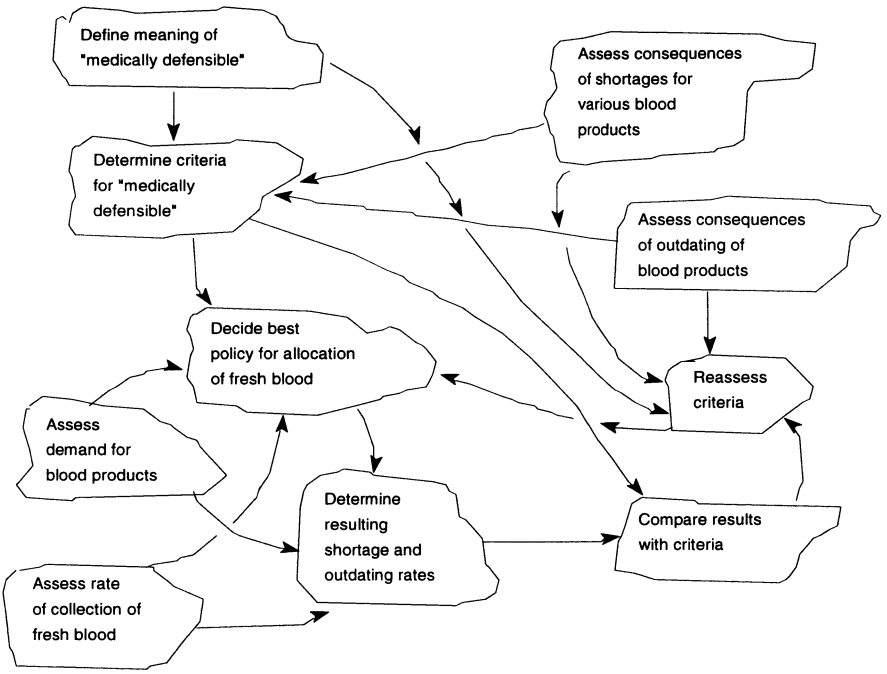
Root definitions can either be **issue-based** or **primary-task** based or a mixture of both. For example, consider the operation of a hospital blood bank. The major concerns raised are the potential shortages of blood products and the wastage of fresh blood through outdating. An issue-based root definition could be as follows:

> "A system which allows the director of the blood bank to reconcile the conflict of potential blood product shortages versus outdating of fresh blood so as to derive a medically defensible policy for managing blood stocks with limited shelf-life, collected from voluntary donors and needed for transfusions to patients in emergency and scheduled operations and for conversion into blood components administered to patients with blood deficiencies."

This root definition specifies the problem user or actor (director of the blood bank), the problem customers (director, patients, donors), the transformation process (reconcile conflict for allocating fresh blood to various uses), the Weltanschauung (medically defensible; blood shortages undesirable; outdating of blood undesirable), and aspects of the environment (demand for blood and blood products, limited shelf life). The problem owner is not stated explicitly. By implication it is the board of management of the blood bank or its director.

At stage 4 a so-called **conceptual model** is developed separately for each root definition. Each model shows the logical relationship between all systems activities needed for the transformation defined in its corresponding root definition to happen. Each conceptual model is usually expressed in the form of six to ten verbs, connected to each other in a logical influence or precedence sequence. Figure 2 shows the conceptual model derived from the root definition for the blood bank.

Figure 2: Conceptual model for blood bank problem


Define meaning of "medically defensible"
Assess consequences of shortages for various blood products
Determine criteria for "medically defensible"
Assess consequences of outdating of blood products
Decide best policy for allocation of fresh blood
Reassess criteria
Assess demand for blood products
Determine resulting shortage and outdating rates
Compare results with criteria
Assess rate of collection of fresh blood

Some activities may be decomposed further into sub-activities at a higher level of resolution, such as "Decide best policy for allocating fresh blood". Some, such as the one just mentioned, could also lead to the construction of an OR/MS model.

As is true for systems, conceptual models do not pretend to represent the real world or some ideal system. They simply show what activities of the system described by the root definition need to occur for the system to achieve a particular transformation process. So each conceptual model must be derived solely from its corresponding root definition. It should also include activities which monitor the performance of the system in terms of the stated transformation process. Conceptual models are not normative of what ought to be done. They do not form the basis for recommendations of possible changes. Their purpose is different, as will be described below.

Each conceptual model may be checked against what Checkland [1981] calls a **formal system model**. It has the following necessary minimal features: a mission, a measure of performance for evaluating how well the mission is achieved, decision making and control processes, an environment with which the system interacts, resources, a guarantee of continuity, and components which themselves may be subsystems with all these properties.

Stage 5 compares each conceptual model with the rich picture, i.e., with what is perceived to exist in the real world. The aim is to develop an **agenda of discussion topics** with the stakeholders of the problem at stage 6. So we look for similarities and differences between conceptual models and the real world. The emphasis should be on `whats', e.g., `what activities are missing or problematic?', rather than in terms of `hows', such as `how is this activity done on the real world?' There may be several `hows' to achieve a given `what'.

The most commonly used method of developing this agenda is through a tabular display [Wilson, 1984, 79-82], as shown in Figure 3, where columns 1 through 4 represent stage 5, while columns 5 and 6 are already part of stage 6.

Figure 3: Tabular display for stage 5 and 6

(1) Activity	(2) Exist or not?	(3) Present mecha-nism	(4) Measure of per-formance	(5) Proposed change	(6) Comment
A list of concep-tual model activities (`whats')	Can a `how' be found in the real world situation for this `what'?	What is the present `how' used?	Measure or crite-rion against which present `how' is said to be good or bad	Increment al changes to im-prove the situation	Evidence about present `how'; potential benefit from change; effect on activity linkages

The **debate** at stage 6 should, whenever possible, involve all types of stakeholders. The purpose of this debate is to subject the implications of possibly conflicting world views to the collective judgment of the group in an open and non-defensive manner. The aim is to develop new ideas for change in the real world that are systemically desirable and culturally feasible. Such changes may be in terms of the procedures used in the organization or operation, in terms of the structure, or in terms of the attitudes. While procedural and structural changes may be relatively easy to agree on, changes in attitude are more difficult to achieve, but may be more crucial if substantive improvements are to be secured. An effective approach for this stage is to use the tabular display of Figure 3 as a basis for the

debate. Conflicts will become evident in the discussion for answering column 6.

This stage will usually result in revisiting earlier stages. It may call for an updating of the problem situation, as new aspects are uncovered in this debate. It may lead to a restatement of the some of the root definitions and the corresponding conceptual models derived from them, which in turn leads to new questions to be subjected to the debate at stage 6. The process is thus highly iterative, with forward and backward linkages.

Regardless of whether any changes are implemented or not, each completed cycle of this process transforms the original problem situation. The new situation should find the stakeholders with a shift in perception and at a higher level of understanding. This becomes then the starting point for another learning cycle, i.e., the methodology cycles back to stage 1. By involving all stakeholders in the process, it is hoped that **change or implementation** (stage 7) is facilitated.

It is also interesting to look at the focus of these various stages. Stage 1 and 2 clearly relate to the real world. Stage 3 and 4, on the other hand, focus on systems concepts, using a systems language. Stage 5 to 7 return to the real world, comparing the systems view with the real world.

Nowhere in this description of the process is the analyst mentioned. In fact, ideally, there is not one analyst, but the stakeholders or a subgroup of them assume the role of analysts. If there is an analyst, her or his role is largely one of offering advice about important "do's" and "don't's" at various stages. As the stakeholders become more confident with the process, the analyst becomes superfluous.

SSM is doubly systemic in the sense that it is a `learning system' using systems models to bring about and orchestrate the debate about purposeful real-world changes which are **systemically desirable** and **culturally feasible**. SSM is an iterative learning cycle, with the original problem situation emerging into another, different problem situation. For Checkland, problem solving within a management or social systems context is a never-ending learning process.

3. IN WHAT SENSE IS SSM AN MCDM METHODOLOGY FOR MULTIPLE-DECISION MAKERS?

Note that the terms `attributes', `objectives', and `preference structure' — so central to MCDM techniques — are absent in the short descriptions of Checkland's SSM given above. The reason is that SSMs confronts `problem situations' — vague feelings or concerns that something is amiss, or leads to inferior outcomes, or needs clarification for improved performance. It does not start with `fully or partially formulated problems', where the relevant objectives, including appropriate attributes for measuring their level of achievement, have already been specified, and where the system, relevant for exploring their response to changes in input controls, is defined, leaving only the exact form of the preference structure and the problems most preferred solution to be determined, as is the underlying assumption for most MCDM techniques. Defining relevant attributes and objectives and an appropriate preference structure are thus an integral part of SSMs. It is thus very much along the lines of R. Keeney's value-focused thinking [1992].

In SSM the aspect of the problem situation that will ultimately determine the relevant objectives and the underlying preference structure for a given problem are the Weltanaschauungen or world views of the various stakeholders, i.e., the problem owner(s), the problem user(s), the beneficiaries and/or victims of the problem solution, and the problem analyst, but in particular of the decisions maker(s), usually the problem owner(s). The world view is the set of values, background, experiences, views, and perspectives that allow people to attribute meaning to their perceptions or appreciations of a given situation. It is a filter through which a given situation, including what we normally see as `facts', are interpreted. SSM thus views the world as subjective, since we never can fully and completely step outside our world view. (`Objectivity' at a given point in time becomes merely a consensus of individual subjectivities!)

It is also clear that different stakeholders may have different and possibly conflicting world views. Failure to recognize this and have means to deal with it are bound to lead to difficulties later on in the problem solving process and are often the main causes for lack of implementation of any findings or recommendations.

In Checkland's SSM the world view is usually summarized in such phrases as "making profit is essential for a firm's survival", or "the operation of a blood bank must be medically defensible, recognizing that both blood shortages and outdating are undesirable". Obviously, these are mere slogans, capturing only the central aspects of the full complexity of a person's world view. Expressed in this form, the connection between the world view and the objectives can be seen more readily.

Checkland would contend that, unless the different world views of the stakeholders are brought into the open and their implications fully explored, a meeting of minds and a lasting solution or resolution of the problem will be difficult to achieve. Only by confronting the various world views and comparing their implications in an open, non-threatening debate can a meeting of minds be achieved, that will lead to a solution that is systemically desirable, i.e., leads to a performance seen as superior by all stakeholders, and is culturally feasible, i.e., is not in direct conflict with their world views and is, therefore, politically acceptable.

This description of some of the salient features of SSM indicate that this methodology is ideally suited for resolution of conflicts arising from different world views, and hence conflicting objectives, of the various stakeholders. The resolution of conflicts is achieved via the debate at stage 6, and the iterative process of the methodology. It is generated by the stakeholders themselves through their constructive debate, rather than presented as a recommendation by a `neutral' analyst. The tabular display approach of Figure 3 is only one possible mechanism for encouraging this debate.

In this debate the stakeholders obtain both cognitive and outcome feedback. Outcome feedback provides information about the possible solution space. Cognitive feedback gives information about the world views of the various stakeholders and how they come about. It leads to a better understanding and appreciation of the various positions. Both help in contributing towards finding a compromise solution. As H. Simon's general model of adaptive motivated

behaviour [J. March and H. Simon, 1958, 47-52] postulates, the level of satisfaction, the level of search, the expected reward, and the level of aspiration are all interdependent. What you want is affected by what you can get and how easy it is to get it, and vice-versa. Cognitive input will relate to all these variables. So, the debate will influence the preference structures of the various stakeholders, and hopefully result in a narrowing of the conflicts among the world views of the various stockholders, increasing the propensity to agree on a suitable compromise solution. The parallel with interactive MCDM methods, which help the decision maker explore and firm up the preference structure as a result of exploring the solution space, is obvious. However, these methods work mainly on outcome feedback, rather than cognitive feedback. For multi-decision maker situations, cognitive feedback is the more crucial input for bringing about a climate of appreciation and understanding of the conflicting world views among stakeholders. Any compromise solution usually requires at least a partial shift in world views.

The methodology does not include any technique for determining and then optimizing some preference function, nor does it give any guidelines as to how to go about that. In fact, the individual preference structures of the various stakeholders are usually never formally externalized and combined into some group preference structure based on a set of plausible assumptions, such as additivity. Except for the recourse to systems thinking, the process used is entirely ad hoc, culminating in a constructive debate, where potential `solutions' are compared holistically, or aspects of several `solutions' are recombined to form a new `solution'. This could be seen as the major weakness of SSM, although Checkland would probably claim this type of complete flexibility to be one of its strength. As stated earlier, this debate, properly conducted, will bring conflicting world views into the open and better understood by all stakeholders. This is more conducive to derive a mutually acceptable compromise solution. It is likely to lead to agreement about minimum feasible achievement levels for various objectives, which then narrows down the solution space from which the compromise solution will have to be found in the subsequent debate. An external analyst or team of analysts is unlikely to be able to fully capture and model the psychological response patterns of all stakeholders, since these response patterns are dynamic. They evolve and are affected by the debate. It is this aspect which makes it possible to come up with suitable compromise solutions without formally expressing the individual preference structures and combining them into some group preference structure.

It may often be the case that the debate reveals a given desirable change to be acceptable or neutral for all world views involved, without the need for first reconciling conflicts in world views and hence objectives. This is similar to the findings of K.R. Hammond et al. with the use of Social Judgement Theory [1975], showing that despite fundamental disagreement on policy issues, there was agreement on choice of action, once the noise of misunderstandings and misconceptions of each other's world views has been eliminated. The compromise solution derived represents some sort of weak unanimity of all stakeholders.

In other instances, some residual conflicts between stakeholders remains, i.e., the process does not generate an overall systemically desirable and culturally

feasible solution, but hopefully it will still lead to a better understanding between the stakeholders and a clearer appreciation of each others' world views. This may then well lead to a redefinition of the problem situation and the choice of a higher-level issue, such as an issue of values or attitudes, for a new application of SSM.

However, nothing prevents the use of some suitable MCDM technique as an aid to both stage 5 and 6. In particular, social judgment theory [Hammond, 1976] could be used to clarify and narrow down conflicts in world views, and overcome aspects of verbal misunderstandings. Similarly, interactive conferencing methods, such as a Decision Room approach would help create a non-threatening interactive environment for cognitive inputs. For problems with a large number of feasible choices that lend themselves to quantification, multi-objective programming methods or multi-attribute utility analysis could be used for the exploration of the tradeoffs between objectives derived from the various world views, and give a better insight into the efficient solution space and the response of various objectives to the set of systemically desirable and culturally feasible choices.

4. SSM AS AN MCDM METHODOLOGY FOR A SINGLE DECISION MAKER

While for multiple-decision makers, the debate at stage 6 is used to come up with changes that are systemically desirable and culturally feasible, for a single decision maker the equivalent to the debate at stage 6 by necessity has to be an internal process, where the decision maker systematically explores and confronts, in an internal dialectic debate, conflicting multiple objectives externalized from her or his world view. There are two ways to proceed.

By analogy to the multiple-decision maker situation, a conflicting world view is decomposed into several world views, each giving rise to a different objective. Each world views is exposed individually to stage 3 to 5. At stage 6 they are then jointly subjected to an evaluation in view of either revising the issue, and/or the various world views and associated root definitions, or adopting changes that are systemically desirable and culturally feasible. If a constructive debate for the multiple-decision maker case is an effective means for finding a compromise solution, the absence of advocates with different and opposing world views poses a serious handicap. An "internal debate" is likely to be less effective in resolving internal conflict. What the SSM process will achieve is more insight into the problem situation and the conflicts. But it will not necessarily lead to finding a good compromise solution. If the problem lends itself for the use of some MCDM technique to help clarify the solution space and the preference structure, it seems almost inevitable that this is the course to be taken as an aid at stage 6.

The second approach is to consider various versions of world views that give different emphasis to the various objectives. For example, the root definitions focus on a single world view or objective, imposing targets or constraints on the other objectives. Checkland [1981] and Wilson [1984] list several examples of this sort. However, this approach basically suffers from the similar drawbacks as discussed above, in the sense that it does not provide any formal mechanism for

setting the targets and for reconciling conflicts and may not give the decision maker sufficient insights into the solution space. Furthermore, the targets may well be set in the dominated part of the solution space.

5. CONCLUSIONS

The aim of Checkland's SSM is to bring about changes in operating procedures, structures, and/or attitudes that are systemically desirable and culturally feasible. Both these aspects are essential for any change to be accepted and implemented. Cultural feasibility usually implies that the solution derived is a compromise over conflicting world views and hence conflicting objectives. From this point of view, SSM is an MCDM methodology, highly effective for bringing about change involving multiple stakeholders. It is a methodology, rather than a technique, in the sense that it consists of an iterative sequence of stages, based on general and flexibly used principles, which involve the use of systems thinking. The resolution of conflict is achieved via a constructive debate which brings into the open conflicting world views and evaluates systemically feasible changes in the light of these world views.

Its suitability for single-decision maker situations with conflicting objectives is somewhat more problematic, since in the absence of advocates with opposing views the debate remains internal to the single decision maker and is, therefore, unlikely to be as effective. If the nature of the problem situation lends itself to the use of some formal MCDM technique, a representative set of potential solutions derived with such aids can then be subjected to stage 6 of the SSM methodology.

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Validation Aspects of a Prototype Solution Implementation to Solve a Complex MC Problem

Anna Ostanello
Politecnico di Torino, Dipart. Sistemi di Produzione ed Economia dell'Azienda
Corso Duca degli Abruzzi 24
10129 Torino, ITALY

Abstract. This paper summarises some methodological considerations concerning the development/validation process of a 'tool' to be used as a support in a complex situation of decision making. A 'bipolar' and multistage model is proposed for such a process, within a Multicriteria Decision Aid (*MCDA*) framework. This model integrates and extends two important models of modelling/validation process literature, and is conceived as a support tool for the analyst's work during a decision aiding intervention.

1 Introduction

In a recent research project conducted by the Author and Collaborators, [1] [4], some issues on 'modelling and solving' a complex multicriteria decision problem were explored. Such a process was conducted interactively with one of the managers of the Client's factory (i.e. the decision maker to be supported, DM), during the first phase of problem conceptualisation-representation. The participating DM was particularly interested in having a conceptually and logically valid model of his problem which possibly included all the considered representation elements, and in obtaining some elements of support for 'strategic' decision making. One of the models developed during such a process, was selected to be the 'core' of a DSS prototype, on the basis of a few validity criteria (cf [1]). This stage of the development process of a DSS prototype is hereafter called *implementation stage*. [Different interpretations of 'implementation' are found in other literature works].

This paper summarises some methodological considerations concerning the *validation process* during the implementation stage, and proposes a *multicriteria (MC) model* for such a process within a Decision Aiding framework (*MCDA*). This model is derived from some literature models based on similar sets of hypotheses regarding 'internal' [to the procedure itself] and 'external' validation [i.e. with reference to the development and decision (or context) processes in an organisational context (cf [6] [7] [9] [12]). Other interesting works on 'validation', found in OR/DA literature (such as, for instance [13] [14] [15]) are essentially based on a cognitive conception of the decision process and, principally, refer to the 'selection problem' of 'valid' *MCDM* tools in situations in which the DM's and analyst's roles are not clearly differentiated. Such a perspective, which is very important in decision aiding, is however only one dimension of a

modelling/validation process within OR/DA intervention (cf [3] [10] [12] [17]); it is not usually sufficient, if considered separatedly from the other process dimensions, to explain how a 'best method' selection often does not fit with other important validity criteria activated (often implicitly more than explicitly) in real cases. A 'best method' selection should therefore be considered as a 'second level' problem to the more general (multicriteria) perspective discussed in this paper which conceives *validation as a multistage, multiple criteria decision process* for the 'OR/DA worker' (cf [9] [11]).

2 Validation Process within an Organisational Context

2.1 Validation as a MCD process

Model building is consensually recognised in literature as a fundamental activity in OR/DA studies. The importance of the 'validation problem' in such a process is also unanimously accepted; however, general agreement does not exist on what one intends as 'validity' (cf [9]). As many different criteria may be deduced, conflict can arise even over the evaluation of the validity of the tools used or of the models developed. The kinds of tools (methods, approaches, procedures and techniques) that are used in a modelling process tend to promote certain views on what constitutes a 'valid model' (cf [7]). This perspective of validation, based on instruments, concerns the 'efficiency dimension' of the problem ("doing things right"); besides this, a basic question however remains unanswered, regarding the 'effectiveness dimension' of the validity problem within a real organisational context ("doing the right things"). Such a perspective takes the research to a point at which the cognitive dimension of the OR/DA study has to be integrated with the so-called 'social' (or organisational/political) dimension of the process (cf [17]).

There are no universal approaches to or criteria for validation. Various perspectives and criteria can be defined. These depend on the stage of the modelling process and the actual phase of the context (or reference) organisational process. Validation can be formally or informally conducted in various ways, such as: directly, by the application of technical tools (cf [7]) or by judgement; or, indirectly, using interactive processes through which 'valid consensual' solutions or action proposals, that regard different aspects of a 'problem representation', are searched for. Validity judgements should depend on the 'situation' being modelled, the work that has been carried out, the conditions under which such work has been developed, and how it has been carried out. They should regard both the 'products' developed and the 'impact' that the process activities had or might have on the reality of the process and on the organisational context. Various authors have recognised the need to focus more research efforts on developing an appropriate validation methodology which could make it possible to distinguish different kinds of validation (sub)processes, and therefore different kinds of criteria to be applied appropriately. Such a methodology could be very useful to orient the OR/DA worker's activities in real

problem contexts, and to support 'valid' selection and application of OR/DA tools in different operational and decisional contexts. A first 'structured' debate on some fundamental issues of model validation has been proposed in [6].

The present paper deals with a hypothesis of an integrated approach to the modelling/validation problem within an organisational context. *Validation* is seen *as a multiple criteria decision process* in which the OR/DA worker plays a decision-maker's role. Validation is therefore not an 'act' or an isolated (or final) stage of the process in which some formal result or a technical 'object', such as a model, a 'problem solution' or a 'prototype', a structured data base or a method to be used, is evaluated with reference to some operational context. Validation is intended as a continuous flow of activities conceived and developed throughout the process, and which concerns different possible 'objects' and contexts (cf Figure 3). *Ex-ante* and *ex-post* criteria intervene to support impact evaluations of the subsequent multiple choices that have to be made during the process. These may concern operational tools that must be applied or other 'objects' of a various nature (cf [8] [10] [11]). 'Object' validity is obtainable by means of a validation sub-process or test that cannot be performed independently of other subsequent steps of the process or the possibility of revising other previously carried out validity tests.

As with any decision process that develops with reference to a social-organisational context, validation implies the presence or influence of various actors independent of their direct involvement. These actors, such as: the client, the decision maker(s), stakeholders, information sources and potential users, play different roles at different stages in the process and in relation to the different 'objects' dealt with, especially in the cases of their participation in modelling activities. The OR/DA worker develops different functions (analyst, modeller, implementer, facilitator, negotiator, or change agent), as pointed out in various literature works (cf [3] [9]). A particular function identifies this subject as an "element of the client's technological system" during intervention (cf [12]) [such a 'technological resource' has to be validated by the organisational actors involved in the process], or even as a DSS through which the client aims to acquire valid data/information [2].

This concept tends to weaken the 'unipolar' hypotheses of many methodological analyses that are based on a 'classical' view of the decision process [in which 'data' are considered as 'given', and therefore *a priori* validated, and modelling activities are restricted to the so-called 'choice phase' and considered as being independent of any relationship with the process/context] and to move the focus of research towards a more integrated and *'bipolar' conception* of the process. This conception considers the roles and activities of the two main actors (analyst and client) as being, normally, separated during the DA intervention, and tends to underline the inseparability of the binomial data/information structuring and decision support. The learning function of the process is therefore pointed out, without over-emphasizing 'the decision need' of the client/decision maker (cognitive conception). Moreover the validation of the instruments used and of the 'formal' or 'implemented products' is considered as being dependent on the

operational and actorial contexts in which they are expected to be applied or introduced.

The concept of *data validity* [7] is of great importance to the modelling process. This involves, in the Author's hypotheses, a more complex meaning than the 'technical' meaning that is usually assumed in literature. Data validity is not an input property that is exogenous to the modelling process. It is instead a result that is continuously searched for and tested during the process using suitable tools for different operational contexts. *Valid data* are therefore 'products', i.e. *sub-models or partial results structured/developed and validated* during the process. The implementation/validation stage of a hybrid procedure is, in particular, conditioned by data validity, as the model proposed in the following pages underlines.

Few approaches or general models to represent and analyse a modelling/validation process from such a perspective exist in literature. The main features of three models based on such hypotheses, i.e. the models of Landry *et al.* [7], Oral and Kettani [9], and Ostanello [12], are summarised in the following pages. These models constitute the framework from which the following MC validation model has been developed.

2.2 Reference models

2.2.1 A network model of MVP

An OR/DA modelling-validation process (MVP) has been conceptualised by Landry *et al.*, [7], as a network of sub-processes called: *conceptualisation*, *formal modelling*, *obtaining a solution* and *implementation* (in the sense of model operationalisation or 'solution' transfer/application in the organisational context). These sub-processes develop, starting from a *problem situation* (*CPS*) as perceived or recognised by the client/decision maker(s), and tend to obtain some expected results: i.e. a valid *conceptual model* (*CM*), a valid *formal model* (*FM*) deduced from *CM*, and a valid *solution* (or *prototype* solution) (*PS*). This has to be made operationally 'valid', then 'implementable' within the organisational context in which the 'problem' exists. The interrelations between these processes are not necessarily linear, as a two-dimensional graph representation of the MVP shows (Figure 1). In this representation, *data validity* is a connection node with a central position that is linked to the other four nodes of the network.

Five *types of validity* are defined in the paper [7]: *conceptual*, *logical*, *experimental*, *operational* and *data validity* (Figure 1). The achievement of some kind of validity may require iteration of several developments of the defined sub-processes; each of these developments might have to be conducted on possible different 'objects' and within different contexts, so as to arrive at a state of acceptance and satisfaction by the interested or involved actors (cf Figure 3). The sub-process sequences may vary, moreover, as a function of the 'main problem object' and the organisational context in which the MVP develops, thus showing various possible levels of cognitive and 'socio'/organisational complexity.

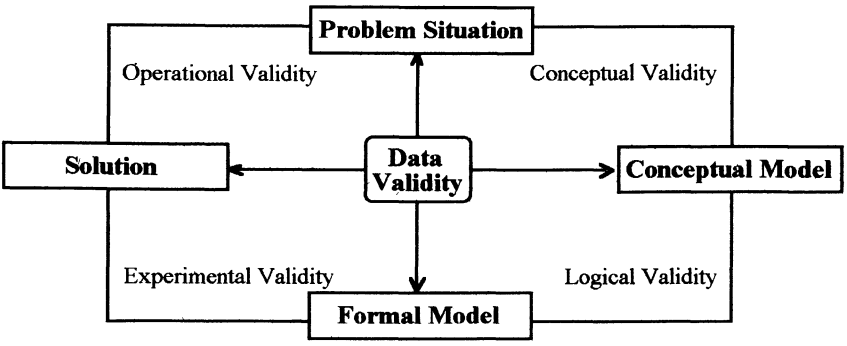

Figure 1: Network representation of the modelling-validation process

2.2.2 A three -dimensional model: 'the quartet' of MVP and its facets

The four basic elements of this model have been named as the *'quartet' of the modelling validation process* by Oral and Kettani [9] (O/K). These Authors introduce a powerful three dimensional view of MVP which permits the process to be represented as a *tetrahedron* (Figure 2). The decomposition of such a tetrahedron into four *facets* ('Prototype', 'Descriptive', 'Pragmatic', and 'Theoretical' facets) allows a more meaningful analysis of the validation concept. Each of these facets has been associated with 'OR problems' or issues and with a type of validation sub-process. Such a representation reveals the need to define other types of validation, such as: *formulational, aptness, legitimational,* and *verificational.*

An explicit representation of 'data validity' is, however, excluded from this tetrahedron, despite the centrality assigned to such an aspect ('criterion') at all stages of the process (cf [9]). Such an exclusion is justified by an assumption made by the proposers of the model regarding a somewhat more general (but conceptually simpler) meaning assigned to the 'node' *CPS,* that has been renamed *Managerial Situation*; this includes various types of 'managerial situations' of a purely cognitive nature, such as 'assessment', 'prediction' or 'analysis', which do not require an immediate solution to or decision on a 'problem situation', as considered in Landry's model. The concept of 'solution' has also been changed into that of *decision* so as to include an operational dimension in the evaluation of results of the 'formal modelling' sub-processes.

The O/K model brings some important contributions to an enlarged view of the cognitive dimension of the modelling/validation process that have not been given here for the sake of brevity. The model has however, in the Author's opinion, a more limited interpretive capability to that of Landry's *et alii* model, as far as the social/organisational dimension of the MV process and the influence of this dimension on the cognitive dimension, or 'simply' the cognitive dimension in a multiactorial context are concerned. In other words, the model does not permit a clear distinction between the 'managerial situation' and the 'problem situation in an organisational context', nor does it establish explicit links with the 'problem

situations' in which 'managers' have to understand problems, identify or develop 'valid solutions' or answers to questions and continuously take decisions, and where it is really a matter of validation of the 'manager's work'. Moreover, the 'managerial situation', as included in the O/K model, can refer to different actors, teams or groups, in the different sub-processes of the MVP, as the multiple roles defined in O/K analysis underline. The conceptual simplification introduced on the basic 'quartet' therefore does not facilitate the evaluative analyses of these sub-processes in their possible different concatenations during real process cases. The analyses of MVP based on a 'bipolar' conception of the actorial system, as introduced in the previous section, can therefore be developed with difficulty. The recurring interactions conducted in the organisational context, during the search for some types of data/information, and to acquire some kind of validity for a given 'object' (such as a partial model, a solution or a proposed action) or the legitimacy of an actor cannot be meaningfully evaluated in terms of validation. The situations in which some steps of inquiry, conceptualisation, structuring/modelling are conducted solely by the OR/DA worker or jointly with the client or with some other stakeholders cannot be distinguished; nevertheless, the validation dimensions and criteria that have been activated in such different situations can greatly vary.

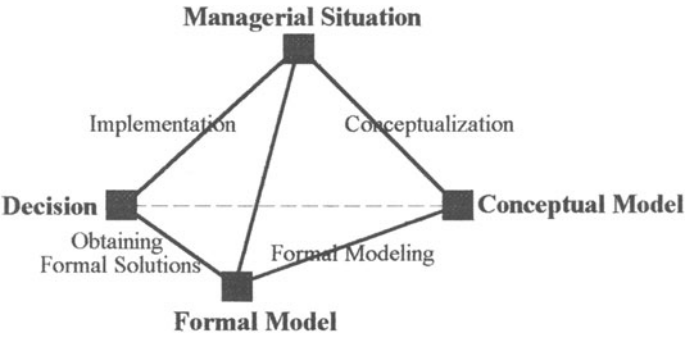

Figure 2: A three-dimensional view of the MVP

2.2.3 A 'bipolar' network model: MVP in an organisational context

A third reference model, proposed in paper [12], relates to an approach that tries to integrate cognitive, behavioural and 'social' (or organisational/political) factors into the conception of a modelling-validation process.

One of the main assumptions of this model regards the analyst's attitude that is oriented to activating 'participative interactions' with the subjects of the organisational environment (client, experts, information sources and stakeholders). The actors' *interaction and participation* are fundamental issues for the validation of the process and its results. Participation is not confined to a traditional conception of expert involvement, but refers to all the key actors, including the client and the analyst himself. The client has to be involved in the

study so as to be in a condition to obtain a better understanding of the problem situation and the main characteristics of his own problem ('helping to improve decision-making' or 'decision aiding'). He must also be in a condition to share the 'action' or problem representation 'structured' by the analyst with the other involved actors. Such a conception also implies that the analyst's behaviour might differ from the traditional role of an expert "who cannot be *contradicted*, within the very-defined limits of his expertise, but who can be *ignored*" [16].

The involved subjects are considered not only as a part of the information system (therefore *a priori* assumed as being collaborative and producers of 'valid' data for the MP), but also as actors who are capable of influencing the process thanks to their systems of values, information and relationships. The analyst is not restricted to cognitive and operative activities, as assumed by the cognitive paradigm. Some organisational and political activities therefore become important to obtain valid information. Some examples of such activities are: the diffusion of selected information to solicit or provoke an actor's reaction, communication to acquire an actors' participation, negotiation with the organisational sources to obtain some data/information that is important to develop a point of view introduced by other stakeholders. On the other hand, formal modelling activities may have some important organisational and political functions if applied not 'as decision making rules' but as useful tools to support a client's learning and to help an actors' communication, cooperation, compromising, negotiation and thus organisational change. This, in particular, implies that the analyst's endowment of tools has to be conceived in a wider sense, from both the cultural and operational points of view, [2]. The MVP is then considered as being closely interrelated with the *organisational context*. This may be synthesized as a 'system' in which *four evolving sub-systems* (or components), *Actors (A), Processes (P), Objects (O) and Technologies (T),* interact [12].

A graphical representation of such a system is given in Figure 3. The *O-system* is a component which allows one to explicitly outline some relationships of a cognitive nature that might develop within and between the other sub-systems. It refers to a concept of a problem 'object' that is similar to that introduced in [5]. The *T-system* includes both hard and soft technologies that exist in the organisation, and formal and informal Information Systems. In relation to this framework, the OR *analyst/modeller* is seen as being included *in the T-system*, i.e. as a 'tool' by which one or a few organisational actors try to promote some organisational change. The temporary intervention of this external subject is an *act* (as defined in process analysis literature) produced by an organisational actor (the client), and may refer to one or several dimensions of the reference organisational process, depending on the finalities the promoter(s) assign to the analyst's process. OR/DA intervention can therefore be seen as a sub-process that involves the context sub-systems in a different manner; its 'core' is constituted by the OR/DA worker and his instrumental/cultural endowment. During the MV process this actor undertakes various kinds of acts, and may use his own tools in a different manner within the interactions with the context components (*A, O, P, T*), which represent the possible *domains of the analyst's activities* and characterise the main *dimensions of validation* to be considered. A more

integrated representation of the modelling-validation process in the organisational context, can therefore be deduced from such a framework.

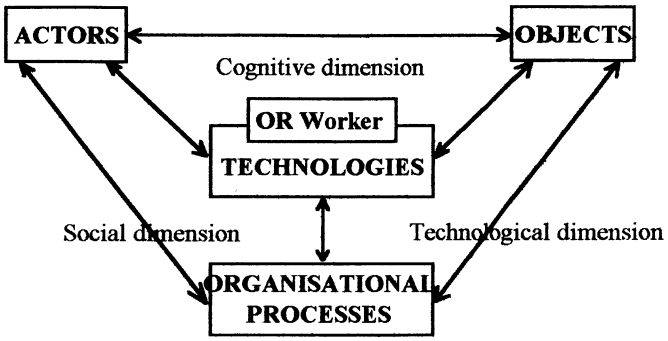

Figure 3: A context model of MVP

3 Basic 'Sextet' and Octahedron of a MVP

The proposed model integrates the main features of the three reference models in an attempt to represent and permit more meaningful analyses of the OR/DA modelling-validation processes. A 'bipolar' view of the actorial system within an organisational context is taken as a basic starting assumption. This allows one to point out both the *multiple criteria* that might be differently activated in various validation sub-processes, as the facets in the Oral and Kettani's model already show, and the possible *different validity dimensions* interwoven in the validation process, as specified in section 2.2.

3.1 Client's and analyst's problem situations and validation tetrahedrons

The MVP representation includes all the basic elements of Landry's *et al.* model [7] and therefore, explicitly, the *data validity.* A sixth constitutive element of the representation is introduced to show the distinction that must be made between the concepts of *Client's problem situation (CPS)* and *Analyst's problem situation (APS)* (Figure 4). Thus, by extending the interesting terminology introduced in Oral and Kettani's work [9], the Author's model is based on the following *'sextet' of basic elements*: *Client's Problem Situation (CPS), Analyst's Problem Situation (APS), Conceptual Model (CM), Formal Model (FM), Data Validity (DV), and Solution (*or *Prototype Solution) (PS).* Some principal interrelations between these elements allows one to define *four tetrahedrons* connected within a *polyhedron (octahedron)* that represents the MV process. Each tetrahedron is associated to one principal sub-process; the four sub-processes are analogous to those introduced in Landry's *et al.* model, and are named: *Analysis and Conceptualisation, Formal Modelling, Obtaining Solution,* and *Implementation within CPS* (or Organisational Implementation). 'Implementation within *CPS'*

refers to a validation sub-process (or a stage) that develops within, or with reference to, some context linked to the client's problem situation; while the concept of 'implementation', as specified in the title and in the introduction to the paper, refers to the obtaining of a 'visible object' developed by means of 'valid' applications of available tools (such as models, algorithms, methods, procedures, and informatic technologies) within the context of the analyst/modeller's problem situation. This last type of 'implementation' is associated to the *Obtaining Solution* tetrahedron, in the Author's model, and is hereafter called *Implementation of a Prototype Solution Tetrahedron (IPST)*. A full description of the MVP model is not given here for the sake of brevity. An analytical presentation, based on the three-dimensional view illustrated in Figure 4, is limited to *IPST*.

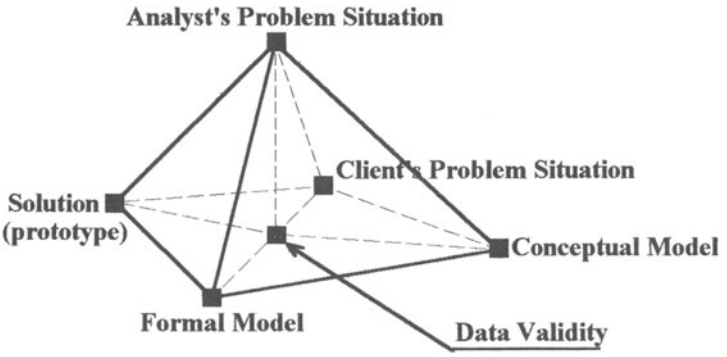

Figure 4: Polyhedron representation of MVP

3.2 Data validation

The centrality of data validity is underlined in all models of the conceptual framework. Various kinds of data/information are needed to 'model and validly solve' real complex problems. 'Valid data' are not usually *a priori* available; on the contrary, the search for such elements is a process that develops during the analyst's intervention, so that 'data' result to be models structured or developed during the process. These 'data' are instrumental in both the cognitive and social dimensions of a MVP: they are used, for instance, to recognise some problem aspects of the client's situation (*CPS*), and may be 'objects' in the analyst's problem situation; they intervene in the construction and definition of a conceptual model (*CM*), and in the development of a formal model (*FM*); they facilitate communication and are essential in negotiations which might have to be conducted with validity purposes in organisational contexts.

Data validation is a process which may take different features and requires different possible approaches and techniques; these depend on various factors, such as: the kind of data, context and purpose(s) of their use, the way they are used and the instruments applied to handle them. Difficulties of a cognitive, technical and organisational/political nature might arise throughout the process.

Several criteria for data validation are found in literature, such as: availability, reliability, appropriateness, relevance, congruence, meaningfulness, usefulness, usability, accuracy, maintainability, and cost. For an important review of these criteria, the reader can refer to paper [9]. *Appropriateness* is a criteria especially applied to data to be used within tool applications. Two main types of appropriateness may be distinguished: *a) internal*, pertaining to axiomatic or logical foundations of an applied tool [most of the 'academic'-technical discussions are focused on this validity type]; *b) external*, related to the possible contributions that the results of a tool application can give to other problem aspects which have to be dealt with. An example of problem related to external appropriateness is given as follow: a new development of a formal result (that has been deduced by a valid procedure) has to be conducted, because the present 'form' is not appropriate for communication in the 'social' context for which such a 'solution' has been developed (social/operational dimension). External appropriateness may therefore be associated to different criteria, that depend on the process dimension which prevails in the evaluative analysis.

4 Implementation Tetrahedron of a Prototype Solution

The 'implementation tetrahedron' of a prototype solution generalises the 'prototype facet' of O/K model [9]. This tetrahedron helps to better specify the evolution of some possible kinds of activities of a 'prototype solution' development, as conducted by the OR/DA worker in relation to the problem conception of the client, and the expected implementation of such a solution within an organisational context (*CPS* and *CM*).

A prototype solution implementation, therefore, principally refers to the analyst's problem situation (*APS*), and is based on the following *'quartet' of basic elements*: *APS, Data Validity (DV), Formal Model (FM),* and *Solution (PS).* These elements have strong 'internal' and 'external' interrelations with the *CPS* and *CM* elements, which have a 'control' function on the solution/implementation process. This quartet can evolve during the process so as to permit the modeller/implementer to obtain or preserve some validation kinds, as summarised in Figure 5.

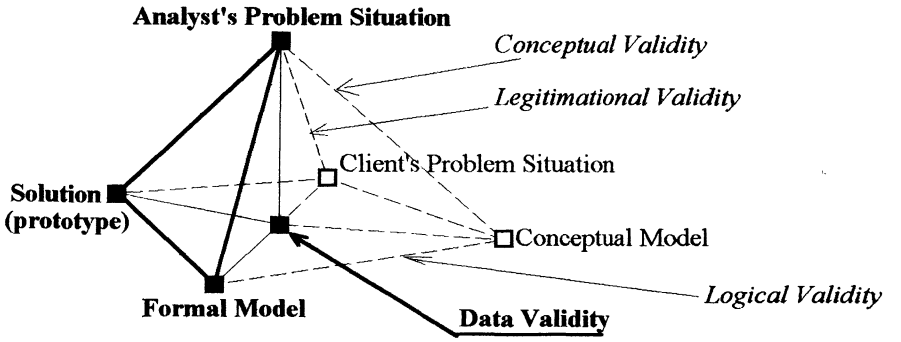

Figure 5: Tetrahedron of a prototype implementation/validation

A *prototype implementation (sub)process* develops regarding a 'well conceived' problem, as much as the available knowledge and information permit. A 'valid' conceptual model is therefore available, in principle, and little or no additional conceptualisation effort is required at this stage. *CM* constitutes a source of information elements which permits the (re)development(s) of formal model(s) and the selection of solution approaches, and is also a source of 'constraints' to keep such a development(s) within the terms of validity criteria. An initial formal model (*FM*) with logical validity is also available. One of the principal goals of this validation stage consists of obtaining a stable level of validity for such a model or for some other re-formulation. Solution techniques and implementation procedures are particularly important at this stage, as is the role of data validity, especially as far as the internal and external/technical appropriateness are concerned.

Four validation 'facets' of *IPST* can be defined on the basis of relationships between the quartet basic elements; these are named: *Formulational validation* (Facet 1), *Implementational validation* (Facet 2), *Experimentational validation* (Facet 3), and *Operational validation* (Facet 4). These 'facets' are more than a generalisation of the three edges of the 'Prototype Facet' defined in Oral and Kettani's model, since they are based on different conceptual assumptions, and are here briefly analysed.

Formulational Validation Facet - The following principal types of validation micro-processes are associated to the edges of Facet 1, that is defined by the *APS, DV, FM* elements: *Formulational validation* (APS-FM edge), *Conceptual data validation* (APS-DV edge), *Operational data validation* (DV-FM edge). 'Formulational validation', as defined by Oral and Kettani [9], refers to the various tests that have to be continuously conducted by the modeller when the validity of a formal model under implementation/solution is the main problem. This process is based on activities connected to the possible iterating sub-processes of re-formulation, and requires an intensive application of criteria for data validation. The process therefore includes possible re-structuring of the available data or developments of different new models. The need to re-model some elements of representation might induce changes in *CM*. 'Formulational validation' also refers to the selection and use of appropriate and efficient solution techniques and effective validation procedures (Facet 2 - Tool validation). Conceptual and logical validity, as defined in paper [7], have the function of 'constraints' to control this stage of the process. One can therefore say that formulational validity is concerned with the capability degree of a formal model to 'correctly and accurately describe' a client's 'well conceived' problem (or some aspects of it). With complex problem situations, a formal model is but a rigorous and stable formulation of a mythical well conceptualised client's problem, [10].

Implementational Validation Facet - The principal types of validation micro-processes associated to the edges of this Facet, defined by the *FM, APS, PS* basic elements, are identified as: *Formulational validation* (APS-FM edge), *Operational/experimental validation* (APS-PS edge), and *Tool validation* (FM-PS edge). 'Implementational validation' principally refers to the appropriateness and efficiency of the applied techniques or solution tools, and also to the

effectiveness of the implemented procedures. Formal model(s) might have to be re-modelled, at this stage, for a 're-solution'. This depends on the level of operational validity the analyst assigns to the solutions. A large amount of literature is devoted to these aspects of validation (for instance, papers [13] and [14]). 'Implementational validation' is especially important if a hybrid (or mixed) procedure has to be applied to find a solution to a client's problem; in such cases, different problems have to be formalised 'in cascade' and solved with different tools, and validation tests have to be conducted in terms of both internal and external appropriateness.

Experimental Validation Facet - Formulational/operational validation (*DV-FM* edge), *Experimental validation* (*DV-PS* edge), and *Tool validation* (*FM-PS* edge) are the principal types of validation micro-processes associated to the 'edges' of Facet 3, which is defined by the *DV, FM* and *PS* elements. 'Experimental validation' is the characterising issue of this process, which is primarily concerned with the input and output of tool applications. Various factors enter such a validation dimension; these may refer, for instance to the *types of solutions,* which depend on the applied solution techniques, and the *nature* of these solutions, which may be descriptive, explicative, prescriptive, explicit (direct) or implicit (indirect), partial or global [9]. The *quality of solutions* is also an important validity aspect, which has recently often been used. This refers to criteria such as: *meaningfulness* (the level of insight gained into the problem situation), *aptness* (the acceptability levels from different possible points of view), *applicability* (the usability level(s) for further developments of the process), *usefulness* (the level of cognitive/operative contribution to support a decision), *robustness* (the sensitivity level to changes in the values of some 'parameters' of a model, or capability to fit into different possible scenarios of C's problem situation included in *CM*), and many others (cf [9]). *Efficiency of solution techniques* is another aspect that is usually considered in experimental validation and can be evaluated in terms of various criteria, such as: *efforts* to make the technique operative, *time* required to obtain a solution of some expected kind, or *cost.* Experimental validity cannot be evoked as a process criterion, if there is no way to obtain solutions from a valid formal model. This implies, in real MCDA cases, that either an available *FM* might have to be modified, so as to facilitate the achievement of a solution using an available efficient/effective tool, or some other solution technique(s) might have to be selected or developed with the aim of 'solving' an available model that has been validated during other process stages. This establishes a connection with logical validity, i.e. the capability of capturing the richness and entirety of a *CM* in a formal representation, and underlines the non-permanent 'stability status' of some validity aspects, obtained at previous stages of the process. The fact that a *FM* has a high level of logical validity does not therefore necessarily imply that it is readily amenable to provide answers to the client/decision-maker's questions.

Operational Validation Facet - The principal types of validation micro-processes associated to Facet 4, defined by the *PS, APS, DV* elements, are: *Operational validation* (*APS-PS* edge), *Logical data validation* (*APS-DV* edge), and *Experimental validation* (*DV-PS* edge). 'Operational validation' deals with

aspects similar to those considered by experimental validation, but refers to a different stage of the analyst/modeller's decision process and has to evaluate the result(s) of his work ('the prototype solution') in relation to the points of view of the client's problem situation or of some other actors or recipients. Criteria relating to the quality of the used data and solutions are especially activated in this situation, and establish connections with other facets of *IPST* or with other tetrahedrons of MVP.

5 Concluding Remarks

Validation has been presented as a multistage and multicriteria decision process, within a MCDA context. This conception of modelling/validation is especially important for complex problems that forcefully take the OR/DA worker into multiactorial contexts. The validation process has therefore to be conceived from a multiactorial and multicriteria perspective, so as to permit the analyst/modeller to constantly evaluate his work throughout the process, with reference to contexts of different kinds, such as technical/operational or socio/organisational, and 'objects' of various types, and therefore to validly manage the evolving of the modelling process with control and planning activities. A global general view of validation cannot be given. Some different methodological approaches may be conceived, each of which can refer to specific assumptions. These necessary validation methodologies have to be considered as operational tools to support the OR/DA worker's orientation within the complexity of a client's context situation. Several trade-offs and compromises might have to be made between the levels of different types of validity. The ultimate judgements have to be based on the modelled situation with all the considered complexities, and also on the way the work has been developed within or in relation to the actorial context which is expected to 'accept the prototype solution' or 'developed action'.

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II

Aggregating Preferences: Outranking and Utility Function Approaches

An invited leading paper by Martel presents an introduction to the subject of aggregating preferences. In this paper Martel makes a review of distinct multicriteria aggregation procedures, as a means to aggregate partial preferences to build a global preference structure for "solving" the problem. The author considers the approaches based on: a single criterion synthesis (utility theory), an outranking synthesis and a local interactive judgement with trial-and-error iterations. In the utility function approach MAUT and AHP are considered.

Czyzak and Slowinski present a multi-criteria ranking method based on concordance-discordance approach to construct a valued outranking relation, in the case alternatives are evaluated using criteria assuming fuzzy values. The use of pseudo-criteria, proposed for Electre III method, is replaced by a comparison of fuzzy performances. Four different measures based on possibility and necessity concepts are applied to compare the fuzzy numbers. Then, one aggregated index is build, and a valued outranking relation is constructed and exploited using the usual distillations procedure of Electre III. A final partial preorder results from the exploitation procedure.

Stam and Silva introduce a nonlinear, multiplicative functional form for combining the ratio-scale ratings to determine the alternatives' overall priority ratings on a set of relevant criteria. Depending on the preference information provided by the decision maker, this procedure can yield overall priority ratings that reflect situations of decreasing, constant or increasing marginal contributions to the achievement of the overall goal. They claim that the method precludes the occurrence of rank reversal under any condition.

The paper by Ozernoy addresses several general concerns and complex features of the problem of selecting the the most appropriate MCDM method in a particular decision context. It argues that discrete alternative MCDM methods have inherent limitations, as the validity of a method cannot be established formally. An approach for substantiating MCDM methods is suggested, and some directions for future research are outlined.

Derot, Gareau, Kiss and Martel present the objectives of the VOLVOX (a consortium composed of six leading canadian firms in the field of information technology and decision making aid) research center. The VOLVOX multicriterion table is an element of a generic shell under development, which allows ordinal or cardinal, punctual or distributional evaluations. The SOLVER (multicriterion aggregation procedure) is based on the outranking synthesis approach and lies within the scope of the ranking problematic.

Aggregating Preferences: Utility Function and Outranking Approaches

Jean M. Martel

Faculté des Sciences de l'administration
Université Laval, Québec
Canada G1K 7P4

1. Introduction

A multicriterion analysis with the aim of sheding light on a decision comprises, as any decision process, several steps. The first steps concerning the definition and structuring the decision problem take shape in a multicriterion analysis by the definition of the set A of the potential actions (feasible alternatives) and of the set A/F of the attributes or criteria from which actions are evaluated.

While the first set can, in any case, be identified without too much difficulty, this is generally not the case for the second one whose construction may require a rather important thinking and structuration effort. Two types of approaches have been proposed to succeed in defining this second set: a "top-down" approach consisting in the organization of a hierarchy of a single point of view (single objective) splits up into sub-objectives, then in measurable attributes (Keeney and Raiffa, 1976; Saaty, 1980). This approach is advocated by those who belong to what is agreed to be called the American school of the MCDM. The other approach "bottom-up" consists in structuring consequences into axes of meanings (operationalization of points of view) from which are built criteria (Roy, 1985). This second approach is specific to the Francophone European school of the MCDA. According to Munda (1993), the criterion is the basis for evaluation: it is a function that associates each action to a description (any real and complete number, or a fuzzy number on a probabilistic[1] distribution of numbers) which indicates its desirability according to consequences related to the same point of view.

We then define the set E of the evaluations (performance, impacts) of actions $a_i \in A$ compared to each attribute or criterion j ($X_j/g_j \in A/F$). When A is finite and explicitly defined (which is the case for the described methods in this chapter), these three sets make up the decision matrix or the table of performance (Table 1).

In the terminologly introduced by Vansnick (1990), this type of structuring can be defined as a "model A.A/F.E" (Alternatives, Attributes/Criteria, Evaluations). Although discrete multicriteria methods use this model as a basis, other models exist for structuring a decision problem: for instance the Analytic Hierarchy Process (Saaty, 1980).

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[1] Term not specifically used by Munda

		A/F				
		X_1/g_1	...	X_j/g_j	...	X_n/g_n
	a_1			.		
	:			:		
A	a_i	...	...	$X_j(a_i)/g_j(a_i)$	...	...
	:			:		
	a_m			.		

Table 1: Table of performance

Undoubtedly, the most delicate step in this structure is to build a consistent family of criteria, i.e. a set of criteria representing all different points of view of the problem at hand while avoiding redundancies (Vincke, 1992). Moreover, according to Bouyssou (1990), a set of criteria should have two important qualities:

1) the "legibility" i.e the family should contain a sufficiently small number of criteria to serve as a discussion basis providing the analyst with inter-criteria information necessary for the implementation of an aggregation procedure,

2) the "operationality" i.e. the family should be considered by all actors as a sound basis for the continuation of the decision-aid study.

This structuring has much more chances to be realistic when different stakeholders to the decision problem participate in concrete terms to its making (Banville et al., 1993; Keeney, 1992); its legitimacy often depends on it. Whatever, in general, the construction of the table of performance (pay off matrix, impact matrix) cannot solve the multicriterion decision problem; it only sets up the problem.

Therefore, the multicriteria aggregation procedures (MCAP) indicate how to aggregate partial preferences (i.e. preferences restricted to the aspects taken into account in the point of view underlying the definition of the criterion (Bouyssou, 1990) to build a global preferences for "solving" the problem. At this level the difficulty comes from the fact that criteria are generally conflicting (characteristic of a multicriterion problem) and heterogeneous. Although a good number of MCAP came into being during these 25 years, none is perfect and quite suitable for all situations found in practice (Schärlig, 1985).

Most of the MCAP lie within the framework of one or another of the three following operational approaches (Roy and Bouyssou, 1993):

1) Approach of a single criterion synthesis (utility theory);
2) approach of an outranking synthesis;
3) approach of a local interactive judgment with trial-error iterations.

As procedures lying within the third approach developped mainly in the frame of the multiple objective mathematical programming (MOMP), it will not be discussed in this chapter that focuses on some discrete multicriteria methods.

There is a fundamental difference between the aggregation procedures that belong to the two first approaches: complete vs partial aggregation procedures. In the following sections we briefly present these two kinds of aggregating preferences.

2. The utility function approach

2.1 The multiple attribute utility theory (MAUT)

The theory behind this approach is based on the hypothesis of existence of a real valued function U defined on the set A of feasible alternatives the decision-maker wishes, consciously or not, to optimize. This function aggregates the different attributes taken into consideration, so that the problem can be formulated as:

$$\max U(X_j(a)) : a \in A$$

where U $(X_j(a))$ is a utility function aggregating the n attributes. Thus, the multicriterion problem is then replaced by a monocriterion one.

The main difficulty with this approach is to estimate this function U. Although in principle nothing prevents a decision-maker "to make holistic judgments over multi-attribute outcomes" (Slovic and Lichtenstein, 1971), it is generally more realistic to try to factorize this function U into a partial utility function. This sends word to Vincke (1992) that essentially two types of problems are studied in the frame of the MAUT:

1) What properties must the decision-maker's preferences fulfill in order to be able to represent them by a function U with a given analytical form?

2) How can such functions be built and how can parameters pertaining to the chosen analytical form be estimated?

The most usual analytical forms are the additive (linear) and the multiplicative form:

$$U(a) = \sum k_j U_j(X_j(a)) \quad \textit{(additive form)}$$

$$U(a) = \frac{1}{K}\left\{\prod\left[1 + Kk_jU_j(X_j(a)\right] - 1\right\} \quad \textit{if} \quad \sum k_j \neq 1. \quad \textit{(multiplicative form)}$$

where k_j is scaling constant $0 < k_j < 1$, $K > -1$ is a non-zero constant and U_j are strictly increasing functions scaled from zero to one. These two analytical forms require

preferential and utility independance conditions (Fishburn, 1970; Keeney and Raiffa, 1976).

There exist several direct methods to build these analytical forms (Fishburn, 1967; Farguhar, 1984); for instance SMART (simple multiattribute rating technique) method due to von Winterfeldt and Edwards (1986) for the additive form. Among indirect methods, there is UTA (utility additive) method due to Jacquet-Lagrèze and Siskos (1982) using as input data the table of performance and preference order of the alternatives (holistic judgments on some alternatives). An ordinal regression model based on a linear programming formulation enables the estimation of the parameters of an additive utility function with linear piecewise marginal utility functions.

The MAUT is based on a complete transitive comparability axiom, preferences are modelled by means of two binary relations I (indifference: reflexive, symmetric and transitive) and P (strict preference: irreflexive, asymmetric and transitive). This method allows complete compensation among all the attributes and is rather demanding on the level of information the decision-maker must provide. MAUT has been developped in a probabilistic context, i.e. at least one attribute is viewed as a random variable.

2.2 The Analytic Hierarchy Process (AHP)

The AHP method (Saaty, 1977) is based on a set of axioms proposed by Saaty (1986). These axioms describe basic tasks in the AHP: formulating and structuring problem as a hierarchy, and eliciting judgments in the form of pairwise comparisons (Harker, 1989). The top level of the hierarchy represents the overall objective, the last level enumerates the m alternatives under consideration and the intermediary levels are for elements (attributes/criteria and sub-attributes/sub-criteria) to be considered in satisfying the overall objective.

At each level of the hierarchy, a pairwise comparison of the elements (attributes/criteria, alternatives; depending on the level within the hierarchy) is performed from the point of view of their contribution (importance, preference) to each of the higher-level elements to which they are linked. The pairwise comparisons are captured on a scale (ratio scale) proposed within the method (a semantic scale to which is associated a numerical 9-point scale). Thus, this elicitation of contribution for a given set of n elements under a higher-level element to which they are linked involves the completion of a n x n matrix. In fact, since the comparisons are assumed to be reciprocal, the decision-maker needs only to answer n(n-1)/2 comparisons. Saaty proposed an eigenvector approach for the estimation of the relative contribution of each element (priority vector) in this reciprocal matrix.

The AHP method does not require the decision-maker to be perfectly consistent (the priority ratio between e_1, and e_3 has not to be exactly equal to the product of the priority ratio between e_1, and e_2 by that between e_2 and e_3) but is rather used to provide a measure of this inconsistency.

When each element of the hierarchy has been evaluated from the point of view of its contribution to the immediate higher-level elements to which they are linked, the overall priority (contribution) of each alternative is calculated by means of a linear additive function (weighted average type).

Recently, this method has also been elaborated in an uncertainty context (Paulson and Zahir, 1994). It is a very popular method and brings up a large amount of comments (Belton, 1986; Dyer, 1990; Holder, 1990; Perez, 1995; ...). Among these comments we find those linked: to the numerical linear scale associated to the semantic scale; to the lack of a point of reference; to the quantification of judgments from the method of Perron-Frobenius' proper vectors and to rank inversion.

Before ending this section on the utility function approach, we want to add that some MCAP lying in this type of approach have been developped to deal with mixed evaluations (qualitative and quantitative): REGIME method and EVAMIX procedure (Nijkamp et al., 1990).

3. The outranking approach

This approach is based on a fundamental partial comparability axiom where incomparability is a key concept. The outranking approach consists first in building on the set of potential alternatives, one (or several) outranking relation(s) to represent the decision-maker's preferences. The second step is the exploitation of this (or these) outranking relation(s) in order to construct the prescription according to a specific problem statement (Vanderpooten, 1990). In an utility function approach, the prescription is immediately derived from the aggregation process. However, when the aggregation process is based on the outranking approach, additionnal treatments are required to construct the prescription.

An outranking relation is a binary relation S defined on A so that a S b (a, b $\in$ A) if, given what is known about the decision-maker's preferences and given the quality of the valuations of the alternatives and the nature of the problem, there are enough arguments to decide that "a is at least as good as b", while there is no essential reason to refute that statement (Roy,1974). The idea behind this relation is that the enrichment of the dominance relation (too poor to be useful because it is based on the unanimity of points of view) can only be done when realistic information is available. With such a relation some incomparable actions (according to a dominance relation) become comparable because realistic information exists, but other actions remain incomparable unlike MAUT and AHP. This relation is not necessarily transitive nor complete.

Because of the semantic content of an outranking relation, the statement a S b covers situations from indifference (a I b, where I is reflexive and symmetric) to strict preference (a P b, where P is irreflexive and antisymmetric) going through weak preference (a Q b, where Q is irreflexive and antisymetric) for an hesitation between a I b and a P b, being sure that not b P a. The incomparability (a R b, where R is irreflexive and symmetric) is observed when the decision-maker hesitates between a P b and b P a (not a S b and not b S a).

The MCAP which have been proposed in the spirit of the outranking approach differ, among other aspects, in the way they build the outranking relations. A common property of these procedures is that they are completely or partially non-compensatory.

In the ELECTRE methods (developped by B. Roy and his collaborators) the statement a S b is accepted if, when comparing a with b, the two following tests are verified: a

concordance test which ensures that a majority of criteria are in agreement with a S b (majority principle) and a non discordance test which ensures that none of the discordant criteria strongly refutes a S b. The precise implementation of these tests is described in Roy and Bouyssou, 1993. These tests can be expressed in a more or less demanding way which results in a more or less strongly established outranking relations (for example ELECTRE I, II and III). In ELECTRE I and II, an absolute discrimination power is allowed to criteria (concept of true-criteria) while in ELECTRE III we use the concept of pseudo-criteria (i.e. criteria to which two thresholds q_j (indifference threshold) and p_j (strict preference threshold) have been assigned; these thresholds are not necessarily constant. In ELECTRE I and II, one or two crisp outranking relation are defined while in ELECTRE III, we define a fuzzy outranking relation. The tests are based on the performance vectors of the two actions, the discrimination thresholds and the importance of the criteria coefficients (except in ELECTRE IV). The importance of a criterion is taken into account by means of these coefficients and the veto threshold.

The exploitation of the outranking relations in regard to the chosen statement of the problem is described in Roy and Bouyssou (1993) and Vanderpooten (1990).

Among the other MCAP belonging to the outranking approach, there are the methods of PROMETHEE (Brans et al., 1986); QUALIFLEX (Paelinck, 1979); ORESTE (Roubens, 1981); and MELCHIOR (Leclercq, 1984) developped for qualitative multicriteria evaluation. For taking into account incertainty, imprecision and inaccurate determination, probability distributions (Martel and Zaras, 1995; Martel et al., 1991; Martel et al., 1986), fuzzy numbers (Munda, 1995) and/or different types of thresholds (like in ELECTRE III) can be used.

4. Contributions within this chapter

This chapter contains four papers. The first one by Czyzak and Slowinski belongs to the outranking approach. Uncertainty and imprecision are taken into account by fuzzy numbers to which four different measures are applied to build one aggregated index. Thus a valued outranking relation is constructed and exploited using the usual distillation procedure of ELECTRE III.

The second paper by Stam and Silva belongs to the utility function approach. They introduce a nonlinear, multiplicative functional form by combining the ratio-scale ratings to determine their overall priority ratings; these overall priorities maintain the ratio-scale property, even after normalization. Another attractive property of this method is that it precludes the occurrence of rank reversal.

The third paper by Derot et al. refers to the outranking approach. They propose a MCAP having several "functionalities" i.e. a MCAP based on ELECTRE and PROMETHEE that can face situations with qualitative, quantitative, distributional and missing evaluations.

The last paper in this chapter by Ozernoy belongs to two approaches and concerns the selection of the most appropriate MCAP for a particuliar decision problem.

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A Concordance-Discordance Approach to Multi-Criteria Ranking of Actions with Fuzzy Evaluations

Piotr Czyżak[1], Roman Słowiński[1]

[1]Institute of Computing Science
Poznań University of Technology
60-965 Poznań, Poland

Abstract: A multi-criteria ranking method based on concordance-discordance approach to construct a valued outranking relation, in the case when actions are evaluated using criteria taking fuzzy values, is presented. The use of pseudo-criteria, proposed for Electre III method, is replaced by a comparison of fuzzy performances. Four different measures based on possibility and necessity concepts are applied to compare the fuzzy numbers. Then, one aggregated index is build, on which a new definition of both concordance and discordance relies. Finally, a valued outranking relation is constructed and exploited using the usual distillation procedure of Electre III. A final partial preorder results from the exploitation procedure. A practical example illustrates the method.

Keywords: multicriteria aid for decisions, fuzzy performances, outranking, application to water supply systems.

1.INTRODUCTION

Decision aid intends to support the decision maker (DM) in making "good" decisions. It means that methods of decision aid serve to recommend some decisions that fit the DM's preferences. In practice, alternative decisions (shortly, alternatives) are usually evaluated from different points of view, which correspond to different criteria. Moreover, in real-life situations, evaluations are neither certain nor precise. There are three main sources of this uncertainty [4]:

- imprecision, because of the difficulty of determining the scores of alternatives on particular criteria,
- indetermination, since the method of evaluation results from a relatively arbitrary choice between several possible definitions,
- uncertainty, since the values involved vary over time.

A known way of handling these phenomena in multi-criteria decision aid consists in using the concept of pseudo-criterion [3,5]. It was used in the methods of ELECTRE family for different multi-criteria problem formulations: choice, sorting and ranking. The model of a pseudo-criterion generalizes that of the true-criterion by distinguishing:

- a zone of indifference defined by an indifference threshold,
- two zones of strict preference, corresponding to a difference whose absolute value is greater than a preference threshold,

- two intermediate zones representing a certain hesitation between indifference and strict preference, and corresponding to an attitude called weak preference.

Although distinguishing of these zones seems rational, experience indicates that somehow "abstract" definition of indifference and preference thresholds, especially in their functional form, may pose some problems to the DM. Being aware of the above mentioned uncertainty, the DM is sometimes less reluctant to affect this uncertainty directly to the scores of alternatives on particular criteria. For the modelling of uncertainty having rather epistemic character, fuzzy sets are particularly well suited.

The aim of this paper is to propose a multicriteria ranking method which allows the uncertain evaluation of alternatives in terms of fuzzy performances on particular criteria. It relies on the use of an outranking relation constructed via concordance and discordance tests. Its scheme is similar to ELECTRE III method, however, existence of fuzzy performances requires the use of some comparison techniques for fuzzy numbers.

In the next section, the principles of concordance and discordance tests are described. In the third section, the methodology of building a valued outranking relation is explained and in the last section an application of the proposed method to the decision aid at the stage of programming water supply systems for rural areas is presented.

2.BUILDING OF THE CONCORDANCE-DISCORDANCE INDICES

Let A=$\{a,b,...\}$ be a finite set of alternatives, $\tilde{g}_j$ - fuzzy criterion, w_j - weight of criterion j (j=1,...,k). We assume that preference of a is increasing with the value of $\tilde{g}_j(a)$.

The ranking of alternatives must take into account:

- their fuzzy performances on all criteria,
- the weight attached to each criterion,
- the possible difficulties of comparing two alternatives when one is significantly better than the other on a subset of criteria, but much worse on at least one criterion from the complementary subset.

For each pair of alternatives $a,b \in$ A, the credibility index d(a,b) is calculated, which expresses the credibility of the affirmation: "a is at least as good as b", which is usually stated as: "a outranks b" or aSb. The formula for calculation of d(a,b) is based on two concepts: concordance and discordance. They characterize, respectively [3]:

- a group of criteria considered to be in concordance with the affirmation being studied, and assess the relative importance of this group of criteria compared with the remainder of the criteria; and

- among the criteria which are in discordance with the affirmation being studied, the ones whose opposition is strong enough to reduce the credibility which would result from taking into account just the concordance, and to calculate the possible reduction in it that would thereby result.

In the case of fuzzy performances, concordance and discordance tests involve comparisons of fuzzy numbers. Using the terms of possibility theory, in order to determine whether fuzzy number $\tilde{g}_j(a)$ is greater than $\tilde{g}_j(b)$ it is recommended to apply four indices of comparison [1]. They express the possibility $\mathrm{PS}(\tilde{g}_j(a) \geq \tilde{g}_j(b))$ and $\mathrm{Ps}(\tilde{g}_j(a) \geq \tilde{g}_j(b))$, and the necessity $\mathrm{NC}(\tilde{g}_j(a) \geq \tilde{g}_j(b))$ and $\mathrm{Nc}(\tilde{g}_j(a) \geq \tilde{g}_j(b))$ that the assertion: "fuzzy score $\tilde{g}_j(a)$ is better than fuzzy score $\tilde{g}_j(b)$" is true.

Formally, they can be defined as follows:

$$Ps(\tilde{g}_j(a) \geq \tilde{g}_j(b)) = \sup_{u \geq v} \min(\mu_{\tilde{g}_j(a)}(u), \mu_{\tilde{g}_j(b)}(v))$$

$$PS(\tilde{g}_j(a) \geq \tilde{g}_j(b)) = \sup_{u} \inf_{v \geq u} \min(\mu_{\tilde{g}_j(a)}(u), 1 - \mu_{\tilde{g}_j(b)}(v))$$

$$Nc(\tilde{g}_j(a) \geq \tilde{g}_j(b)) = \inf_{u} \sup_{v \leq u} \max(1 - \mu_{\tilde{g}_j(a)}(u), \mu_{\tilde{g}_j(b)}(v))$$

$$NC(\tilde{g}_j(a) \geq \tilde{g}_j(b)) = 1 - \sup_{v \geq u} (\mu_{\tilde{g}_j(a)}(u), \mu_{\tilde{g}_j(b)}(v)).$$

The indices of comparison take the values from the interval [0,1].

A partial concordance index $C_j(a,b)$ can be defined as a result of aggregation of the above possibility and necessity measures. In the simplest case, the aggregation may be arithmetic one:

$$C_j(a,b) = \frac{1}{4}(\mathrm{PS}(\tilde{g}_j(a) \geq \tilde{g}_j(b)) + \mathrm{Ps}(\tilde{g}_j(a) \geq \tilde{g}_j(b)) + \mathrm{NC}(\tilde{g}_j(a) \geq \tilde{g}_j(b)) + \mathrm{Nc}(\tilde{g}_j(a) \geq \tilde{g}_j(b))) \quad (1)$$

$C_j(a,b) \in [0,1]$ and has to be calculated for each pair of alternatives $(a,b) \in A$ and for $j=1,\ldots,k$ (cf. Fig.1).

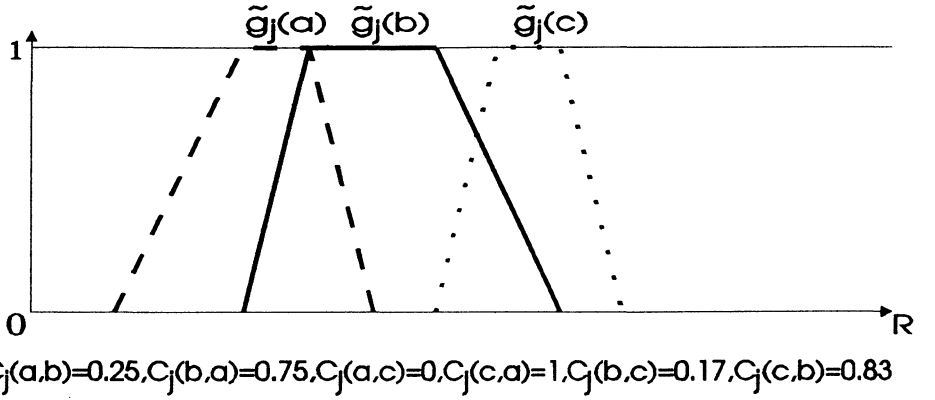


Fig.1.C_j for fuzzy performances of alternatives *a,b,c.*

The comprehensive concordance index $C(a,b)$ is obtained by the following weighted-sum aggregation of $C_j(a,b)$:

$$C(a,b) = \frac{\sum_{j=1}^{k} w_j C_j(a,b)}{\sum_{j=1}^{k} w_j}, \qquad (2)$$

where w_j is a relative weight assigned to criterion j by the DM.

To build a discordance index, the philosophy of veto preference is applied. Veto preference gives criterion j the power to take all credibility away from the affirmation being studied even when opposed to all the other criteria in concordance with the affirmation. Veto preference $b\mathrm{PV}_j a$ takes place when the performance of b is better than the performance of a on criterion j even if the performance of a is augmented by v_j, called a veto threshold of criterion j (Fig.2).

According to this, the discordance index can be calculated as:

$$D_j(a,b) = \frac{1}{4}(\mathrm{PS}(\tilde{g}_j(a) \geq \tilde{g}_j(b) + v_j) + \mathrm{Ps}(\tilde{g}_j(a) \geq \tilde{g}_j(b) + v_j) + \qquad (3)$$
$$+ \mathrm{NC}(\tilde{g}_j(a) \geq \tilde{g}_j(b) + v_j) + \mathrm{Nc}(\tilde{g}_j(a) \geq \tilde{g}_j(b) + v_j).$$

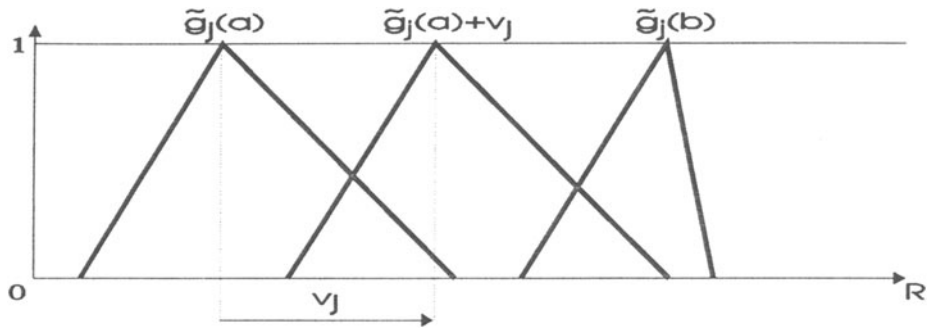


Fig. 2. Interpretation of the veto preference.

3. VALUED OUTRANKING RELATION AND FINAL PREORDER

The degree of credibility $d(a,b)$ of the outranking relation aSb is obtained from the comprehensive concordance index (2) weakened by discordance indices (3) (up to the points of its annulment).

Let $F(a,b)=\{j : D_j(a,b)>C(a,b), j=1,...,k\}$. Then

$$d(a,b)=C(a,b), \qquad \text{if } F(a,b)=\varnothing$$

$$d(a,b)=C(a,b)*\prod_{j\in F}\frac{1-D_j(a,b)}{1-C(a,b)}, \qquad \text{if } F(a,b)\neq 0$$

$$d(a,b)=0, \qquad \text{if } \exists j : D_j(a,b)=1.$$

Let us notice that due to the coherence of this approach with the original scheme of ELECTRE III, a joint consideration of fuzzy criteria and pseudo-criteria is also possible within the same scheme.

The information synthesized in the credibility index $d(a,b)$, can be exploited in the same way as in ELECTRE III. Let us recall that ELECTRE III does not necessarily lead to a complete preorder of alternatives, but, more generally, to a partial preorder derived from the valued outranking relation. In partial preorder some alternatives may not be related, i.e. they remain incomparable. The fact that, in certain cases, it is impossible to come to conclusions about the relation between alternatives is not a weakness of the method. It is a natural consequence of not only the imperfect nature of the data (which must not be made to express more information than they contain), but also of the precautions taken in aggregating the criteria (precautions taken with the purpose of avoiding expressing unjustified comprehensive preferences.)

This final preorder is a set-theoretic intersection of two complete preorders. These complete preorders are constructed by using a technique called 'distillation' to process, in two different ways, the information synthesized in the numbers $d(a,b)$. The two methods differ with regard to the essential

problem: where to put an alternative who is difficult to compare with one or more of the others.

The partial preorder remains nevertheless dependent on the fuzziness affected to the performances, on veto thresholds and on the weights. Let us precise that the fuzzy performances are given by the domain expert who takes into account the uncertainty inherent to evaluation of alternatives. The veto thresholds and the weights constitute a preferencial information acquired from the decision maker. In the definition of all these data there is an element of arbitrariness, thus a thorough robustness analysis is recommended to bring out which order of alternatives is convincingly justified by the model.

4.APPLICATION TO THE GROUNDWATER MANAGEMENT SYSTEM

Let us consider the groundwater management system described in [2]. On a circular island there are some groundwater resources, accumulated in a single water layer, which are to be used for water supply of a distribution center (e.g. a town) located in the middle of the island. The management problem consists in finding the location of the wells and corresponding pumping rates, satysfying the groundwater flow law (the hydrodynamic model), all the constraints on the hydraulic heads and the minimal demand at the water distribution center. Moreover, the solution should reflect the DM's preferences and should yield the best compromise among several criteria.

Set A of 13 potential locations of water supply wells, denoted by a1,a2,...,a13, is considered.

To evaluate their quality, three criteria are taken into account:

g_1 - the total pumping rates over the aquifer;

g_2- operating costs including water pumping and water transport costs;

g_3 - risk interpreted as the possible fluctuation in water quantity at the exploitation stage.

The first objective is maximized while two others are minimized.

Because the groundwater management problem involves long-term planning, the performances presented in Table 1 have only an approximative character. In practice, all these performances are uncertain. Let us assume that, using the past experience, the experts can estimate some ranges of variation of these performances together with possibility distribution within them.

So, the performances will be fuzzy numbers $\tilde{g}_i(a)$ for $a \in A$ and i=1,...,3. Triangular form of all fuzzy numbers is assumed, i.e. number $\tilde{g}_i(a)$ is a triple $(g_i(a),a_i(a),b_i(a))$ where $g_i(a)$ is the "most possible" value, $a_i(a)$ and $b_i(a)$ are nonnegative left and right "spreads" of $\tilde{g}_i(a)$.

The fuzzy performances of alternatives are defined as follows:

- the "most possible" values of fuzzy performances are equal to the values defined in [2] (cf. Tab.1),
- the left and right spreads depend on criteria in the following way:

- for total pumping rate they are constant for each alternative and equal to 2;
- for costs they are equal to 5% of the "most possible" value of each performance;
- for risk they are equal to 10% of the "most possible" value of each performance plus 0.2.

Table 1. The "most possible" values of fuzzy performances for potential water supply wells.

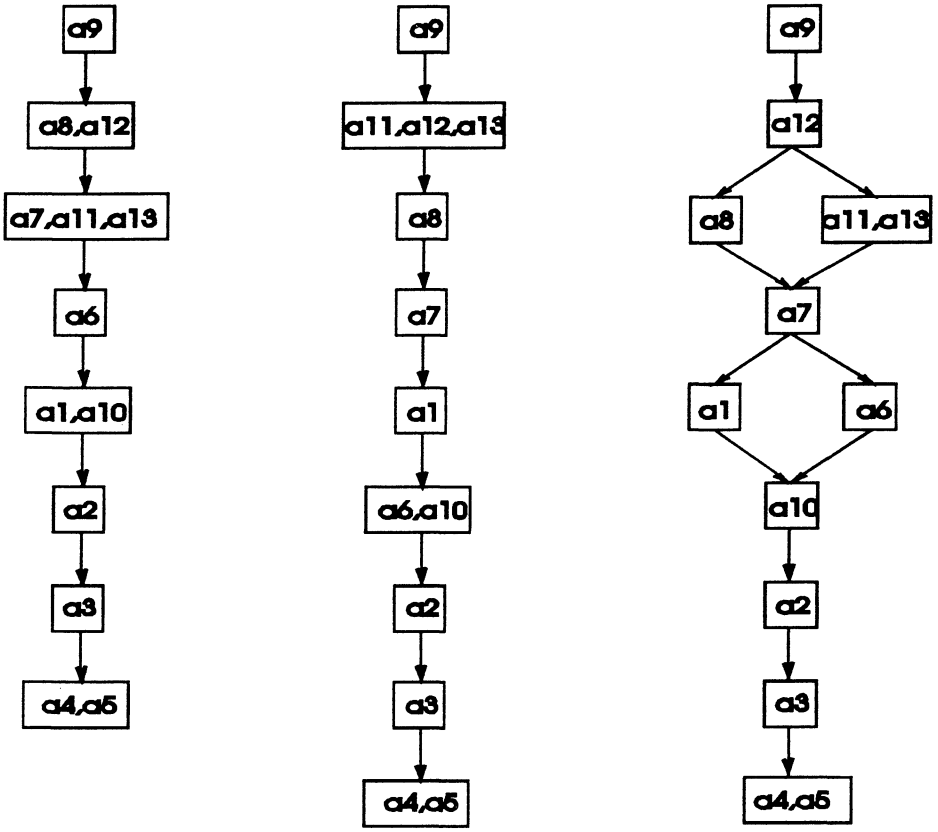
Alternative	Pumping rate	Costs	Risk
a1	32.1	23.8	2.51
a2	36.8	32.1	3.43
a3	41.3	40.3	4.35
a4	44.9	48.6	5.27
a5	44.9	56.8	6.19
a6	27.2	15.5	3.43
a7	22.3	8.1	3.22
a8	17.4	4.5	2.80
a9	12.5	2.3	2.09
a10	27.2	27.3	0.49
a11	22.3	19.4	0.23
a12	17.4	16.9	0.17
a13	12.5	18.7	0.12

Moreover, the veto preference thresholds are different for each criterion and two of them are given in a functional form:

$$v_1=35,\ v_2=0.6g_2(a)+6,\ v_3=0.5g_3(a)+3 \text{ for } a\in A.$$

Importance weights assigned by the DM to the criteria are $w_1=2$, $w_2=4$ and $w_3=3.5$.

Applying the proposed method to the above problem, we have got two complete preorders as a result of distillation and the final partial preorder being a set-theoretic intersection of the two complete preorders (Fig.3).



(a) (b) (c)

Fig.3. Descending (a) and ascending (b) preorders, and the final ranking (c).

In Tab.2 we present a comparison of results obtained by our method to the results obtained by methods: ELECTRE III, Multiattribute Utility Function (MUF) and UTA on the same problem.

As one can see, there are three alternatives (a9, a11, a12) selected as "good" ones by each method. Moreover, alternative a8 was selected as a "good" one by MUF, UTA and by our method. On the other hand, a5 has been chosen as a "poor" one by each method and, in the case of the methods MUF, UTA and our method, a3 and a4 are two others "poor".

Concluding, let us point out that the four different multicriteria methods that have been applied lead to similar solutions. It proves, first of all, that our method is not giving any surprising results. Secondly, that the use

of one of these techniques depends on the nature of the decision situation, i.e. the presence and the character of the uncertainty and imprecision of data and the form of preferential information acceptable for the DM.

Table 2. Comparison of the final rankings obtained by different methods.

Class	Rank	ELEC-TRE III	MUF	UTA	Our method
good	1	a11,a9	a9	a9	a9
	2		a8	a8	a12
	3	a2,a12	a11	a12	
	4		a12	a11	a8,a11,a13
medium	5	a3,a8	a10	a13	a7
	6		a7	a7	
	7	a7	a1	a10	a1,a6
	8		a6	a1	
	9	a4,a6,a10	a13	a6	a10
	10		a2	a2	a2
poor	11	a1,a13	a3	a3	a3
	12		a4	a4	
	13	a5	a5	a5	a4,a5

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A Multiplicative Method for Synthesizing Preference Ratings

Antonie Stam[1]
A. Pedro Duarte Silva[2]

[1] Department of Management, Terry College of Business, The University of Georgia, Athens, GA 30602, U.S.A.
[2] Universidade Católica Portuguesa, Centro Regional do Porto, Curso de Administração e Gestão de Empresas, Rua Diogo Botelho 1327, 4100 Porto, Portugal

Abstract: Consider a discrete alternative multicriteria problem, in which the attractiveness of the alternatives is measured by absolute, ratio-scale preference ratings on a set of relevant criteria. We introduce a nonlinear, multiplicative functional form for combining the ratio-scale ratings to determine their overall priority ratings, which, depending on the preference information provided by the decision maker, can yield overall priority ratings that reflect situations of decreasing, constant or increasing marginal contributions to the achievement of the overall goal. Previous methods for combining priorities using a nonlinear functional form have either been based on interval-scale value function representations, or have assumed constant marginal contributions to the overall goal. As noted previously by other researchers, our method the overall priorities maintain the ratio-scale property, even after normalization, and precludes the occurrence of rank reversal under any condition. Our methodology includes the weighted geometric mean method for determining composite priority ratings as a special case.

1. INTRODUCTION

In this paper, we introduce a multiplicative method for synthesizing (combining) direct ratio-scale measurements (ratings) of discrete alternatives with respect to multiple criteria that extends previous multiplicative methods proposed by Barzilai and Golani (1994) and Lootsma (1994). One of the most widely used method for synthesizing ratio scale preferences is the Analytic Hierarchy Process (AHP) (Saaty 1980). However, the AHP normalization has been shown to be problematic (Dyer 1990; Schoner, Wedley and Choo 1992). In particular, Schoner, Wedley and Choo (1992) have shown that under most conditions the normalization used in the AHP leads to combined priority weights that violate the ratio-scale property.

In addition to the technical problems of this normalization, the linear pro-rating of the preference weights of the AHP, where the total weight associated with a given criterion level is distributed proportionally among the alternatives according to their achievement with respect to this criterion, may *not* accurately reflect the way in which the criterion levels in fact contribute to the achievement of the overall goal.

In multiattribute utility theory, the marginal contribution of an individual local criterion to the overall goal is frequently assumed to be a decreasing function of the level of achievement of this criterion. The hypothesis of decreasing marginal contributions has been verified and confirmed in many applications and experimental studies, and corresponds directly – in interval scale preference representation – to a value function (assuming one exists) which is convex with respect to the origin (Keeney and Raiffa 1976; Yu 1985). Similarly, increasing marginal contributions correspond to a concave value function, and constant marginal contributions imply a linear value function.

We develop a methodology which synthesizes the local ratings into global ratings by means of a multiplicative functional representation, and will formally show that the priority weights synthesized using our method can always be normalized without loss of the ratio scale property. Ríos Insua (1990) has shown that in the AHP nondominated alternatives that are dominated by linear combinations of other nondominated alternatives can never receive the highest rating. An interesting property of our method is that, due to its nonlinear synthesis method, the most highly rated solution is not limited to such solutions. In this paper, we limit ourselves to the case of absolute ratings, but the approach can easily be extended to the case of pairwise comparisons and inconsistent preference judgments.

2. A CLASS OF MULTIPLICATIVE SYNTHESIS FUNCTIONS

Suppose that the overall preference rating V (output) is determined as a function of n continuous direct ratings (inputs) a_j, with respect to the criteria C_j, $j = 1, \ldots, n$. We express the general form of the multiplicative function for combining the direct ratings as in (1):

$$V = \Pi_k \, a_k^{\beta_k}, \quad a_k > 0, \, \beta_k > 0, \, k = 1, \ldots, n. \tag{1}$$

Without loss of generality, we assume that $a_j > 0$ and $\beta_j > 0$, for each j, *i.e.*, the preference ratings are strictly positive, and each criterion does indeed contribute to the overall preference ratings. The function in (1) is similar to the Cobb-Douglas production function, which is widely used in macro-

economic modeling. The ratings a_j can be viewed as "inputs" and the overall rating V as "output." Theorem 1 high-lights some relevant characteristics of the way in which the input factors in (1) affect the value of total output.

Theorem 1: Assume the function in (1) describes the way in which the overall ratings V depend on the absolute ratings a_j. Then: (a) if the rating a_j of a single criterion C_j changes by a factor of ϕ_j, and the ratings a_m of all other local criteria C_m, $m \neq j$, remain unchanged, then V changes by a factor of $\phi_j^{\beta_j}$; (b) the marginal contribution of a_j to V is decreasing if $\beta_j < 1$, it is constant if $\beta_j = 1$, and it is increasing if $\beta_j > 1$; (c) if *all* a_j change *simultaneously* by ϕ percent, then V will change by $\phi^{\Sigma_k \beta_k}$ percent.

Proof: Suppose that the rating a_j of only one criterion, say C_j, changes, so that (1) can be written as $V = \theta a_j^{\beta_j}$, where $\theta > 0$ is comprised of those components in (1) involving a_m, $m \neq j$, which are unchanged. It is easy to see that V changes by $\phi_j^{\beta_j}$ percent if a_j changes by ϕ_j percent, which proves (a). The partial derivative of (1) with respect to a_j, $\partial V/\partial a_j = \theta \beta_j a_j^{\beta_j - 1}$, is strictly positive, so that V is always increasing in a_j. The second derivative, $\partial^2 V/\partial a_j^2 = \theta \beta_j (\beta_j - 1) a_j^{\beta_j - 2}$ is negative if and only if (iff) $\beta_j < 1$, implying that the *marginal* contribution of a_j is decreasing; it is zero iff $\beta_j = 1$, indicating a constant marginal contribution; and positive iff $\beta_j > 1$, in which case the marginal contribution of a_j is increasing as a function of V. This completes the proof of part (b).

In the more general case where the ratings with respect to more than one criterion may change simultaneously, let $a'_m = \phi_m a_m$, where $\phi_m > 0$, for each m. It follows from (1) that the revised rating V' equals $V' = \Pi_k (a'_k)^{\beta_k} = \Pi_k \phi_k^{\beta_k} a_k^{\beta_k}$. If each local rating changes by the same factor, so that $\phi_m = \phi > 0$, for all m, then $V' = \phi^{\Sigma_k \beta_k} V$, which completes the proof of part (c).

Returns to scale refer to the percentage change in overall ratings resulting from a certain percentage change in each of the priority ratings a_j. Denote $\Sigma_k \beta_k$ by $\Sigma_k \beta_k = \alpha$. Theorem 1 shows that, provided that the attributes are valued according to their marginal value, $\alpha > 1$ would imply that the overall ratings V will change by a higher proportion than the a_j. Similarly, $\alpha = 1$ would imply that a given change in each a_j will result in a proportional change in V, and higher ratings a_j will contribute less than proportionally to the overall rating if $\alpha < 1$. At this point, it is important to note that, while the marginal contribution of all criteria combined can exhibit constant returns to scale, individual criteria usually exhibit decreasing marginal contributions to the overall goal.

We believe that the multiplicative function in (1) is attractive in terms of synthesizing preference ratings, for several reasons. First, it allows us to analyze nonlinear preference structures. Second, as we will show below in Theorem 2, the preference ratings calculated using (1) are insensitive to scaling, and preserve the ratio scale property. In the case of constant marginal contributions to the overall goal, the latter property has been noted previously by Lootsma (1993) and Barzilai and Golani (1994).

3. SYNTHESIZING RATIO-SCALE PRIORITIES

Let the criterion set be denoted by $\mathcal{G} = \{C_1, ..., C_n\}$, and define the set of alternatives by $\mathcal{A} = \{A_1, ..., A_m\}$. Denote the direct preference ratings of A_i with respect to $C_k \in \mathcal{G}$ by a_{ik}, and let β_k be the weight which reflects the contribution of $C_k \in \mathcal{G}$ to the achievement of overall goal. In our nonlinear synthesis, v_i, the overall rating of A_i, is calculated as in (2):

$$v_i = \Pi_k \, a_{ik}^{\beta_k}, \qquad i = 1, ..., m. \tag{2}$$

It is easy to show that the weighted geometric mean is a *special case* within the general class of multiplicative synthesis functions in (2), with $\Sigma_k \beta_k = 1$. The β_k can be interpreted as the relative preference ratings of the criteria.

As noted above, in general the nature of the returns to scale depends on the magnitude of $\alpha = \Sigma_k \beta_k$. In the AHP, the corresponding sum $\Sigma_k w_k$ is normalized to unity. Previous formulations of the Multiplicative AHP (Barzilai and Golani 1994; Lootsma 1993, 1994) have also assumed $\alpha = 1$. Our proposed methodology is flexible, and allows for a normalization of α to any arbitrary positive scalar. Selecting α equal to unity would imply that the decision maker's preference structure is consistent with the assumption of constant returns to scale. While the constant returns to scale assumption may in fact be reasonable in many cases, there may be situations where increasing ($\alpha > 1$) or decreasing ($\alpha < 1$) returns to scale are appropriate.

Nevertheless, a requirement of $\alpha = 1$ is not as restrictive as it may look initially. As long as C_k is a legitimate (*i.e.*, relevant) criterion, β_k will be strictly greater than zero, and $\alpha = 1$ implies that each individual β_k is strictly less than one. From Theorem 1 it follows that under these conditions the impact of each individual criterion C_k decreases as its level of achievement increases, which appears to reflect the nature of a wide variety of decision problems, and certainly applies more frequently than a constant marginal impact, as exhibited in the traditional AHP.

In any case, one needs to verify the value of α empirically by eliciting information from the decision maker, for instance by assessing the relative change in the overall preference as a result of a relative change in the achievement of each $C_k \in \mathcal{G}$. One way to achieve this is by asking the decision maker by how much his/her overall preference level V would increase, given the current levels of achievement of all criteria, if the achievement level of each $C_k \in \mathcal{G}$ would increase by ϕ percent. Denote the magnitude of the change in V indicated by the decision maker as the answer to this question by ΔV, and the relative change by $R = \Delta V / V$. An implicit assumption in our method is that R/ϕ will be constant, if the model is valid for ΔV values of any magnitude. In practice, if the current model does not accurately reflect the tradeoffs for large ΔV, R/ϕ should at least be constant for reasonably small changes in the achievement levels of C_k. It may be helpful to ask the decision maker to evaluate the above question for several different values of ϕ, so that the validity of the assumption that R/ϕ is constant can be verified. Once the appropriate value of $\alpha = R/\phi$ has been established, the β_k should be scaled such that $\Sigma_k \beta_k = \alpha$.

Note that if after scaling some of the individual criterion weights β_k exceed unity, our method reflects the infrequent, but possible situation where the marginal contributions of some criteria increase with the level of criterion achievement.

In Theorem 2 we show that our synthesis method preserves the ratio scale property. As mentioned above, for $\alpha = 1$ this property has been noted previously by Barzilai and Golani (1994) and Lootsma (1993, 1994).

Theorem 2: Suppose that that the decision maker has indicated the appropriate value of $\alpha = \Sigma_k \beta_k = R/\phi$, where R and ϕ are as defined before. Then: (a) if each direct rating a_{ik} is multiplied by a factor $d_k > 0$, so that $a'_{ik} = d_k a_{ik}$, for each i, k, then the modified overall alternative ratings v'_i are proportional to the original ratings v_i, $i = 1, \ldots, m$; and (b) if the original criterion weights β_k are multiplied by a factor $g > 0$, *i.e.*, if $\beta''_k = g\beta_k$, then the overall priority weights v''_i will be identical to v_i, $i = 1, \ldots, m$, after re-scaling $\Sigma_k \beta''_k$ to α.

Proof: Equation (2) applied to the modified local priority ratings a'_{ik} yields $v'_i = \Pi_k \ (d_k a_{ik})^{\beta_k} = \Pi_k \ d_k^{\beta_k} a_{ik}^{\beta_k} = \Pi_k \ d_k^{\beta_k} v_i = c' v_i$, $i = 1, \ldots, m$, where $c' = \Pi_k \ d_k^{\beta_k}$. Since c' is independent of i, the revised composite priority weights v'_i are proportional to the original ones, v_i, $i = 1, \ldots, m$, proving part (a). Re-scaling the modified sum $\Sigma_k \beta''_k$ to the appropriate value of α requires multiplying it by a factor $1/g$. Hence, the revised exponent of a_{ik} in the expression for $v''_i = \Pi_k a_{ik}^{\beta_k / g}$ is β_k, $i = 1, \ldots, m$, as before, and thus the analysis is unchanged, and $v''_i = v_i$, for all i.

The result in Theorem 2 is important, as it provides a strong justification for using our method for synthesizing the preference ratings, in that it shows that the ratio scale properties of local priority weights are always preserved in terms of the composite priorities. In fact, from the proof of Theorem 2 we know that the proportionality factor c' by which v_i changes as a result of re-scaling the direct ratings a_{ik} by d_k equals $\Pi_k d_k^{\beta_k}$. The only restriction on the d_k values is $d_k > 0$. In contrast, Schoner, Wedley and Choo (1992) have shown that the synthesis of the original AHP preserves the ratio scale property *only* if $d_1 = d_2 = ... = d_n$, and that the traditional AHP loses the ratio scale property for *any other* combination of d_k values. We note that in part (b) of Theorem 2 it would not make sense to allow for different factors g_k in the transformation of the w_k, rather than a single factor g, as different factors would change the *relative importance* of the individual criteria, which would obviously alter the preference structure itself, and would as a consequence generally result in a change in the v_i values.

Theorem 2 implies that we can normalize the v_i values using any proportional transformation, without affecting the results of our analysis. We will use the normalization $\Sigma_i v_i = 1$, facilitating easy interpretation of the relative preferences as percentages or proportions of total value.

In Theorem 3 we establish that our method applied to direct priority ratings cannot lead to rank reversal.

Theorem 3: If (2) is used to synthesize the local direct ratings into overall priority ratings, then the rankings and ratings of the alternatives are not affected if a new alternative is added to or one of the existing alternatives is deleted from the set of alternatives.

Proof: Let the original set of alternatives be given by $\mathcal{A} = \{A_1, ..., A_m\}$. Suppose that a new alternative, A_{m+1}, is added to the problem. Inspecting (2), we see that the composite ratings v_j and v_q of alternatives A_j, $A_q \in \mathcal{A}$ do not involve any information regarding A_{m+1}, so that their ratings and rankings are not affected by the new alternative. Moreover, from Theorem 2 we know that the composite ratings v_i are insensitive to proportional transformations, so that a normalization of $\Sigma_{i=1}^{m+1} v_i = 1$ does not alter the relative values of v_j and v_q. Since this argument holds for any A_j, $A_q \in \mathcal{A}$, we have proven the claim that rank reversal cannot occur in the presence of direct ratings.

From Theorem 3, we see that the rankings of the alternatives using our method are not affected by the addition or deletion of alternatives, as both the criterion weights and the direct ratings a_{ik} (and consequently their rankings) are independent of additional alternatives.

4. EXAMPLE

Our example, due to Dyer and Wendell (1985), has previously been used as an illustration by several researchers for illustration purposes (see, *e.g.*, Harker and Vargas 1987). Harker and Vargas (1987) use this example to address the issue of rank reversal. The direct ratings a_{ik} of alternative $A_i \in \{A_1, ..., A_4\}$ with respect to the criteria $C_k \in \{C_1, ..., C_4\}$ are given in Table 1. In this example, the β_k-values are assumed to be the same for all criteria (*i.e.*, $\beta_p = \beta_q$, for all p, q), reflecting that each criterion is equally important. We compare the results of the synthesis using (2) for several different values of α, reflecting increasing ($\alpha = 1.5$), constant ($\alpha = 1.0$) and decreasing returns to scale ($\alpha = 0.5$), respectively.

Table 1: Absolute Preference Ratings

	C_1	C_2	C_3	C_4
A_1	1	9	1	3
A_2	9	1	9	1
A_3	8	1	4	5
A_4	4	1	8	5

Table 2: Composite Preference Ratings, Four-Alternative Model[a]

Composite Ratings (AHP)		Composite Multiplicative Ratings		
		$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 1.5$
i	r_i	v_i	v_i	v_i
1	0.264 (1)	0.215 (4)	0.184 (4)	0.156 (4)
2	0.243 (4)	0.247 (3)	0.242 (3)	0.236 (3)
3	0.246 (2-3)	0.269(1-2)	0.287(1-2)	0.304(1-2)
4	0.246 (2-3)	0.269(1-2)	0.287(1-2)	0.304 (1-2)

a: The rankings of the alternatives are given within parentheses.

Table 3: Composite Preference Ratings, Three-Alternative Model[a]

Composite Ratings (AHP)		Composite Multiplicative Ratings		
		$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 1.5$
i	r_i	v_i	v_i	v_i
1	0.320 (3)	0.282 (3)	0.258 (3)	0.194 (3)
2	0.336 (2)	0.339 (2)	0.339 (2)	0.335 (2)
3	0.344 (1)	0.379 (1)	0.403 (1)	0.471 (1)

a: The rankings of the alternatives are given within parentheses.

The composite ratings and rankings of our method with $\alpha = 0.5$, $\alpha = 1$ and $\alpha = 1.5$ are given in Table 2. We also provide the composite ratings for the traditional AHP, denoted by r_i, as a benchmark. In this example, our method with $\alpha = 1$ actually coincides with the weighted geometric mean of the direct ratings a_{ik}. Table 3 gives the corresponding results for the modified problem where alternative A_4 has been removed from the set of alternatives. The rankings associated with our multiplicative synthesis method (A_3 and A_4 tied for first, A_2 third and A_1 last) are identical for the three different levels of α, and are consistent across the three- and four-alternative formulations. However, the AHP rankings reverse when A_4 is removed, as the ranking of A_1 falls from first to third. Note that Theorem 3 shows that such a reversal could never occur within our method. Interestingly, for $\alpha = 1$ the composite ratings of our method are proportional across the three- and four-alternative problems.

5. CONCLUDING REMARKS

In this paper, we propose a method for combining ratio-scale priorities into overall ratings which uses a nonlinear, multiplicative synthesis function, and can represent a wide class of preference structures, including increasing, constant and decreasing marginal contributions to the overall goal. In many decision problems the impact of a criterion to the overall goal decreases as a function of the level of its achievement. A multiplicative function like (2), in which all β_k values are less than one (as is the case, *e.g.*, if their sum adds to one), models this kind of decision making behavior.

Among others, our method allows for alternatives dominated by linear combinations of other nondominated alternatives to be identified as optimal, while additive models which assume that the marginal contribution of each criterion is constant (such as the AHP) do not. Moreover, since our method maintains the ratio-scale property of the priority weights, it is not vulnerable to rank reversal, as long as the alternatives are rated using absolute measurements, and can accurately reflect the preference structure for many real-life decision situations.

Due to its flexible representation of various types of marginal contributions to the overall goal of the decision problem, our model appears to be applicable to a wide range of decision situations, and that its assumptions may be met more frequently than those of the additive model underlying the traditional AHP. Future research should concentrate on rigorously studying the theoretical and empirical properties of our proposed method. The extension of our methodology to the case of relative preference judgments based on pairwise comparisons can be found in Stam and Duarte Silva (1994).

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Some Fundamental Problems in the Selection and Justification of Discrete Alternative MCDM Methods

Vladimir M. Ozernoy

School of Business and Economics, California State University, Hayward,
Hayward, CA 94542, U.S.A.

Abstract. The selection of the most appropriate, or "best," discrete alternative MCDM method is important in many applications. The problem of selecting the "best" MCDM method is itself an MCDM problem that does not have an obvious solution. This paper addresses several general concerns and complicated features of the selection problem. The paper argues that discrete alternative MCDM methods have inherent limitations, as the validity of a method cannot be established formally. An approach for substantiating MCDM methods is suggested, and some directions for future research are outlined.

1. Introduction

It has long been recognized that the selection of the most appropriate MCDM method from among several alternative feasible methods is itself an MCDM problem that does not have an obvious solution. Several MCDM methods may appear to be useful for a particular decision problem. However, different methods often represent radically different approaches to decision making, and choosing among methods may depend on the particular problem under consideration, the decision maker's personal tastes, and many other factors. These methods are difficult to classify, evaluate, and compare, because they are based on a variety of assumptions about the decision maker's preferences and use different types of preference information. The problem of selecting the "best" MCDM method cannot be alleviated by carefully structuring the decision situation or by developing a universal "perfect" MCDM method.

Over the past two decades, several approaches to the selection problem have been suggested. They are based on various taxonomies of MCDM methods and include an informal and intuitive approach, classification trees, and MCDM expert systems. However, these approaches have not resulted in a set of guidelines that can be used to make it possible for a decision maker to select the most suitable MCDM method from among several which may be feasible in a given decision situation.

The purpose of this paper is to illuminate the complexities of the selection problem, as well as provide insight and facilitate the possibility of better understanding, evaluating, and improving decision making. The paper is based on the assumption that for significant MCDM decision problems the following premises are relevant: (1) the

selection of the "best" MCDM method is important, and (2) the selection of the "best" MCDM method should be based on consideration of the decision situation, method's assumptions, and information requirements of the method.

This paper discusses a number of experimental comparisons of several discrete alternative MCDM methods and MCDM decision support systems that have been conducted by various MCDM researchers. The paper suggests that disputes concerning the performance of discrete alternative MCDM methods cannot be resolved from experiments. The need for formal analysis of the selection problem is substantiated and the utilization of MCDM expert systems is suggested.

In this paper, the terms "MCDM method" and "MCDM problem" are used for brevity, where appropriate, instead of "discrete alternative MCDM method" and "discrete alternative MCDM problem," respectively.

The paper is organized as follows. Section 2 discusses the selection problem and the difficulties in the selection of the "best" MCDM method. Section 3 looks at experimental comparisons of MCDM methods. Section 4 justifies the need for MCDM expert systems and briefly reviews two expert system prototypes. A possibility for improving MCDM expert systems is outlined in Section 5. Finally, Section 6 provides some conclusions.

2. The Selection Problem

Based on a typology suggested by Zionts (1985), MCDM methods can be classified into those for solving discrete alternative MCDM problems and those for solving multiobjective mathematical programming (MOMP) problems. While much of MCDM research is devoted to developing and/or investigating MOMP methods, the focus in this paper is on discrete alternative MCDM methods for solving deterministic discrete alternative MCDM problems.

The differences that substantiate the need for the consideration of discrete alternative MCDM methods as a distinct area of research and applications are discussed in detail in Zionts (1985), Hwang and Yoon (1981), and many other publications. Briefly put, MOMP methods address problems whose constraints find explicit mathematical expressions and in which alternatives are implicit and perhaps infinite in number. The purpose of MOMP methods is to "design" the best alternative by considering the various interactions within the design problem constraints which best satisfy the decision maker through obtaining some acceptable attribute levels. Most MOMP methods assume that the decision maker can compare two nondominated alternatives and select the preferred one. In contrast, the purpose of discrete alternative MCDM methods is to provide a procedure for comparing two nondominated alternatives. The distinguishing feature of discrete alternative MCDM problems is that there is a finite number of alternatives (although it could be a very large number), the constraints are implicit, and the required type of ordering of a set of feasible alternatives should be obtained and justified.

In solving real-world discrete alternative MCDM problems, neither alternatives nor attributes are usually fixed a priori; rather, these aspects are changed and continuously reformulated in the process of the structuring of the decision situation. Nevertheless, the decision maker's representation of the decision problem is eventually described by a finite set of alternatives and a number of attributes. It is then that an MCDM method is selected and used. Choosing the "best" MCDM method is therefore a problem that is as important as the structuring of the decision situation.

Often, a major problem for both the decision maker and analyst is how to deal with many existing MCDM methods in making sound and defensible decisions. The selection of the most appropriate method is important in a variety of situations — for example, when the decision is controversial and must be defended against attacks on the soundness of the method and the decision based on it.

Many authors maintain that the key issues in choosing the "best" MCDM method are the method's accuracy in representing the decision maker's preferences and the quality of the prescriptive insight that will be gained from its use. Unfortunately, the criteria for the method's accuracy and the quality of the prescriptive insight are difficult to define precisely. All the methods claim that they correctly represent the decision maker's preferences. The results of sensitivity analysis are usually referred to as insights that can be gained from it.

While solving an MCDM problem, the analyst usually selects a method developed by him or her, a method the analyst has the most faith in, or a method the analyst is familiar with and has used before. When using a particular MCDM method, the analyst forms the decision maker's preference structure in accordance with the requirements of the method. For example, if multiattribute utility functions are used, the decision maker answers some lottery and tradeoff questions to specify single-attribute utility functions, and to determine the scaling constants.

If a particular MCDM method is selected, the role of the analyst is to elicit the required preference information from the decision maker and to alleviate or incorporate inevitable inconsistencies in the information provided by the decision maker. However, Slovic et al. (1977) indicate that "many of the aids require assessments of quantities the decision maker has never thought about, and that these apparently simple assessments may be psychologically more complex than the original decision." One may argue as well that the justification of the most appropriate MCDM method may be psychologically more complex than the selection of the "best" alternative or rank-ordering the set of feasible alternatives.

An MCDM method is a conditional statement. Its antecedent (or left-hand-side) is a formal MCDM model, i.e., a formal description of the decision problem, including the description of decision alternatives and decision maker's preferences. Its consequent (or right-hand-side) is a formal recommendation for action (Holtzman, 1989). Thus if the

decision maker believes that the decision context (i.e., a formal description of the decision domain and the decision situation) and the method selected adequately represent the decision problem, the decision maker should accept the recommendations of the method.

It was noted in Holtzman (1989), that formal decision methods are a special class of formal deductive methods because they yield recommendations for actions. By construction, nothing in the decision method's antecedent (i.e. in the formal description of the decision problem) is prescriptive. Hence, the definition of a formal MCDM method must contain at least one prescriptive axiom (an action axiom) that defines a set of sufficient conditions for action. The prescriptive axiom must retain an element of faithful trust. Even though one can argue that a given prescriptive axiom is reasonable (i.e., that it "makes sense"), the correctness of the prescriptive axiom cannot be proven unless it is embedded in a larger axiomatic system that itself contains a prescriptive axiom. An MCDM method does not deal with the validity of the prescription that constitutes its recommendation; it simply deduces the logical implications from the decision-maker's assumptions.

Some MCDM methods are justified by their axiomatic base, some methods are substantiated by their intuitive appeal and ease of use, while other methods are offered without any justification. Each MCDM method has its own champions, and it is hard to persuade someone who has developed a method to use other methods. On the other hand, neither the decision maker nor the analyst have full knowledge of all the available methods, and such knowledge could potentially enhance the capabilities of both the decision maker and analyst in improving the decision-making process. The selection problem is too complex to be dealt with solely by intuition and cannot be adequately solved without being formally addressed. Thus, the need for formal analysis of the selection problem is justified, and the utilization of a systematic, logical, and defensible approach for the selection of the "best" MCDM method can complement an informal and intuitive approach.

3. Experimental Comparisons of MCDM Methods

A number of experimental comparisons of several discrete alternative MCDM methods and various MCDM decision support systems have been conducted, and the results compared along several criteria. See, for example, Roy and Bouyssou (1985), Hobbs (1985), Hobbs (1986), Goecoechea (1989), Goecoechea and Stakhiv (1989), Hobbs et al. (1992), and Goecoechea et al. (1992). These studies, however, have not provided any recommendations that could enable a decision maker to select the "best" MCDM method for an application.

Hobbs (1985) conducted an experimental comparison of MCDM methods in power plant site selection. He suggested that in comparing MCDM methods, the user should be concerned with their appropriateness, ease of use, validity, and results. He also indicated that experiments are superior to reasoning in determining (1) user's

perceptions of the appropriateness of methods, (2) the difficulty of using different methods, (3) the relative validity of methods as actually applied, and (4) whether choice of method makes a practical difference. The most important conclusion of this experiment was that decisions can depend on choice of method and even on theoretically irrelevant factors such as question phrasing. Hobbs et al. (1986) suggested that users would be prudent to apply more than one method, with the different approaches acting as checks upon each other.

Hobbs et al. (1992) and Goecoechea et al. (1992) executed well-designed experiments involving water planners from the U.S. Army Corps of Engineers and graduate students. These experiments were unique in the MCDM literature in that they asked a large sample of actual users to analyze a real-world problem. In both studies, the purpose was one of evaluating 13 urban water supply plans for meeting the long-range water supply needs of the Washington, D.C. metropolitan area. The plans were evaluated using 19 attributes, which were grouped under the following areas of concern: risk management, flexibility, cost, environment, socioeconomic equity, and growth.

In the Hobbs et al. (1992) experiment, four MCDM methods and three weighing techniques were evaluated. The methods evaluated include goal programming, additive value functions (using MATS-PC decision support system), multiplicative utility functions (using a software package called SmartEdge), and ELECTRE I. Three techniques for choosing weights were direct rating, indifference tradeoff, and the AHP.

Several hypotheses were tested and discussed in Hobbs et al. (1992). They were grouped into three categories: appropriateness of use, validity, and differences in results. The findings of the experiment are rather surprising. It turned out that a) users were generally skeptical of the value of MCDM methods and often preferred unaided judgments, b) no one method was consistently preferred by the users to others, and c) the choice of method could make a significant difference in decision.

In the Goecoechea et al. (1992) experiment, four MCDM decision support systems were used. These were MATS-PC, Expert Choice, Ariadne, and ELECTRE. The participants considered a simplified version of the original problem, in which they evaluated 10 out of the 13 plans using 10 attributes. Based on the results of the non-parametrical statistical analysis, the authors offered several conclusions. Some of them resemble conclusions derived by Hobbs et al. (1992), while some others contradict the findings of the Hobbs et al. (1992) experiments. As an example, Goecoechea et al. (1992) claim that the participants, individually, were generally able to achieve high agreement in the evaluation and ranking of plans when using four MCDM decision support systems.

Hobbs (1985) formulated several criteria for the design of MCDM experiments and pointed out that no experiment can satisfy all the criteria simultaneously because experimental design is also a multicriteria problem. Indeed, findings of experiments described and analyzed in Hobbs et al. (1992) and Goecoechea et al. (1992) suggest that

none of these experiments was conclusive and that disputes concerning the method performance cannot be resolved by experiments. It is hard to imagine what new findings significantly changing the picture could be obtained from additional experiments. In fact, different users will always prefer different methods, and the choice of method or MCDM decision support system can affect the evaluation.

4. MCDM Expert Systems

Over the last few years, the need for the use of expert system technology for the selection of the most appropriate MCDM method has been recognized and several applications have been developed. An MCDM expert system integrates the user's preferential knowledge with "value-free" MCDM knowledge and thus addresses some of the major difficulties in decision support. Specifically, an MCDM expert system incorporates a decision-maker's preference structure "explicitly into the context of the decision problem at hand, perhaps through progressive preferential knowledge acquisition procedures" (CONDOR report, 1988), and this is "likely to have a decided impact on the acceptability, and, hence, the perceived capability of an expert system."

The impetus for the development of MCDM expert systems comes from at least three directions: (1) the needs of users for procedures and tools that will help them make better informed decisions, (2) the possibility of developing a comprehensive system step-by-step and then easily updating it, and (3) the availability of microcomputer-based expert system development tools, giving MCDM researchers the capability to use expert system technology.

A number of MCDM expert system prototypes have been developed during the last few years. Ozernoy (1989) developed a conceptual framework for a rule-based MCDM expert system that would assist the decision maker or analyst in justifying the selection of the most appropriate discrete alternative MCDM method in a given decision situation. A prototype MCDM expert system called MCDM Advisor is described in Ozernoy (1991) and Ozernoy (1992b). MCDM Advisor was developed using a microcomputer based expert system shell called VP-Expert (Hicks and Lee, 1988). The knowledge base contains eight MCDM methods, including both compensatory and noncompensatory methods. These are: disjunctive method, conjunctive method, dominance, lexicographic method, weighted-additive evaluation function with partial information, simple multiattribute rating technique (SMART), aspiration level interactive model (AIM), and ELECTRE-I. The experiments with MCDM Advisor described in Ozernoy (1991) and Ozernoy (1992b) indicate that the basic structure of the system and the interrelationships of its components will permit the development of a comprehensive MCDM expert system.

In the MCDM knowledge base, backward chaining is used to find the appropriate method. With this approach, the system starts with the hypothesis that a particular method is appropriate for a given decision situation, then reasons backward, looking for facts and rules to support it. If the first hypothesis fails, the system switches to another.

Hong and Vogel (1991) developed an architecture for designing a knowledge-based MCDM decision support system. The system determines MCDM methods appropriate to a given problem described by the decision maker. The architecture includes the following six components: (1) the user interface, (2) the model manager, (3) the knowledge base, (4) the data base, (5) the SmartSheet program, and (6) the inference engine. The SmartSheet spreadsheet program searches through the alternatives in the spreadsheet-based data base.

The system architecture is implemented by a prototype system, integrated spreadsheet-based inferencing system (ISBIS). The knowledge base employs the following seven MCDM methods: dominance, conjunctive method, disjunctive method, elimination-by-aspects method, lexicographic method, additive method, and additive-difference method.

The ISBIS inference engine uses the forward chaining algorithm. Seven attributes are selected that may uniquely define the characteristics of MCDM methods employed in the knowledge base. The selection of the appropriate method depends on the user's responses, which indicate whether a particular attribute is essential in the method selection.

5. Discussion

At the present time, the analyst forces an MCDM method on a client. This is usually a method that the analyst has developed, has the most faith in, or has used before. When the user selects an existing MCDM decision support system, it might happen that the MCDM method most appropriate for a particular decision situation is not implemented. Then, the user is likely to select and utilize an existing "stand-alone" system that is not necessarily suitable. As an example, imagine an MCDM decision support system that provides an easy-to-use user interface but is based upon an MCDM method that is neither well motivated nor well explained. Such a system may seem to have advantages over an MCDM decision support system based on a sound MCDM method but with less sophisticated user dialogue support. Clearly, the selection of the most appropriate MCDM decision support system is an important issue that cannot be adequately addressed without clear understanding of how the "best" MCDM method should be selected.

A different approach to the selection problem, which is based on the development of a prescriptive MCDM expert system, is possible. This approach takes into consideration that a user might not be indifferent to various methods.

An MCDM expert system will help the user make better informed decisions in MCDM problems by facilitating the selection and justification of the most appropriate MCDM method. It will therefore give the user greater confidence in his or her decisions. The system will also provide a training facility for the inexperienced user.

The major methodological difficulties in the development and implementation of MCDM expert systems are as follows:

(1) The MCDM knowledge is complex and cannot be neatly structured, therefore the reasoning behind a particular result becomes convoluted and not valuable to the user. The user needs more information about the questions being asked and the statements being made.

(2) If a user's answer to a question is "What does it mean?" or "I do not know," the dialog stops and the appropriate method cannot be specified without the help of an analyst.

An effective solution to the limitations of MCDM expert systems is offered by hypertext systems. Hypertext is a computer technique that permits the user to explore above and beyond the words displayed on the screen. As the user digs down into the subject, layer after layer, the user finds new concepts, new terms, and new related ideas that may not have previously occurred to him or her. The combination of MCDM expert systems and hypertext systems offers the following potential advantages:

1. An MCDM expert system with appropriate hypertext will permit the user to learn quickly the additional information needed to carry out the successful consultation.

2. An MCDM expert system will be able to a) request the required information from the user, b) provide for explanation of terms in a question, c) recommend the most appropriate method for a given decision problem, and d) justify its recommendations by displaying its line of reasoning.

3. In terms of accuracy, reliability, and consistency of its conclusions, the performance of an expert system with appropriate hypertext is expected to be comparable with that of a human expert. Moreover, such a system has the potential of achieving results which may otherwise be unobtainable because the system does not mimic any one expert in selecting the most appropriate method. Instead, it can be designed as a sort of intelligent encyclopedia that can capture the knowledge of many MCDM experts.

6. Conclusion

MCDM expert systems address some of the major difficulties in multiple criteria decision making, and thus they will be useful in helping to improve the decision-making process. In particular, an MCDM expert system will perform some of the important functions of the decision analyst, helping people make better informed decisions and giving them greater confidence in their decisions. Also of significant importance will be a better understanding of the usefulness and limitations of a wide range of MCDM methods.

The use of an MCDM expert system cannot be expected to "solve" the selection problem, nor is it intended to do that. Rather, it is intended to provide insight which may not be obvious because of the complexity of the selection problem, and to help the user make logically consistent and responsible choices.

An MCDM expert system could be helpful and educational as it would demonstrate the method's assumptions, information requirements, summary of published criticisms of the method, findings of experiments, references, etc. The process of making decisions using an MCDM expert system will be a learning experience as valuable as the final decision itself. An MCDM expert system with appropriate hypertext brings the user closer to real communication.

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The SOLVER of Volvox Multicriterion Table

Benoît **DEROT**[1], Jean **GAREAU**[1], Laszlo Nandor **KISS**[2] and Jean-Marc **MARTEL**[2]

(1) - Centre de Recherche VOLVOX (1991) Inc., 1210, Sherbrooke Ouest, MONTRÉAL,
P.Q., H3A 1H6, Canada
(2) - Université Laval, Faculté des Sciences de l'Administration, SAINTE-FOY,
P.Q., G1K 7P4, Canada

ABSTRACT: VOLVOX is a consortium composed of six leading canadian firms in the field of information technology and decision making aid. The CGI group acts as manager in this project whose goal is to develop, in collaboration with several partners especially from university milieu, and to commercialize systems of decision making aid DSS in the environmental field. In addition to developing DSS responding to specific needs, the consortium intends to focus on a technological generic platform (a generic shell) VOLVOX. This shell contains several elements, one of which is the construction of a multicriterion table. The VOLVOX multicriterion table allows ordinal or cardinal, punctual or distributional evaluations. The SOLVER (multicriterion aggregation procedure) is based on the outranking synthesis approach and lies within the scope of ranking problematic. It appeals to the pseudo-criteria and incomparability notions. It can also act as a negotiation and group consensus support. The purpose of this paper is to present succinctly the objectives of the VOLVOX research center as well as the main caracteristics of the solver.

1. INTRODUCTION

The Volvox Multicriterion Table Solver discussed hereinafter falls within the scope of a much more ambitious and mobilizing project initiated by the Bruntlant Commission when it introduced the "lasting development" principle to ensure the safeguard of our planet in the years to come. This principle was ratified and supported by most governments including

Canada's.

If we are to maintain our growth expectancy concerning economic development, we must be in a position to measure, anticipate and control the impacts of human activities on the environment. That entails that we be able to carefully examine wide scope ventures that are most likely to have an impact on the natural environment, such as industrial plants, hydro electric or petrochemical installation management and urban development.

Keeping up with environmental situations requires immediate access to great amounts of diversified information not only found as texts or numbers but also as maps, three-dimensional images and videos of occurring far away events. To manage and evaluate such information, it is essential that a wide variety of information technology tools be made available, thus greatly improving the decision-making process.

Over the past years, several devices have been developed to make decision-making easier. However, when these aids were applied for decision-making in environment-related activities, their limits quickly came apparent mostly because of the multidisciplinary and multidimensional nature of the problems as well as the array of decision-makers.

Nowadays, sophisticated information systems are able to provide faster and more precise analyses related to each environmental field. Unfortunately, there are so many fields and, therefore, so many different tools that instead of simplifying the decision-making process this situation has produced the opposite effect.

The solution points out that a uniform approach in which decision-makers would find a set of integrated tools is most likely to help them in taking the right decision in due time. With that in mind, a group of leading canadian high technology firms, called the Volvox Consortium, have joined forces to develop computerized systems of decision making aid (DSS) superior to existing tools tanks to judicious integration of the most advanced technologies.

The next section will provide the reader with a succinct description of the VOLVOX project and its multicriterion table. In the following part, we will present the main characteristics of the solver itself, a multicriterion aggregation procedure. We will then briefly talk about an actual application and, finally,

we will conclude with our evolution perspectives for the solver.

2. THE VOLVOX PROJECT

VOLVOX is a consortium composed of six leading canadian firms in the field of information technology and decision making aid. The CGI group acts as manager in this project whose goal is to design and to develop a DSS shell, in collaboration with several university partners. The VOLVOX DSS shell is flexible in the sense that it offers the opportunity of putting together specific DSS for particular clients using a library of predefined and reusable components. Thus reducing the time and costs needed to develop each specific DSS since existing components are simply associated instead of having to design new ones. The VOLVOX shell also has an impact on the reliability of each specific DSS because the grouped components have been tested and ratified for previous specific orders.

The VOLVOX DSS shell is mainly composed of a library of programs[1] and an application platform that provides the necessary support for their integration. Following are the three types of activities pertaining to decision making: find information on the problem, analyze the collected data and make the decision. These activities do not necessarily occur in sequence. The DSS shell library provides support to these three activities on a single or collective basis.

The Librarian and the geographical Atlas are the two components used for data collection. The Librarian provides support for several complementary and high standard documentary research metaphors enabling the user to interrogate intuitively a series of multimedia CD-ROM, diversified and scattered databases. The geographical Atlas is the Librarian counterpart for geographic related data. It supports the simultaneous operation of vector and matrix maps and allows for association of multimedia CD-ROM documents and geographical items which maps are made of.

When it comes to information analysis activities,

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[1] The programs' components have been written in MS-Visual C++ under the Windows environment.

simulation and visualisation tools are called for. With simulations, the DSS shell provides:

- an integration framework for modelizing environmental events from a library of qualitative and quantitative simulation models; and
- access to a parallel computation frame to execute simulation models requiring a powerful computation system.

The DSS shell visualisation tool generates three-dimensional (3-D) settings using topographic maps. These 3-D settings can then be altered with sophisticated tools. For instance, the user might want to add a high structure to estimate to visual impact of such a construction project. With his keyboard, the user may move around and modify the settings as he pleases.

The third type of activities are supported by the decision-making aid component which offers functional features to manage the user-selected approach to carry out the project, as well as an individual and collective decision-making environment. This environment is based on the decision-making process concept and leads to the construction of a multicriterion table.

2.1. THE MULTICRITERION TABLE

The decision-making process concept is to DSS what the work flow concept is to information systems. A work flow process describes the different steps required to transform an order (ex.: expert opinion) into a usable output (ex.: analysis report). The different steps and their links form a circuit-free-oriented graph that describes sequential and parallel groups of steps. Their definition implies the description of the inputs and outputs (ex.: intermediate version of documents), the list of the involved users and the tools they most likely will call upon at this stage (ex.: editware).

The user constructs his own decision-making process by following a series of pre-established steps: problem description (identification, analysis, evaluation criteria specification), listing of the viable alternatives (generation, organization, prioritization), multicriterion evaluation of these alternatives and choice of solution (negotiation, selection). The type of required inputs and the outputs supplied for each

group of steps should be entered into the system so as to enable it to check the consistency of the user-constructed decision-making process by executing a type check. Specific tools to help in decision-making are related to each group of steps to follow (ex.: an help editor) for problem analysis or a preparation tool for negotiating the solution.

The Multicriterion table (MCT) is used for the presentation of the results at the different steps taken. The MCT includes the set of items $\mathcal{A}$ containing the potential alternatives (columns); the set **A** containing the criteria/attributes (rows) and the set **E** containing the evaluations of alternatives a_i, i = 1, 2, ...,m in relation with the criteria g_j, j = 1, 2, ..., n in the "body" of the table. The table also includes the criteria's relative importance coefficients w_j, j = 1, 2,..., n and the different thresholds (indifference q_j , preference p_j, and veto v_j , j = 1,2,..., n) that provide the discrimination power of the criteria. A module assists the user in defining the different required parameters.

In its present version, the relative importance coefficients w_j go through a ranking process followed by a multi-criteria rating process, with visual support. Some help-examples are given to decision-maker to show the signification of discrimination thresholds. The table can process criteria for which the associated scales are either defined on an ordinal or cardinal level. That will be specified when filling out the table, but note that the MCT may include cells without any evaluation. These sets and parameters make up the main elements of the problem definition and the decision-maker's preference structure.

3. THE VOLVOX SOLVER

The solver is a multicriterion aggregation procedure (MCAP) whose object is to rank potential alternatives to a problem by taking into account, in a simultaneousmanner, evaluations of all criteria as well as their relative importance.

Because of the special features that characterize the MCT, such as the non compensatory nature of the criteria, the ordinal and cardinal nature of the evaluations, the notion of pseudo-criteria, etc., we

think that an ELECTRE type outranking synthesis approach was the most appropriate MCAP (Roy and Bouyssou, 1993). The main reasons justifying our choice are, in fact, the MCT's special features and the realistic way in which this MCAP links the preferences, that is somewhat similar to the democratic principle so much natural when faced with a decision having environmental impacts.

The MCAP developed for the VOLVOX solver is based on the ELECTRE III method. Concordance and discordance indexes must be determined as well as an outranking degree for each pair of alternatives $(a_i, a_i*) \in \mathcal{A} \times \mathcal{A}$. Bear in mind, however, that the MCT may include ordinal and cardinal evaluations and some evaluations may be missing.

When the solver is put on line, an input set χ is created from the MCT and transferred to the solver for processing, using an algorithm procedure $\Gamma_{\S 1}$. This is what we get:

$$\chi = \Gamma_{\S 1}[\, m,\ n,\ M_{[mxn]},\ K_{[n]},\ \Xi_{[n]},\ R_{[n]},\ Q_{[n]},\ P_{[n]},\ V_{[n]},\ W_{[n]}]$$

where

m = number of alternatives (a_i, i = 1,2...,m)

n = number of criteria (g_j, j = 1,2,...,n)

$M_{[mxn]}$ = real m x n cardinal matrix containing the alternatives' evaluations in relation to the criteria (i.e.the set **E**)

$K_{[n]}$ = binary n cardinal vector indicating the nature of the criteria's evaluation with

$$\kappa_j = \begin{cases} 0 & \text{if the evaluation in relation to the } j^{th} \text{ criterion is ordinal type} \\ 1 & \text{if the evaluation in relation to the } j^{th} \text{ criterion is cardinal type} \end{cases}$$

and $\kappa_j \in K_{[n]}$; j = 1,...,n;

$\Xi_{[n]}$ = binary n cardinal vector indicating the orientation of the criterion, with

$$\xi_j = \begin{cases} 0 & \text{if the } j^{th} \text{ criterion is to be minimized} \\ 1 & \text{if the } j^{th} \text{ criterion is to be maximized} \end{cases}$$

and $\xi_j \in \Xi_{[n]}$; $j = 1,\ldots,n$;

$R_{[n]}$ = real n cardinal vector containing the range for every criterion and $\rho_j \in R_{[n]}$; $j = 1,\ldots,n$;

$Q_{[n]}$ = real n cardinal vector containing the indifference thresholds and $q_j \in Q_{[n]}$; $j = 1,\ldots,n$;

$P_{[n]}$ = real n cardinal vector containing the preference thresholds and $p_j \in P_{[n]}$; $j = 1,\ldots,n$;

$V_{[n]}$ = real n cardinal vector containing the veto thresholds and $v_j \in V_{[n]}$; $j = 1,\ldots,n$;

$W_{[n]}$ = real n cardinal vector containing the criteria's relative importance coefficients w_j, and $w_j \in W_{[n]}$; $j = 1,\ldots,n$.

The following relations must be verified between the thresholds:

$0 \leqslant q_j \leqslant p_j < v_j$; $j = 1,\ldots,n$

and the relative importance coefficients are normalized, i.e. $\sum_{j=1}^{n} w_j = 1$. In the present version of the solver, $q_j = p_j = 0$ and $v_j >>> \rho_j$, i.e. no discordance, for all ordinal criteria , that is when $\kappa_j = 0$.

These inputs are processed using a second algorithm procedure $\Gamma_{\S 2}$ to determine the ranking $\mathbf{R}_{[\nu]}$ from the best to the worst alternative with possible *ex aequo*. Thus,

$$\mathbf{R}_{[\nu]} = \Gamma_{\S 2}[\chi]$$

where

$\mathbf{R}_{[\nu]}$ = a ν cardinal set containing the ranking of the alternatives, with $\nu \leqslant m$.

3.1. CONSTRUCTION OF OUTRANKING RELATIONS

For every pair of alternatives $(a_i, a_i*) \in \mathcal{A} \times \mathcal{A}$, a global concordance index $C(a_i, a_i*)$ is calculated as follows:

$$C(a_i, a_i*) = \sum_{j=1}^{n} w_j \cdot \delta_j(a_i, a_i*),$$

where

$$\delta_j(a_i, a_i*) = \begin{cases} 1 & , \text{ if } -\Delta_j \leq q_j \\ \dfrac{\Delta_j + p_j}{p_j - q_j} & , \text{ if } q_j < -\Delta_j \leq p_j \\ 1 & , \text{ if } p_j < -\Delta_j \end{cases}$$

and $\Delta_j = g_j(a_i) - g_j(a_i*)$; $g_j(a_i) \in M_{[mxn]}$; $j = 1,\ldots,n$; $i = 1,\ldots,m$.

Since some criteria call for maximization while others require minimization, we have introduced two temporary *sentinelle* variables k and k* in our calculations of Δ_j. If $\xi_j = 0$, meaning that this criterion is to be minimized, then $k = a_i*$ and $k^* = a_i$, while if $\xi_j = 1$ (criterion to maximize) $k = a_i$ and $k^* = a_i*$; this way $\Delta_j(a_i, a_i*) \equiv \Delta_j = g_j(k) - g_j(k^*)$, $\forall \xi_j$.

A discordance index $D_j(a_i, a_i*)$ is also calculated for every pair of alternatives at each criteria level as with ELECTRE III:

$$D_j(a_i, a_i*) = \begin{cases} 0 & , \text{ if } -\Delta_j \leq p_j \\ \dfrac{-\Delta_j - p_j}{v_j - p_j} & , \text{ if } p_j < -\Delta_j \leq v_j \\ 1 & , \text{ if } v_j < -\Delta_j \end{cases}$$

It should be noted that in the present version of the solver, $D_j(a_i, a_i*) = 0$ for every pair of alternatives in the case of an ordinal criterion, that is when $\kappa_j = 0$.

This version also considers that missing evaluations are interpreted as irrelevant evaluations and therefore should not advantage nor disadvantage an alternative. For instance, if the evaluation $g_j(a_i)$ is missing, then $\Delta_j(a_i, \mathfrak{b}) = \Delta_j(\mathfrak{b}, a_i) = 0$ for all $\mathfrak{b} \in \mathcal{A}$, which implies:

$$\delta_j(a_i, \mathfrak{b}) = \delta_j(\mathfrak{b}, a_i) = 1 \text{ and } D_j(a_i, \mathfrak{b}) = D_j(\mathfrak{b}, a_i) = 0$$

Using the concordance and discordance indexes, we generate outranking degrees $\sigma(a_i, a_i*)$ for every pair of alternatives. These outranking degrees are obtained using the following (Rousseau and Martel, 1993):

$$\sigma(a_i, a_i*) = C(a_i, a_i*) \cdot \prod_{j=1}^{n} [1 - (D_j(a_i, a_i*))^3].$$

3.2. EXPLORATION OF OUTRANKING RELATIONS

The obtained outranking relations set may be represented as an outranking matrix. Working with outranking relations implies synthesizing that matrix in such a way as to provide a recommendation in the form of alternative ranking. The solver's present version makes that synthesis following the notion of outgoing flow and incoming flow from PROMETHEE (Brans and al., 1984).
For calculations, we use:

$$\sigma^+(a_i) = \sum_{b \in A} \sigma(a_i, b) \quad \text{and} \quad \sigma^-(a_i) = \sum_{b \in A} \sigma(b, a_i)$$

and we get a first total ranking of the alternatives from the net flow (as in PROMETHEE II) $\sigma(a_i) = \sigma^+(a_i) - \sigma^-(a_i)$. The ranking of alternatives so obtained $\mathbf{R}_{[V]}$ is the one displayed on the screen, along with the sensitivity analysis ranking.

4. HYDRO SHERBROOKE CASE

The first version of the DSS VOLVOX shell was used in putting together a specifically designed DSS to measure the impact the construction of high voltage power lines would have on the environment. To meet the increasing electricity demand in the city of Sherbrooke (Québec, Canada), it was decided that an additional 120 - 25 kv transformation post would be built.

This project included three steps. The first step was to determine a corridor. The second step was to identify possible positions of the actual electric cables. The results of these two steps were presented in a public meeting. This led to the proposition of another position for the cables. This was compared (step 3) with those identified in step 2. For each of these steps, a set of criteria was established to determine the environmental, human, economic,... impacts.

5. EVOLUTION PERSPECTIVES

The present version of the VOLVOX solver already includes many functional features: different levels of scales depending on the criteria, missing evaluations, calculation of outranking degrees, the use of these valued binary relations and a sensitivity analysis. Definitely, such features integrated into one software is an innovative concept compared to available commercial software in the same category.

We are, at present, working on a new version in which there will be even more functional features. For instance, the solver would be able to deal with distributional evaluations (Martel and al., 1986), to serve as group consensus support (Martel and Kiss, 1993) and to enhance the sensitivity analysis (Mareschal, 1988). We are also searching for improved ways of obtaining inter-criteria information, establishing discrimination thresholds as well as a better processing ordinal and missing evaluations. As well, we are working on including veto thresholds for ordinal evaluations and, concerning the missing evaluations, we wish to introduce the notion of "not available" instead of "irrelevant" which is more likely to lead to incomparability.

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III

Multiobjective Mathematical Programming

An invited leading paper by Nakayama makes introductory remarks for multi-objective mathematical programming ahead the papers in this chapter. It addresses the issues of value judgements and how to treat them.

Gal and Hanne give a tutorial survey on the theoretical aspects of multiobjective mathematical programming. The paper is divided into three parts: review of the historical roots and the developments of vector optimization; discussion of main streams of research; and directions of future development based on the knowledge of past and present trends.

Arbel and Korhonen propose a new approach to generating interior search directions for multiple objective linear programming problems. The proposed approach is based on using achievement scalarizing functions and aspiration levels to convert a problem with multiple objectives to a problem with a single objective that is amenable for a solution by the interior point affine-scalling primal algorithm.

The paper by Ogryczak shows how predefined priorities (the so-called preemptive goal programming) can be incorporated into the reference point method. The proposed preemptive reference point method preserves the most important properties of the standard reference point method and it has been developed using goal programming implementation techniques.

Cambini, Martein and Cambini introduce new classes of vector generalized concave functions and clarify their role in investigating local and global efficiency, as well as establishing sufficient optimality conditions for a vector optimization problem.

The paper by Nakayama points out some remarks in using the automatic trade-off method and the exact trade-off method, previously proposed for making the trade-off analysis in a easy way in his "satisficing trade-off method".

Zeleny develops the notion of optimum as a well-defined balance among multiple criteria. Eight different, separate and mutually irreducible optimality concepts are proposed, where the traditional single-objective optimality is only a special case.

Multi-objective Mathematical Programming

Hirotaka Nakayama
Department of Applied Mathematics
Konan University
8-9-1 Okamoto, Higashinada, Kobe 658, JAPAN
Fax: +81-78-452-5507, Tel: +81-78-435-2534
e-mail: nakayama@edu2.math.konan-u.ac.jp

Abstract: Although traditional mathematical programming techniques has been remarkably developed for the last half century, they do not necessarily give satisfactory results in practical applications. Because many practical problems often have several objectives conflicting with each other, while traditional mathematical programming techniques can treat only one objective function. The history of multi-objective mathematical programming began almost at the same time as nonlinear programming. In particular, it has been developed very actively since the beginning of 70's. Now, several effective methods have been being applied to a wide range of real problems. This paper makes an introductory remarks for multi-objective mathematical programming ahead the following papers in this chapter.

1. Problem of Value Judgment

Roughly speaking, there are two stages in the decision process: at the first stage, alternatives are evaluated by several criteria; at the second stage, the final decision is made through the selection taking into account the evaluation by these criteria.

At the first stage, sometimes the traditional modeling, e.g. the structure modeling and the impact modeling are made (Sawaragi *et al.*, 1985). The structure modeling makes the strucure of the problem clear, and the impact modeling enables the measurement by the criteria based on various kinds of equations and simulations. Traditionally in these modelings, the problem of value judgment was excluded because it is not the subject of "science". However, it is no exaggeration to say that the decision making is the problem of value judgment. Avoiding the problem of value judgment, decision problems are not resolved. Therefore, if the evaluation at the first stage can be made by only some physical measure, the problem of value judgment is left to the next stage. In many engineering problems, for example, physical models are possible at the first stage, and hence the problem of value judgment usually appears in selecting a "reasonable" alternative as a final decision. On the other hand, in many problems in which the physical measurement is not pos-

sible, some subjective measurement are introduced. In these problems, the value judgment sometimes can be treated also at the first step of decision process. However, the multi-objective mathematical programming treats cases in which the essential value judgment is made at the second stage of decision process even though some kinds of value judgment are made at the first stage.

The second stage exactly tackles the problem of value judgment. Usually, the decision can not be so easily made through the multi-dimensional evaluation. Decision makers often worry about the trade-off over the criteria. To this end, theories and techniques of multiple criteria decision making have been developed. Among them, the multi-objective mathematical programming treats problems which are formulated as mathematical programming with multiple objective functions. Namely, in the multi-objective mathematical programming, the objective functions and the constraint functions are given explicitly. Depending on their function forms and the characteristics of variables, they are categorized into multi-objective linear programming, multi-objective convex programming, multi-objective non-convex programming, multi-objective integer programming, multi-objective mixed integer programming, multi-objective stochastic programming, multi-objective fuzzy programming, and so on.

The most important feature of these multi-objective mathematical programmings in a practical sense is how to treat the value judgment of decision makers. In treating the value judgment of decision makers, we have two difficulties. The one is the multiplicity of value judgment: the value judgment varies depending on decision makers. The other is the inconsistency: the value judgment of decision maker varies even in the decision process, e.g., the present one contradicts with the previous one. This occurs naturally because the available information to decision makers increases during the decision process. Of course, the inconsistency of judgment results sometimes from the limitation of capability for the judgment. In the traditional mathematical programming, the value judgment can be treated only in a form of objective function and constraint functions. Once they are decided, they can not be so easily changed depending on the situation. Namely, it lacks in flexibility and robustness for multiplicity and dynamics of value judgment. On the other hand, many kinds of techniques in multi-objective mathematical programming have been developed to overcome these two difficulties.

2. How to Treat Value Judgment

At the beginning of the history of multi-objective mathematical programming, some axiomatic approaches such as the utility (or value) theory were applied. Whether the utility (value) function is assumed to be given explicitly or implicitly in these methods, the preference of decision maker is presumed to satisy several mathematical axioms. However, these axioms do

not usually hold consistently during the decision process due to the reason mentioned above. Namely, the axiomatic approaches have not only a drawback that they can not treat the inconsistency of value judgment of decision makers but also the one that it is usually difficult to assess the preference associated with decision makers.

On the other hand, methods without utility (value) functions were developed. Clearly, the philosophy in methods using utility (value) functions is based on the optimization. (The utilitarian try to maximize his utility). It has been observed, however, that this hypothes does not necessarily hold in many practical problems as Simon (1957) pointed out. Due to these reasons, the optimization gave place to the philosophy called the satisficing in developing methods of multi-objective mathematical programming. The satisficing does not require any axioms on the preference of decision makers, and it is usually formulated to find a solution which the decision maker considers OK. How does the decision maker consider a solution OK? Usually, a solution is considered OK if it attains the aspiraton level of decision maker. Here, it should be noted that the simple satisficing is not sufficient in multi-objective mathematical programming, because the solution by satisficing leaves the possibility that there is another solution which improves all criteria. Therefore, it is desirable to find a solution which attains the aspiration level of decision maker and also leaves no possibility improving all criteria simultaneously, namely Pareto optimal. This philosophy is called the quasi-satisficing in some literature (Wierzbicki 1981). Many methods in this category use some "reference point" such as the ideal point, aspiration level and so on as a probe. For a given reference point, a Pareto optimal solution is found by some scalarization techniques. If the decision maker considers the solution OK, then the decision problem is solved. If the decision maker does not consider it OK, then he/she has to make a trade-off. Because if he/she wants to improve some of criteria, then he/she must agree with some relaxation of another criteria. It becomes very important in developing methods of multi-objective mathematical programming how we support the decision maker to make the trade-off easily.

In addition, methods of multi-objective mathematical programming must not force a solution on decision makers. It is important to convince decision makers of a solution. To this end, decision support systems using multi-objective mathematical programming have to give decision makers a learning effect. In many interactive multi-objective mathematical programming techniques, decision makers can learn how far their wishes are from the reality (i.e, the feasible region) and how much they have to sacrifice some of objective functions in order to improve other objective functions through the trade-off analysis, and so on. We can also say that the multi-objective mathematical programming is a human interface for decision makers to learn their decision making problems toward a better decision.

It goes without saying that the trade-off analysis is almost synonymous

to the totally balancing among criteria. As was mentioned above, this is the most important feature in multi-objective mathematical programming. It depends on the concept of solution to decision problems to develope methods in multi-objective mathematical programming. In this chapter, Zeleny discusses this point. Nakayama gives some remarks on the trade-off analysis. Ogryczak reports a reference point method for problems with some special preference structure. Arbel and Korhonen discuss the possibilty of applying the interior search method in LP effectively to multi-objective problems.

The mathematical theory is also an important subject in multi-objective mathematical programming: It extends several important mathematical characteristics of traditional single-objective mathematical programming, e.g., duality, stability, sensitivity analysis very elegantly to those of multi-objective mathematical programming. Although many of them may not affect directly to methods in multi-objective mathematical programming, they are beautiful and hopefully expected to play an important role at least in some theoretical applications, e.g., economics, sociology and so on. Gal and Hanne give a tutorial survey on the theoretcal aspects of multi-objective mathematical programming. A.Cambini, Martein and R.Cambini clarify some optimality conditions for generalized concave multi-objectvie programming.

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On the Development and Future Aspects of Vector Optimization and MCDM

A Tutorial

Tomas Gal[1] and Thomas Hanne[2]

[1,2]Department of Economics, Operations Research, FernUniversität Hagen, Lütkenheider Str. 2, 58084 Hagen, Germany, E-mail: [1]tomas.gal@fernuni-hagen.de, [2]thomas.hanne@fernuni-hagen.de

Abstract. The paper is divided into three parts: 1) Historical sketch, 2) Main streams of research, 3) Future development. In the first part, a concise review of the historical roots and the developments of vector optimization in the sense of decision making under several goals (objective functions) is presented. In the second part various main streams are discussed. Two basic aspects of vector optimization can be distinguished: the pure theoretical, mainly mathematical one, and the more practical, methodologically oriented one. A brief survey of vector optimization theory is given. In the third part we try to outline directions of future development based on the knowledge of past and present trends.

1 Introduction

In vector optimization the concepts of optimality and optimization are generalized from scalar problems to the vector-valued case. From the first beginnings in different areas to today's matured scientific community that strongly relates to the more methodological and application-oriented "MCDM" philosophy about a century has passed. Since the first works in econonomically and mathematically influenced fields several thousand papers have been published. Therefore, we only can discuss some of the most relevant pioneering work in vector optimization / MCDM. For a more detailed treatment of special aspects, we refer to the quite comprehensively given bibliography (until the early eighties) by Achilles, Elster, and Nehse (1979) and Nehse (1982).

2 Historical Sketch

The roots of vector optimization (VO) in the sense of optimization and decision making under several goals lead back to the last century. Stadler (1979, 1986) surveys the origins of VO in quite different disciplines. The present historical sketch is partially based on Stadler's work.

One of the most important forerunners of VO and its decision-oriented applications can be found in economics, especially in utility theory. Central to this analysis was the assumption of utility functions. Utility was a much older concept in philosophy and economics used by Bentham, Smith, Ricardo

and others (See Stigler (1950) for a historic survey on utility theory.). From today's viewpoint they had some quite naive perceptions of utility: 1. People can measure the utility of a possible state of the world to themselves on a cardinal scale. 2. All the individual utilities can easily be aggregated to a social one by summation. 3. The maximization of the social utility is the foremost task of the government.

With the foundation of general equilibrium theory in economics by Walras (1874-77), Pareto (1906), and Edgeworth (1881) in the late 19th century and with Debreu (1959) in the midth of the 20th century, economics experienced a remarkable mathematization together with more differentiated views of the old utility concepts. Pareto introduced the concept of ordinal utility into economic theory (which was, incidentally, not consistent, see Hicks and Allen (1934)). We owe him, what is today called Pareto criterion or Pareto optimality which means a situation where nobody's utility can be increased without reduction of another one's. Edgeworth (1881) analyzed situations with 2 person-2 commodity distributions (and generalizations). Assuming differentiable, isotonic, concave utility functions $U_i(x_1, x_2)$ for the persons $i = 1, 2$ he derived the optimality condition

$$\frac{\partial U_1}{\partial x_1}\frac{\partial U_2}{\partial x_2} = \frac{\partial U_1}{\partial x_2}\frac{\partial U_2}{\partial x_1}.$$

All points (x_1, x_2) satisfying this condition form the contract curve which today we would call the set of efficient solutions.

Relevant mathematical preparatory work for VO in ordered spaces and order preserving maps has been done by Cantor (1895, 1897) and Hausdorff (1906, 1914, 1927). Hausdorff (1927) showed that there exist (linear) orders, for instance the lexicographic order first examined by Cantor (1895, p. 502) and more extensively by Hausdorff (1906, 1914), for which there do not exist corresponding utility functions.

In this paper we use the following order relations in vector spaces R^n:

$$\begin{aligned} x \leqq y \quad &\text{iff} \quad x_i \leqq y_i \quad \forall i \in \{1, ..., n\}, \\ x \leq y \quad &\text{iff} \quad x \leqq y \quad \text{and} \quad x \neq y, \\ x < y \quad &\text{iff} \quad x_i < y_i \quad \forall i \in \{1, ..., n\}. \end{aligned}$$

(We also use the symbol $\leqq$ instead of $\leq$ for scalars in order to treat them as a special case of vectors.)

Another early mathematically oriented treatment of VO problems was the introduction of vectorial norms by Kantorovitch (1939) which led to a generalization of approximation theory from the scalar to the vector-valued case.

As another important preparatory work for VO, game theory was mainly motivated by economic considerations. Game theory was initiated by Borel

(1921) although von Neumann (1928) is normally regarded to be (and regarded himself to be) the founder. His book together with Morgenstern (1943) is probably the most cited game theoretical reference. John von Neumann proved the fundamental minimax theorem which states that every two-person zero-sum game with mixed strategies has a minimax solution when the players only consider the expected payoff:

Let A be an $(m \times n)$-dimensional matrix. Let $\Delta_m = \{(x_1, ..., x_m) \in R^m : x_i \geqq 0, \sum_{i=1}^m x_i = 1\}$, $\Delta_n = \{(x_1, ..., x_n) \in R^n : x_i \geqq 0, \sum_{i=1}^n x_i = 1\}$. Then

$$max_{y \in \Delta_m} min_{x \in \Delta_n} y'Ax = min_{x \in \Delta_n} max_{y \in \Delta_m} y'Ax.$$

The corresponding optimal arguments (x_0, y_0) lead to a saddle point of the payoff function such that $y'Ax_0$ assumes its maximum at $y = y_0$ and $y_0'Ax$ assumes its minimum at $x = x_0$ (p. 95).

A generalization of the minimax theorem to vector-valued games was given by Blackwell in 1956. He considers a $n_1 \times n_2$-matrix M with elements $m_{ij} : X \to [0, 1]$ being probability distributions over a compact, convex set $X \subset R^N$ (vector-valued random variables). M is interpreted as the payoff matrix of an (infinitely) iterated two-player game. The strategy of player $k, k \in \{1, 2\}$, is a sequence $f_k = (f_{0k}, f_{1k}, ...)$ of functions $f_{ik} : (x_1, ..., x_i) \to \Delta_{n_k}$ with f_{0k} arbitrary in Δ_{n_k}.

The interpretation of the game is that the players 1, 2 select (i, j) according to the distributions of their initial mixed strategies f_{01}, f_{02}. The players are told the result x_1 of the first play which was determined according to the distribution m_{ij}. Considering this result they again select (i, j) according to the distributions f_{11}, f_{12} leading via the distribution m_{ij} to the new result x_2. In this way all former results can be used by their strategies to determine their present behavior.

Blackwell examines the long-run behavior of the game. Considering a set $S \subset R^N$ he asks: "... can I guarantee that the center of gravity of the payoffs in a long series of plays is in or arbitrarily near S, with probability approaching 1 as the number of plays becomes infinite?" He defines S as *approachable* with $\hat{f}_1$ in M, if, for every $\epsilon > 0$, there is n_0 such that for every f_2,

$$prob(\delta_n \geq \epsilon \quad \text{for some} \quad n \geq n_0) < \epsilon,$$

where δ_n denotes a distance measure of $\sum_{i=1}^n x_i/n$ from S and $x_1, ..., x_n$ are the variables determined by $\hat{f}_1, f_2$. S is *excludable* with $\hat{f}_2$ in M, if there exists $d > 0$ such that, for every $\epsilon > 0$, there exists an n_0 such that, for every f_1,

$$prob(\delta_n \geq d \quad \text{for all} \quad n \geq n_0) > 1 - \epsilon,$$

where $x_1, ..., x_n$ are the variables determined by $f_1, \hat{f}_2$. S is approachable (excludable) in M, if there exists $\hat{f}_1$ ($\hat{f}_2$) such that S is approachable (excludable) with $\hat{f}_1$ ($\hat{f}_2$).

Considering the matrix $\hat{M}$ whose elements $\hat{m}_{ij}$ are the expected values of the distributions m_{ij} Blackwell derives the following main results:

Let $T(q)$ denote the convex hull of the points $\sum_{j=1}^{n_2} q_j \hat{m}_{ij}, i \in \{1, ..., s_1\}$. A closed convex set is approachable iff it intersects every set $T(q)$. If it fails to intersect $T(q_0)$, it is excludable with $\hat{f}_2 : f_{n2}(x_1, ..., x_n) = q_0$.

For $n = 1$, let v, v' be the values of the games with matrices M, M'. If $v' < v$, a closed set S is approachable if it intersects the closed interval $[v', v]$ and excludable otherwise. If $v' > v$, a closed set S is approachable if it contains the closed interval $[v, v']$ and excludable otherwise.

Another work central to game theory and important to the concept of MCDM was done by Nash (1951). He examined n-person games denoting mixed strategies as n-tuples $s = (s_1, ..., s_n)$ where s_i is the mixed strategy of player i. Using a substitution expression

$$(s; t_i) := (s_1, ..., s_{i-1}, t_i, s_{i+1}, ..., s_n)$$

he introduced the definition of an equilibrium point: "An n-tuple s is an equilibrium point iff for every i

$$p_i(s) = max_{\text{all}\, r_i}[p_i(s, r_i)]"$$

where p_i is the payoff function of player i. He uses this noncooperative solution, today called a Nash equilibrium, to prove that every finite game has an equilibrium point.

In the early fifties the first works on vector optimization theory, especially in the context of linear production theory, have been published. Gale, Kuhn, and Tucker (1951) analyze matrices with relations $>$ and $\geq$ similarly defined to the vector-valued case. A matrix $\hat{D}$ is called *maximal* within a class of matrices $\mathbf{D}$ iff there is no $D \in \mathbf{D}$ such that $D > \hat{D}$. Then they consider the following two problems:

1. Find a maximal matrix $\hat{D}$ with $Cx \geqq \hat{D}y$ for some $x \geqq 0, y > 0$, such that $Ax \leqq By$.

2. Find a minimal matrix $\tilde{D}$ with $B'u \leqq \tilde{D}'v$ for some $u \geqq 0, v > 0$, such that $A'u \geqq C'v$.

For these two problems Gale, Kuhn, and Tucker state a first duality theorem: A matrix $\hat{D}$ is a solution of problem 1 iff it is a solution ($\tilde{D}$) for problem 2. They also prove the existence of a solution if there exist some $x \geqq 0, y > 0$ with $Ax \leqq By$ and some $u \geqq 0, v > 0$ with $A'u \geqq C'v$.

Finally, they relate these results to game theory: A matrix $\hat{D}$ is a solution for problem 1 (and 2), iff the value of the game with the payoff matrix

$$\begin{pmatrix} A & -B \\ -C & \hat{D} \end{pmatrix}$$

equals 0 and optimal mixed strategies are

$$\begin{bmatrix} u \\ v \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix}$$

such that $v > 0$ and $y > 0$.

Koopmans (1951) is another classic contribution to the development of VO. Introducing the term efficiency, Koopmans analyzed linear vector optimization in the context of production theory. Let y be the net output vector of m commodities $y_1, ..., y_m$ and x the input vector of n activities with the levels $x_1, ..., x_n$. They are linearly connected such that

$$y = Ax \quad \text{where} \quad A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix}$$

is called the *technology matrix*. A commodity vector is *possible* in a technology A iff there exists an activity vector $x, x \geqq 0$ with $y = Ax$. A vector $\hat{y} \in R^m$ is called *efficient* iff $\hat{y}$ is possible and if there exist no possible $y \in R^m$ such that $y - \hat{y} \geq 0$.

A necessary and sufficient condition for a vector $\hat{y}$ to be efficient is that $\hat{y}$ has a positive normal p to A defined by

$$p'A \leqq 0, \quad p'\hat{y} = 0, \quad p > 0.$$

Koopmans also gives an interpretation of p (in an efficient point $\hat{y}$) as a vector of prices p_i of the commodities $i = 1, ..., n$ in $\hat{y}$. Then he states a scalarization result characterizing efficient solutions (p. 66):

A necessary and sufficient condition for an activity vector $\hat{x}$ to lead to an efficient point $\hat{y} = A\hat{x}$ in the commodity space is that there exists a vector p of positive prices ($p > 0$) such that the profit on all activities carried out at a positive level is zero.

Koopmans also analyzes the connectedness of the set of efficient points for the linear vector optimization problem, gives a refinement of the problem distinguishing between primary factors, intermediate commodities, and final commodities, and obtains a plenty of other production theoretical results.

The term *vector maximum problem* was introduced by Kuhn and Tucker (1951). Considering a differentiable function $\phi : R^n_+ \times R^m_+ \to R$ they denote the partial derivatives evaluated at a point $(x_0, u_0) \in R^n_+ \times R^m_+$ by

$$\phi^0_x = [\partial\phi/\partial x^i]^0, \phi^0_u = [\partial\phi/\partial u^h]^0$$

Then they define the three conditions:

$$\text{(a)} \quad \phi^0_x \leqq 0, \quad \phi^{0\prime}_x x_0 = 0, \quad x_0 \geqq 0,$$

$$\text{(b)} \quad \phi_u^0 \overset{\geq}{=} 0, \quad \phi_u^{0\prime} u_0 = 0, \quad u_0 \overset{\geq}{=} 0,$$

necessary for ϕ to have a saddle point in (x_0, u_0), and

$$\text{(c)} \quad \phi(x, u_0) \overset{\leq}{=} \phi(x_0, u_0) + \phi_x^{0\prime}(x - x_0).$$

Kuhn and Tucker consider a VO problem (P)

$$\min f(x) = (f_1(x), ..., f_p(x))$$

$$\text{s. t.} \quad x \in X = \{x : g(x) = (g_1(x), ..., g_m(x)) \overset{\leq}{=} 0\},$$

where all the functions f_i and g_j are assumed to be continuously differentiable and $X \subset R_+^n$.

Kuhn and Tucker give a first definition of properly efficient solutions in order to exclude certain anomalies among the efficient solutions and to characterize the remaining solutions by the scalarization given below: A vector $\hat{x}$ is called *properly efficient* of the VO problem (P) iff $\hat{x}$ is efficient and if there is no h such that

$$< \nabla f_i(\hat{x}), h > \overset{\leq}{=} 0 \quad \text{for all} \quad i \in \{1, ..., p\},$$

$$< \nabla f_i(\hat{x}), h > < 0 \quad \text{for some} \quad i \in \{1, ..., p\},$$

and

$$< \nabla g_j(\hat{x}), h > \overset{\leq}{=} 0 \quad \text{for all} \quad j \in \{j : g_j(\hat{x}) = 0\}.$$

These conditions are also called the *Kuhn-Tucker constraint qualifications.*

They then derive a scalarization result characterizing properly efficient solutions: To assure the proper efficiency of a point x it is necessary that there exists a $v > 0$ and a u satisfying conditions (a) and (b) for ϕ defined by

$$\phi(x, u) = v' f(x) + u' g(x).$$

The additional validity of condition (c) is sufficient for x to be properly efficient. Next, Kuhn and Tucker state an equivalence with a saddle-point problem: Let f, g be concave for $x \in X$. Then x is a properly efficient solution of (P) iff there exist some $v > 0$ and some u such that (x, u) is a solution of the saddle-value problem for

$$\phi(x, u) = v' f(x) + u' g(x).$$

Slater (1950) gives a similar characterization together with a generalization of concavity and another (weaker) concept of efficiency (originally called maximality): f and g are assumed to be continuous. Let $u, v \overset{\geq}{=} 0$ and assume that, for $x_1, x_2 \in X$,

$$v' f(x_1) + u' g(x_1) = v' f(x_2) + u' g(x_2).$$

Then, (f, g) is called *almost concave* iff

$$v'f(x) + u'g(x) \geqq v'f(x_1) + u'g(x_1)$$

for all x on the segment joining x_1, x_2.

$\hat{x} \in X$ is called *Slater-efficient* iff there is no $x \in X$ such that $f(x) > f(\hat{x})$.

Slater states that for (f, g) almost concave, g regular (which means that there is some $x \geqq 0$ with $g(x) > 0$), and Φ defined by $\Phi(x, z, y) = y'f(x) + z'g(x)$, that $\hat{x} \in X$ is Slater-efficient iff there exist $\hat{y} > 0, \hat{z} \geqq 0$, such that

$$\Phi(x, \hat{z}, \hat{y}) \leqq \Phi(\hat{x}, \hat{z}, \hat{y}) \leqq \Phi(\hat{x}, z, \hat{y}),$$

for all $x \geqq 0$ and all $z \geqq 0$.

Geoffrion (1968) gives a more usable definition of proper efficiency which does not require differentiability: x^0 is called *properly efficient*, iff x^0 is efficient and if there exists $M \in R, M > 0$, such that for each i and each $x \in X$ satisfying $z_i(x) > z_i(\hat{x})$, there exists a j such that $z_j(x) < z_j(\hat{x})$ and

$$\frac{z_i(x) - z_i(\hat{x})}{z_j(\hat{x}) - z_j(x)} \leqq M.$$

If all the functions f_i and g_j are convex, then a properly efficient solution in the sense of Kuhn and Tucker is also properly efficient in the sense of Geoffrion. The converse of the statement requires the validity of the Kuhn-Tucker constraint qualifications.

Geoffrion characterized the properly efficient solutions by the following scalarization result: Let $\lambda \in \delta_p, \lambda > 0$. If $\hat{x}$ is optimal for $max_{x \in X} \lambda' f(x)$ then $\hat{x}$ is a properly efficient solution of the VO problem (P).

If X is convex, and $f_k, k = 1, ..., p$ are concave on X then the converse is true: For every properly efficient solution $\hat{x}$ there exists a $\lambda \in \delta_p, \lambda > 0$ such that $\hat{x} = max_{x \in X} \lambda' f(x)$.

As we have seen, VO was influenced from utility theory and economics, game theory, mathematical work on order relations and vectorial norms, linear production theory, and nonlinear programming. Since the early seventies these approaches have led to the foundation of a new scientific community associated with the decision- and method-oriented term MCDM - multiple criteria decision making. The more mathematical development of VO has led to a generalization of VO theory especially by means of cone dominance.

3 Main Streams of Modern VO

As the main streams of modern VO theory the following fields can be regarded: 1) cone dominance theory, 2) efficiency, 3) duality, and 4) stability.

3.1 Cone Dominance Theory

In 1974 Yu generalized VO theory using cones for the characterization of efficiency (first introduced by Koopmans (1951)) which has become a standard concept for dealing with VO. It is applicable in finite and infinite dimensional spaces. A survey on VO in infinite dimensional spaces has been given by Dauer and Stadler (1986). Here, we summarize some basic concepts of this theory:

Let Y be a vector space. $K \subset Y$ is called a *cone* iff $\alpha x \in K$ for all $x \in K, \alpha > 0$. K is *convex cone* iff K is convex and a cone. A cone K is called *pointed* iff $K \cap -K = \{0\}$. Note that in literature different definitions are given. Stadler (1979), for instance, requires for a wedge (= cone) α to be $\geqq 0$.

Cones are used to induce ordering relations. Thus, the definitions of $\overset{\leqq}{=}, \leq$, and $<$ can be generalized:

$$\begin{aligned} x \overset{\leqq}{=}_K y \quad &\text{iff} \quad y - x \in K, \\ x \leq_K y \quad &\text{iff} \quad y - x \in K, \quad x - y \notin K, \\ x <_K y \quad &\text{iff} \quad y - x \in \text{int}K, \end{aligned}$$

where int denotes the topological interior. $\overset{\leqq}{=}_K$ is the K-induced partial preorder on Y.

Many authors demand that K is pointed. Then $\leq_K$ can be defined simply as

$$x \leq_K y \quad \text{iff} \quad y - x \in K \backslash \{0\}$$

and $\overset{\leqq}{=}_K$ becomes a partial order. If K is given, then we also write $\overset{\leqq}{=}, \leq, <$ instead of $\overset{\leqq}{=}_K, \leq_K, <_K$. If $K = R^p_+$ then these definitions of the relations $\overset{\leqq}{=}$, $\leq$, $<$ are the same as the formerly used ones. Based on this theory, efficiency, duality, and other concepts can be defined accordingly.

3.2 Efficiency

With above concepts we can generalize the term efficiency. We now assume that K is a nontrivial closed convex cone. $\hat{y} \in A \subset Y$ is called *efficient (maximal)* in A iff

$$(\hat{y} + K) \cap A = \{\hat{y}\}.$$

$\hat{y}$ is called *weakly efficient* in A iff

$$(\hat{y} + \text{int}K) \cap A = \{\hat{y}\}.$$

For the generalization of proper efficiency several definitions have been given:

Borwein (1977) uses tangent cones for his concept of proper efficiency. Let $S \subset Y$ and $y \in S$. The *tangent cone to S at y*, $T(S, y)$, is the set of limits $h = lim_{k \to \infty} t_k(y^k - y)$, where (t_k) is a sequence in R^+ and (y^k) is a

sequence in S with limit y. Here, an element $\hat{y} \in Y$ is called *properly efficient (maximal)* iff $\hat{y}$ is efficient (maximal) and $T(Y - K, \hat{y}) \cap K = \{0\}$.

Another generalization of proper efficiency (in finite-dimensional spaces) is due to Benson (1979). Benson uses the concept of projecting cones: Let $S \subset Y$. The *projecting cone of* S is defined as $P(S) := \{\alpha y : \alpha \in R, \alpha > 0, y \in Y\}$. $\hat{y} \in Y$ is said to be *properly efficient (maximal)* iff $\hat{y}$ is efficient (maximal) and $\mathrm{cl}P(Y - K - \hat{y}) \cap K = \{0\}$ where cl denotes the closure.

In Sawaragi et al. (1985), p. 44, a survey of the relationships between the concepts of proper efficiency of Borwein, Benson, Henig (1982), Kuhn/Tucker, and Geoffrion is presented. Gal (1986) gives another brief survey on efficient sets.

3.3 Duality

The basic idea of duality theory is as follows: For a given vector minimization problem (P)

$$min_{x \in G} f(x),$$

one considers a vector maximization problem (P')

$$max_{x \in G'} f'(x)$$

such that certain properties of (P) and (P') are satisfied. (P) and (P') are called *weakly dual* if for any (feasible) solution $\hat{x}$ of (P) and any (feasible) solution $\tilde{x}$ for (P') $f(\hat{x}) \not\leq f'(\tilde{x})$ (*weak duality relation*) is satisfied. That means, any solution y in the objective space which is feasible for (P) and (P') ($y \in f(G) \cap f'(G')$) is also an efficient solution for (P) and (P'). We call (P') *strongly dual* to (P) if (P) and (P') have the same efficient set in the objective space. Jahn (1987) has extended duality theory to partially ordered sets $(X, \leq)$ where the dual set of a nonempty primal set $P \subset X$ is defined as $D := \{d \in X : p \in P \Rightarrow p \not\leq d \vee d \leq p\}$. For P and D weak and strong duality can be proven. (For the opposite direction of the converse strong duality proof an additional assumption is necessary.)

It is possible to distinguish between different main directions in duality theory depending on how (P') is constructed. Let us first consider an approach involving modified Lagrange-mappings (Ivanov, Nehse (1985)). Let (P) be defined as $f(x) \to \min$, such that $x \in \{x : g(x) \overset{\leq}{=} 0\}$, where $f : R^n \to R^p$ and $g : R^n \to R^m$ are convex and differentiable. A dual (called modified Wolfe-dual) problem (P') can be formulated by $F(x, L) \to \max$, such that $(x, L) \in \{(x, L) \in R^n \times R^{p \times m} : \exists u > 0 \text{ with } u'L \overset{\geq}{=} 0', u'F_x(x, L) = 0'\}$ where F is the Lagrange mapping defined by $F(x, L) := f(x) + Lg(x)$ and $R^{p \times m}$ is the linear space of $p \times m$ matrices. The strong duality for (P) and (P') can be proved, too.

Using the duality theory given by Van Slyke and Wets (1968), Jahn (1983) studies a dual problem $\max\{y \in Y : \exists t \in C_{Y^*}^{\#}, u \in C_{Z^*} \text{ with } t(f(x)) + u(g(x)) \geq t(y) \forall x\}$. Here, $C_{Y^*}^{\#}$ is the quasi-interior of the ordering cone in the

topological dual space of Y and C_{Z^*} is the ordering cone in the topological dual space of Z. Jahn can prove weak, strong, and strict converse duality theorems under suitable assumptions.

Other approaches to duality use conjugacy in vector optimization (Sawaragi, Nakayama, Tanino, 1985, Chapter 6). For instance, Tanino and Sawaragi (1980) consider the perturbed extension

$$(\mathrm{P_u}) \qquad \min_x \phi(x,u), u \in R^m$$

of an VO problem $\min_x f(x)$ with $\phi : R^n \times R^m \to R^p \cup \{+\infty\}$ such that $\phi(x,0) = f(x) \forall x \in R^n$. The *conjugate map* of ϕ is defined as the point-to-set map

$$\phi^*(T,\Lambda) = max\{Tx + \Lambda u - \phi(x,u) \in R^p : x \in R^n, u \in R^m\}$$

with $T \in R^{p \times n}, \Lambda \in R^{p \times m}$. The dual problem with respect to given perturbations is then defined as:

$$(\mathrm{D}) \qquad \max_{\Lambda} -\phi^*(0,\Lambda).$$

For $(\mathrm{P_u})$ and (D) they prove a weak duality theorem: For any $x \in R^n, \Lambda \in R^{p \times m}, \phi(x,0) \notin -\phi^*(0,\Lambda) - R^p_+/\{0\}$. Iff (P) is *stable* (in the sense that the perturbation map $W : u \mapsto Min\{\phi(x,u) : x \in R^n\}$ is subdifferentiable at 0), then for each solution $\hat{x}$ of (P) there exists a solution $\hat{\Lambda}$ of (D) such that $\phi(\hat{x},0) \in -\phi^*(0,\hat{\Lambda})$. Conversely, if for $\hat{x} \in R^n, \hat{\Lambda} \in R^{p \times m}$ the condition $\phi(\hat{x},0) \in -\phi^*(0,\hat{\lambda})$ is satisfied, then $\hat{x}$ is a solution of (P) and $\hat{\Lambda}$ is a solution of (D).

Then, they consider a constrained VO problem $\min f(x) \quad$ with $\quad g(x) \leqq 0$ and a corresponding perturbed problem $\min_x \phi(x,u)$ with

$$\phi(x,u) = \begin{cases} f(x), \text{if} \quad g(x) \leqq 0, \\ +\infty, \text{else.} \end{cases}$$

The conjugate map ϕ^* is now defined via a Lagrangian expression (for $\lambda \in R^m$):

$$-\phi^*(0,\lambda) = min\{\phi(x,u) - < \lambda, u > \in R^p : x \in R^n, u \in R^m\}$$

For $u = g(x) + v, v \geqq 0$ the dual problem can be written as

$$(\mathrm{D}) \qquad \max \bigcup_{\lambda \leqq 0} min\{f(x) - < \lambda, g(x) > : x \in R^n\}.$$

For (P) and (D) similar duality theorems as for the above unconstrained case can be derived. Moreover, if (P) is stable then $\hat{x} \in R^n$ is a solution of (P)

iff there exists $\hat{\lambda} \in R^m$ such that $(\hat{x}, \hat{\lambda})$ is a saddle point of the Lagrangian $L(x, \lambda) = min\{\phi(x, u) - < \lambda, u >: u \in R^m\}$.

3.4 Stability

Stability is usually analyzed using perturbation maps. Sawaragi, Nakayama, and Tanino (1985) consider a VO problem with two parameters: $u \in U$ specifies the set of feasible solutions and the objective function. $v \in V$ specifies the domination structure in the objective space. The parametrized set of feasible solutions is defined as $Y(u) := \{y \in R^p : y = f(x, u), x \in X(u)\}$. The (efficient) solution sets in objective space and decision space are defined as $N(u, v) := E(Y(u), D(v))$ (where $D(v)$ denotes the parametrized domination structure) and $M(u, v) := \{x \in X(u) : f(x, u) \in N(u, v)\}$. Keeping one parameter $\hat{v} \in V$ resp. $\hat{u} \in U$ fixed they define the solution sets $N_1(u) := N(u, \hat{v}), M_1(u) := M(u, \hat{v})$ resp. $N_2(v) := N(\hat{u}, v), M_2(v) := M(\hat{u}, v)$. For these sets sufficient conditions for upper and lower semicontinuity can be derived. Other aspects of stability theory and sensitivity analysis are discussed by Wolf (1989).

As we have seen the main streams which cristalized during the development of VO and MCDM in the last decades can be subdivided into cone dominance theory and efficiency concepts, stability and duality theory.

4 Future Developments

Concerning the future development one can only guess what the main streams will be. This trial to predict future developments is based on the trends as shown up in the last decades of the twentieth century. The future developments of VO will be influenced, obviously, by 1) theoretical and mathematical aspects and 2) method-oriented and practically motivated trends. Both of this main areas can be subdivided.

The first, mathematical aspect is influenced by 1) generalization, 2) specialization, and 3) interrelations between VO and other mathematical fields:

With respect to 1), the generalization of concepts and results is a main aspect of the evolution of VO during the last twenty years. In the historical development concrete optimization problems have been replaced by concepts of abstract optimization. From considering finite dimensional, real, Euclidian vector spaces first, more abstract, possibly infinite dimensional spaces have been analyzed. Assumptions concerning convexity, concavity, or smoothness of involved sets and functions have been generalized (e.g. by concepts of quasi or pseudo convexity). It may be useful and fruitful not to assume properties such as convexity or even connectedness of the feasible set at all. Especially the generalization of the involved order relations from classical partial orders in finite dimensional vector spaces to arbitrary cone induced orders or other domination structures led to considerable progress in VO theory and application. We do think that these trends will remain and imprint the future development of theoretical VO research.

With respect to 2), specialization is the treatment of VO problems with a special structure, to obtain more concrete results on the efficient set etc. Some problems like the linear VO problems which dominated the origins of VO (and still are an important area of research) or the multiple objective (linear) fractional programming problems brought up a considerable number of publications. Other problems have been treated only occasionally. Most special problems in optimization theory and decision analysis can be generalized from a single scalar to a multiple, vector valued definition of goals. Many problems have already been reformulated that way. For instance, dynamic programming has benefited from these tendencies. The main theorem has been generalized so that not optimal but efficient partial strategies build up the whole strategy. (Markov) decision chains and other control problems fall into the same category. Summarizing, specialization has already influenced several areas investigating special optimization problems. Other areas, for instance in the field of combinatorial optimization have benefited only scarcely from VO.

A third but possibly most interesting trend in the future development of VO is based on the development of relationships between VO and other areas in mathematics: The application of VO in other areas as well as the use of other areas for VO. An example for the application of VO is the treatment of vectorial norms which has induced new and increasing interest in the application in approximation theory. An important example where VO has benefited from other work is Ekeland's (1974) variational principle which offers a lot of potential for further research.

As has been said above there are mutual influences between theory and methods of VO / MCDM. Today, operations research (OR) has received several methodological expansions leading beyond the former frontiers of applicability:

A major recent development in the framework of the second aspect is the increasing importance of computer and software technology. The rise of computer technology accompanied VO since the early seventies when the first VO software (goal programming even earlier) was implemented. Especially, since the midth of the eighties when personal computers and workstations became a standard equipment in universities, the implementation of methods became much easier. This also facilitated the development of new algorithms. Interactivity was regarded as a standard feature of many programs. Problems in the application of VO methods by non-specialists as well as improvements in programming support and interface tools like window-based operating systems and graphic processing improved the usability of newer software. Especially for non-interactive software, the integration of a report generator is an essential feature for post-optimal analysis and the interpretation of the results. These features as well as trends towards an integration of different methods and the usage of knowledge led to the decision support system (DSS) philosophy.

Artificial intelligence (AI) has become an important research area provi-

ding tools for managerial decision making. We see many fusions with OR as well as replacements of OR. Practical applications of AI have already led to a sizeable number of expert systems and knowledge-based systems.

Fuzzy set theory has become a new paradigm for dealing with ill-structured systems. Fuzzy control showed impressive success in applications. Analogue to the fuzzy approach, we find statistical, probabilistic and possibilistic approaches dealing with incomplete information or uncertainty. A newer approach is the theory of rough sets (Pawlak (1992)).

Finally, the generic term 'parallel problem solving from nature' summarizes several new disciplines. (Artificial) neural networks are the most prominent representative. They have proven to be successful for applications in engineering as well as in economics. The new discipline saw an enormous growth, esp. during the eighties. Approaches in evolutionary computation (genetic algorithms, evolution strategies) followed shortly afterwards. A first approach of using evolution strategies for vector optimization was elaborated by Kursawe (1992). Neural networks have been considered for MCDM by Malakooti and Zhou (1994) and Hanne (1994). Also simulated annealing has to be mentioned in the context of 'parallel problem solving from nature'.

Especially, the fuzzy and nature-analogous approaches proved to be successful in technical applications and well-defined hard problems: E. g. fuzzy in control problems, neural networks for the computerization of typical human tasks like pattern recognition, genetic algorithms for combinatorial optimization, and evolution strategies for nonlinear programming.

All these new methods are highly relevant for the future development of MCDM methods. Some of them (esp. DSS approaches, fuzzy set theory) have already gained a remarkable echo in this area of application. In many cases this will lead to new demands for theoretical work. Especially, the fuzzy set approach has already effected theoretical research. How, for instance, is efficiency defined and treated in a fuzzy context?

Summarizing, the future VO research will be influenced by mathematical trends, especially the generalization of concepts and results, and the elaboration of new methods and software. In our opinion the future development should focus on four aspects: 1. Empirical research on the axioms of rational behavior and work on a better foundation of MCDM towards a deeper understanding of the decision processes, 2. improvement and elaboration of solution methods from all areas of MCDM including intelligent and self-learning approaches, 3. deeper elaboration of a DSS philosophy and improved software, 4. report generators.

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Generating Interior Search Directions in Multiple Objective Linear Programming Problems Using Aspiration Levels

Ami Arbel[1] and **Pekka Korhonen**[2]

[1] Industrial Engineering Department, Tel-Aviv University, Tel-Aviv, 69978, ISRAEL, *e-mail*: ami@eng.tau.ac.il, *Fax*: +972-3-642-9540

[2] Helsinki School of Economics, Runeberginkatu 14 -16, 00100 Helsinki, FINLAND, *e-mail*: korhonen@hkkk.fi, *Fax*: +358-0-4313-535

Abstract. We propose in this paper a new approach to generating interior search directions for multiple objective linear programming (MOLP) problems. The proposed approach is based on using achievement scalarizing functions and aspiration levels to convert a problem with multiple objectives to a problem with a single objective that is amenable for a solution by the interior point affine-scaling primal algorithm. The interior solution trajectory takes an interior starting solution and moves it to a nondominated solution on the efficient frontier that corresponds to the optimum of the achievement scalarizing function.

1. INTRODUCTION

Many solution approaches have been developed for addressing MOLP problems. Since this vector maximization problem does not generally have a unique solution, the intervention of a decision maker (DM) is required to find the "most preferred" solution. The resulting class of solution approaches is generally referred to as *interactive methods for multiple criteria optimization* (for an excellent review of this class of algorithms see, e.g., [7]).

Current MOLP procedures enable the DM to search for a solution in the so-called nondominated set of solutions. This is based on a common sense assumption that comparisons between dominated solutions may seem irrelevant from the DM's point of view. However, a search in a dominated (interior) set might be a valid approach at least for two reasons: (1) No simplex-type algorithm is needed to move from point to point in a criterion space, and (2) No premature termination is expected, because the DM is unlikely to stop while better solutions are available.

In large-scale MOLP-problems, a simplex-type algorithm, which is needed to move from facet to facet for finding a path in a nondominated set, requires too many simple pivoting operations to be a practical interactive approach. We propose here a new approach which generates *interior* search directions that guide the search for a solution of the MOLP problem. At each iteration, a decision maker is asked to specify aspiration levels for objectives, and using a

Chebyshev-type achievement scalarizing function [9-10] the aspiration levels are projected onto a nondominated set. This formulation leads to a single objective linear programming model. Using a variant of Karmarkar's algorithm [4], known as the interior-point affine-scaling linear programming algorithms (see e.g., [3], [8]) provides the mean for finding a solution path from a current (interior) solution point to a solution point corresponding to the optimum of the achievement scalarizing function.

First attempts at using the affine-scaling primal algorithm for MOLP problems were reported in [1-2], while the use of achievement scalarizing function was first proposed by Wierzbicki [9-10] in his reference point approach. In this paper, we combine these two ideas and develop an interactive approach which is controlled through the specification of aspiration levels. The resulting approach leads the DM through the interior of the feasible region to the optimum of an achievement scalarizing function, which depends on the aspiration levels specified for the objectives by the DM.

The paper is arranged as follows. In section 2 we provide a short review of the affine scaling primal algorithm and the some fundamentals of achievement scalarizing functions. Section 3 describes our proposed approach. Section 4 illustrates it with an example, and Section 5 provides a summary and some suggestions for future research.

2. ESSENTIAL MATHEMATICAL BACKGROUND

The proposed MOLP algorithm presented in this paper is based on combining the affine-scaling primal algorithm with the achievement scalarizing functions and aspiration levels. We provide a brief review of these two concepts in this section.

2.1. Affine-Scaling Primal Algorithm

The affine-scaling primal algorithm is the interior point linear programming algorithm chosen for implementing our proposed multiple objective algorithm. While it is not the best algorithm in its class, it has the distinct advantage of being simple to describe and easy to implement. In this section we provide a brief review of the single-objective algorithm.

In developing this algorithm we consider a linear programming problem in standard form:

$$\begin{aligned} &\min \quad c^T x \\ &\text{subject to:} \\ &\qquad \mathbf{A}x = b \\ &\qquad x \geq 0, \end{aligned} \tag{2.1}$$

where $x \in \mathbf{R}^n$, $b \in \mathbf{R}^m$ and where the constraint matrix $\mathbf{A}$ is of dimension $m \times n$.

The iterative process starts from a solution, x_0, that is both interior and feasible, that is, $\mathbf{A}x_0 = b$, $x_0 > 0$. The algorithmic steps are aimed at deriving a

step direction vector, dx, that moves the current iterate to a new iterate, x_{new} while maintaining feasibility and reducing the value of the objective function. These feasibility and descent requirements respectively, lead to the following two conditions for dx:

$$\mathbf{A}x_{new} = b \tag{2.2}$$
$$c^T x_{new} \leq c^T x_0. \tag{2.3}$$

If the relation between the new iterate and the current iterate is given through $x_{new} = x_0 + dx$, then the two requirements of (2.2) and (2.3) lead to

$$\mathbf{A}dx = 0 \tag{2.4}$$
$$c^T dx \leq 0 \tag{2.5}$$

Condition (2.4) requires that the step direction vector be in the null space of $\mathbf{A}$, and using the projection operation given by

$$\mathbf{P} = \mathbf{I}_n - \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}\mathbf{A}, \tag{2.6}$$

the step direction vector is then given by $dx = -\mathbf{P}c$. It is easily verified that condition (2.4) is satisfied. To check that condition (2.5) holds as well we note that

$$c^T dx = -c^T \mathbf{P}c = -c^T \mathbf{P}^2 c = -\| \mathbf{P}c \|^2 \leq 0, \tag{2.7}$$

because $\mathbf{P}$ is symmetric and idempotent: $\mathbf{P} = \mathbf{P}^2$.

To further develop the algorithm we introduce the concept of *scaling* which gives the resulting algorithm its name. Performing a scaling operation allows the transfer of the current iterate in the original space to a new location in the scaled space that is of unit distance from all the "walls" of the constraints polytope. We summarize next the details involved with the derivation of the affine-scaling primal algorithm.

Consider a diagonal $n \times n$ scaling matrix $\mathbf{D} = diag(x_1, x_2, \ldots, x_n)$, where x_i is the i-th component of the decision vector x. Using the transformation $x_1 = \mathbf{D}^{-1}x$, we convert the original linear programming problem of (2.1) to the *scaled* problem given by

$$\begin{aligned} &min \quad c_1^T x_1 \\ &\text{subject to:} \\ &\qquad \mathbf{A}_1 x_1 = b \\ &\qquad\quad x_1 \geq 0 \end{aligned} \tag{2.8}$$

where the scaled constraint matrix $\mathbf{A}_1$ is given by $\mathbf{A}_1 = \mathbf{A}\mathbf{D}$ and the scaled cost vector is $c_1 = \mathbf{D}c$. Projecting now the cost vector c_1 on the null space of $\mathbf{A}_1$, the step direction vector, dx_1, is then given by

$$dx_1 = -\mathbf{P}_1 c_1 = -\mathbf{D}[c - \mathbf{A}^T(\mathbf{A}\mathbf{D}^2\mathbf{A}^T)^{-1}\mathbf{A}\mathbf{D}^2 c] \tag{2.9}$$

and using the relation between the scaled space and the original space given through $dx = \mathbf{D}dx_1$, the step direction vector, dx, in the original space is then given by

$$dx = -\mathbf{D}^2[c - \mathbf{A}^T(\mathbf{A}\mathbf{D}^2\mathbf{A}^T)^{-1}\mathbf{A}\mathbf{D}^2\mathbf{c}]. \tag{2.10}$$

This step direction vector can also be written through

$$dx = -\mathbf{D}^2[c - \mathbf{A}^T y], \tag{2.11}$$

where y is the estimate of the *dual vector* that is obtained by solving the symmetric system of equations given by

$$\mathbf{A}\mathbf{D}^2\mathbf{A}^T y = \mathbf{A}\mathbf{D}^2\mathbf{c}. \tag{2.12}$$

The new iterate is then given by

$$x_{new} = x_{old} + \rho\alpha dx, \tag{2.13}$$

where α is a step size that guards against violating the inequality constraints of (2.1) and where $0<\rho<1$ is a step size *factor* that keeps the current iterate interior in polytope. This sequence of scaling the current iterate and projecting the cost vector continues until the *relative duality gap* defined through

$$gap = \frac{|\mathbf{c}^T\mathbf{x} - b^T y|}{1 + |c^T x|} \tag{2.14}$$

falls below a pre-specified tolerance level.

2.2. Achievement Scalarizing Functions

We consider the following multiple objective linear programming (MOLP) problem through the rest of this paper:

$$\begin{aligned} &\text{"}max\text{"} \quad \mathbf{C}x = v \\ &\text{subject to:} \\ &\qquad x \in \mathbf{S} = \{x \mid \mathbf{A}x = b,\ x \geq 0\} \end{aligned} \tag{2.16}$$

where $x \in \mathbf{R}^n$, $b \in \mathbf{R}^m$, the constraint matrix $\mathbf{A}$ is of dimension $m \times n$ and of full rank m, and the objective matrix $\mathbf{C}$, whose rows are the single objectives, is of dimension $q \times n$.

In (2.16), $x^* \in$ S is *efficient* iff there does not exist another $x \in$ S such that $\mathbf{C}x \geq \mathbf{C}x^*$ and $\mathbf{C}x \neq \mathbf{C}x^*$. And $x^* \in$ S is *weakly efficient* iff there does not exist another $x \in$ S such that $\mathbf{C}x > \mathbf{C}x^*$. Let $V = \{v \mid v = \mathbf{C}x, x \in S\}$ be the set of *feasible* criterion vectors (i.e., the feasible region in criterion space). Vectors $v \in$ V corresponding to efficient points are called *nondominated* criterion vectors and vectors $v \in$ V corresponding to weakly efficient points are called *weakly nondominated* criterion vectors. The set of all (weakly) efficient points is called the *(weakly) efficient set*, and the set of all (weakly) nondominated criterion vectors is called the *(weakly) nondominated set.*

The search in the set of nondominated solutions can be controlled by means of an *achievement (scalarizing) function* as suggested by Wierzbicki [8-9]. Such a function projects any given (feasible or infeasible) point $g \in \mathbf{R}^q$ onto the set of

(weakly) nondominated solutions. The simplest form proposed by Wierzbicki [9] is the following:

$$s(\boldsymbol{g},\boldsymbol{v},\boldsymbol{w}) = max\ [(g_i - v_i)/w_i,\ i \in \{1,2,\ldots,q\}], \tag{2.17}$$

where $\boldsymbol{w} > \boldsymbol{0}$ is a q-vector of weights, $\mathbf{g} \in \mathbf{R}^q$ an arbitrary vector whose components are called *aspiration levels*, and $\boldsymbol{v} \in V = \{\mathbf{C}\boldsymbol{x} \mid \boldsymbol{x} \in S\}$. By minimizing $s(\boldsymbol{g},\boldsymbol{v},\boldsymbol{w})$ subject to $\boldsymbol{v} \in V$, we find a weakly nondominated solution vector $\boldsymbol{v}^1$ (see, e.g., Wierzbicki [9-10]). If the given solution $\boldsymbol{g} \in \mathbf{R}^q$ is feasible, then for a solution vector $\boldsymbol{v}^1$, $\boldsymbol{v}^1 \geq \boldsymbol{g}$. If the solution is not unique, to generate only nondominated (instead of weakly nondominated) solutions, more complicated forms or a lexicographic formulation has to be used to guarantee uniqueness. For simplicity, we assume that the solution is always nondominated.

Given $\boldsymbol{g} \in \mathbf{R}^q$, the minimum of an achievement scalarizing function can be found by solving the following problem:

$$\begin{aligned} &min \quad \varepsilon \\ &\text{subject to:} \\ &\qquad \varepsilon \geq (g_i - \boldsymbol{c}_i\boldsymbol{x})/w_i\ ,\ i = 1,2,\ldots,q \\ &\qquad \boldsymbol{x} \geq \boldsymbol{0}, \end{aligned} \tag{2.18}$$

where $\boldsymbol{c}_i$, $(i = 1,2,\ldots,q)$, refers to the i-th row of the matrix $\mathbf{C}$. Using surplus variables, $\boldsymbol{z}$, the problem (2.18) can further be written as:

$$\begin{aligned} &min \quad \varepsilon \\ &\text{subject to:} \\ &\qquad \boldsymbol{x} \in \mathbf{S} \\ &\qquad \mathbf{C}\boldsymbol{x} + \varepsilon\boldsymbol{w} - \boldsymbol{z} = \boldsymbol{g} \\ &\qquad\qquad \boldsymbol{x}, \boldsymbol{z} \geq \boldsymbol{0}. \end{aligned} \tag{2.19}$$

The MOLP-problem is now reduced to a scalar optimization problem. Without loss of generality, we assume that $\varepsilon > 0$. It means that for a given aspiration level vector $\boldsymbol{g} \in \mathbf{R}^q$ there exists no $\boldsymbol{v} \in V$ such that $\boldsymbol{v} \geq \boldsymbol{g}$. Using formulation (2.19) each aspiration level vector is projected onto the set of (weakly) nondominated set. The DM can vary an aspiration level vector and the system will provide a nondominated solution for further evaluation. The DM stops when convinced that the most preferred solution is found.

3. THE PROPOSED APPROACH

Using the achievement scalarizing function shown in (2.19) allows augmenting the various matrices/vectors and leads to the following single objective linear programming problem

$$\begin{aligned} &min \quad \gamma^T\boldsymbol{u} \\ &\textit{subject to:} \\ &\qquad \mathbf{E}\boldsymbol{u} = \boldsymbol{d} \\ &\qquad\quad \boldsymbol{u} \geq \boldsymbol{0}, \end{aligned} \tag{3.1}$$

where $\boldsymbol{u}=[\boldsymbol{x}\ \varepsilon\ z]^T \in \mathbf{R}^{n+q+1}$, $\mathbf{E}$ is an $(m+q)\times(n+q+1)$ matrix and $\boldsymbol{d} \in \mathbf{R}^{m+q}$ and are given by

$$\mathbf{E}=\begin{pmatrix} \mathbf{A} & \boldsymbol{0} & \mathbf{O} \\ \mathbf{C} & \boldsymbol{w} & -\mathbf{I}_q \end{pmatrix} \qquad \boldsymbol{d}=\begin{pmatrix} \boldsymbol{b} \\ \boldsymbol{g} \end{pmatrix} \tag{3.2}$$

and where the coefficient vector of the objective function is denoted by γ and is all zeros except at the $(n+1)$ position where its coefficient is one.

We propose the use of an interior point approach to solve problem (3.1). In the original reference point approach, for each aspiration level vector a nondominated solution corresponding to a (precise) optimum of an achievement scalarizing function is generated for the DM's evaluation. However, especially in the beginning of the search process, finding precise solutions is not important because at this stage the DM is more concerned with gathering information about the problem rather than trying to find the most preferred solution. The interior point approach provides a natural tool to take into account a need to vary the precision of solutions generated.

Ideally, the algorithm should allow the DM to experiment with different aspiration levels. Setting one aspiration allows the solution trajectory to move through the interior close to a nondominated solution. If this candidate solution is not desirable, the DM can re-state his aspirations and move the solution trajectory to the vicinity of another nondominated solution. This process terminates when the candidate boundary solution is acceptable to the decision maker. The algorithm summarized next describes the sequence of steps needed to come close to a nondominated solution for a single given aspiration level. Future work should consider modifying this procedure by allowing the specification of multiple aspiration levels and devising a structured approach for its specification.

Summary of the Proposed Algorithm:

The algorithm is comprised of two loops: the outer loop controls the number of solution cycles with a given aspiration, while the inner loop runs the primal algorithm until the duality gap becomes small. The algorithm proceeds now as follows:

Step 0: Define a starting feasible and strictly interior solution vector x_0, (i.e., $\mathbf{A}x_0=\boldsymbol{b}$, and $x_0>\boldsymbol{0}$) to the original MOLP-problem (2.16). Display objective values $v_0=Cx_0$ to the DM and set $g_0 = v_0$. Form the augmented system as shown in (3.1) and (3.2), and adjust ε_0 and z_0, accordingly. Specify the maximum number of solution cycles for each aspiration level, $h_{max} > 0$, and set the cycle counter, $h:=0$, and the iteration counter, k at $k := 0$.

Step 1: Set $h:=h+1$ and $\mathbf{g}_h := \mathbf{g}_{h-1}$. Ask the DM to specify (new) aspiration levels g_h for the q objectives. If the DM is not willing to do it and k mod $h_{max} \neq 0$, then ***Stop.***

Set $\varepsilon_k > 0$ and $z_k > \mathbf{0}$ such that $\mathbf{C}x_k + \varepsilon_k w - z_k = g_h$.

Define the solution vector of the augmented system :

$\boldsymbol{u}(k) = [x_k \ \varepsilon_k \ z_k]^T > \mathbf{0}$

Step 2: Define the scaling matrix: $\mathbf{D} = diag(u_1(k), u_2(k),..., u_{n+q+1}(k))$, increment iteration counter $k:=k+1$, and solve for the $(m+q)$-dimensional *dual estimate* vector $y(k)$ from the symmetric system of equations given by

$(\mathbf{E}\mathbf{D}^2\mathbf{E}^T)y(k) = \mathbf{E}\mathbf{D}^2\gamma.$

Step 3: Evaluate the step direction vector, $\boldsymbol{du}(k)$, through

$\boldsymbol{du}(k) = -\mathbf{D}^2[\gamma - \mathbf{E}^T y(k)]$

and take a step toward the next iterate, $\boldsymbol{u}(k)$, given by

$\boldsymbol{u}(k) = \boldsymbol{u}(k\text{-}1) + \rho\alpha\boldsymbol{du}(k), \qquad 0 < \rho < 1$

and where the step size, α, is found from the *ratio test* given by

$\alpha = min\{ -\frac{u_i(k\text{-}1)}{du_i(k)}, \forall\, du_i(k) < 0, 1 \le i \le n \},$

Step 4: If k mod $h_{max} = 0$, then goto ***Step 1***; ***else***, evaluate the relative *duality gap*

$$gap = \frac{|\gamma^T \boldsymbol{u}(k) - \boldsymbol{d}^T y(k)|}{1.0 + \| \gamma^T \boldsymbol{u}(k) \|}$$

if the duality gap is small goto ***Step 1***;. Otherwise goto ***Step 2***.

4. AN ILLUSTRATIVE EXAMPLE

We demonstrate our proposed approach on the following example whose solution was reported earlier using simplex-based approaches [5-6]. Consider the MOLP problem described through:

$$\begin{aligned} &max \quad f_1(\boldsymbol{x}) = x_1 \\ &max \quad f_2(\boldsymbol{x}) = x_2 \\ &max \quad f_3(\boldsymbol{x}) = x_3 \end{aligned}$$

subject to:

$$\begin{aligned} 3x_1 + 2x_1 + 3x_3 &\le 18 \\ x_1 + 2x_2 + x_3 &\le 10 \\ 9x_1 + 20x_2 + 7x_3 &\le 96 \\ 7x_1 + 20x_2 + 9x_3 &\le 96 \\ x_1 \ , \ x_2 \ , \ x_3 &\ge 0 \end{aligned}$$

Adding the necessary slack variables, this problem becomes

$$\begin{array}{ll} \text{“}max\text{”} & \mathbf{C}\boldsymbol{x} \\ \textit{subject to:} & \mathbf{A}\boldsymbol{x} = \boldsymbol{b} \\ & \boldsymbol{x} \geq \boldsymbol{0}, \end{array}$$

where:

$$\mathbf{A} = \begin{pmatrix} 3 & 2 & 3 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 & 0 \\ 9 & 20 & 7 & 0 & 0 & 1 & 0 \\ 7 & 20 & 9 & 0 & 0 & 0 & 1 \end{pmatrix} \quad \boldsymbol{b} = \begin{pmatrix} 18 \\ 10 \\ 96 \\ 96 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Converting this system of constraints to the form amenable for the scalarizing function approach we arrive at the augmented system given by

$$\begin{array}{ll} \text{“}min\text{”} & \gamma^T \boldsymbol{u} \\ \textit{subject to:} & \mathbf{E}\boldsymbol{u} = \boldsymbol{d} \\ & \boldsymbol{u} \geq \boldsymbol{0}, \end{array}$$

where the augmented decision vector is now $\boldsymbol{u} = [\boldsymbol{x}\ \varepsilon\ \boldsymbol{z}]^T \in \mathbf{R}^{11}$, and

$$\mathbf{E} = \begin{pmatrix} \mathbf{A} & \boldsymbol{0} & \mathbf{O} \\ \mathbf{C} & \boldsymbol{w} & -\mathbf{I}_3 \end{pmatrix} \quad \boldsymbol{d} = \begin{pmatrix} \boldsymbol{b} \\ \boldsymbol{g} \end{pmatrix} \quad \gamma = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0]^T$$

The optimization problem is now: *min* ε, subject to the augmented system of constraints. As discussed in [4-5], assuming a utility function give by $u(\boldsymbol{x}) = min(3x_1, 5x_2, 3x_3)$, this problem has a unique optimal solution (excluding slack variables) given by $\boldsymbol{x}^* = [2.5\ 1.5\ 2.5]^T$. Note that this solution is on a face of the polytope and not at an extreme point.

To demonstrate our approach, we define $\boldsymbol{w} = [1\ 1\ 1]^T$ in the achievement scalarizing function (2.17) and assume an aspiration level vector $\boldsymbol{g} = [3.5\ \ 2.5\ \ 3.5]^T$. The nondominated solution corresponding to these specifications is the optimal solution vector corresponding to the utility function $u(\boldsymbol{x})$. Assuming, therefore, that the DM makes one specification of aspiration levels, the results of the proposed algorithm for a step size factor of $\rho = 0.9$ are shown in *Table 1*. For brevity we show only the original decision vector's components. We also evaluate the relative duality gap in the last column.

As we see from *Table 1*, our approach converges fast to the optimal solution that is known in this case. In the absence of a utility function and in an interactive environment, the true optimum is not known and there is no need to go onto the nondominated surface except at termination. Therefore, remaining interior and close to the surface, we can stop after a few iterations and ask the DM to re-specify an aspiration level vector. If the DM prefers to stay with the current aspiration, we continue to the optimum of the scalarizing function and end on the surface of the polytope. The plots of the relative duality gap and the evolution of decision variables are shown in Figures 1 and 2 respectively.

Table 1. Solution results ($\rho = 0.9$)

Iteration	x_1	x_2	x_3	*Gap*
0	1.0000	1.0000	1.0000	–
1	1.0851	1.0431	1.0851	0.2958
2	2.0440	1.1160	2.0440	0.2554
3	2.4455	1.4883	2.4455	0.0541
4	2.4924	1.5053	2.4924	0.0213
5	2.4967	1.4989	2.4967	0.0032
6	2.4994	1.5006	2.4994	0.0012
7	2.4998	1.4998	2.4998	0.0002
8	2.5000	1.5000	2.5000	0.0001
9	2.5000	1.5000	2.5000	0.0000
10	2.5000	1.5000	2.5000	0.0000

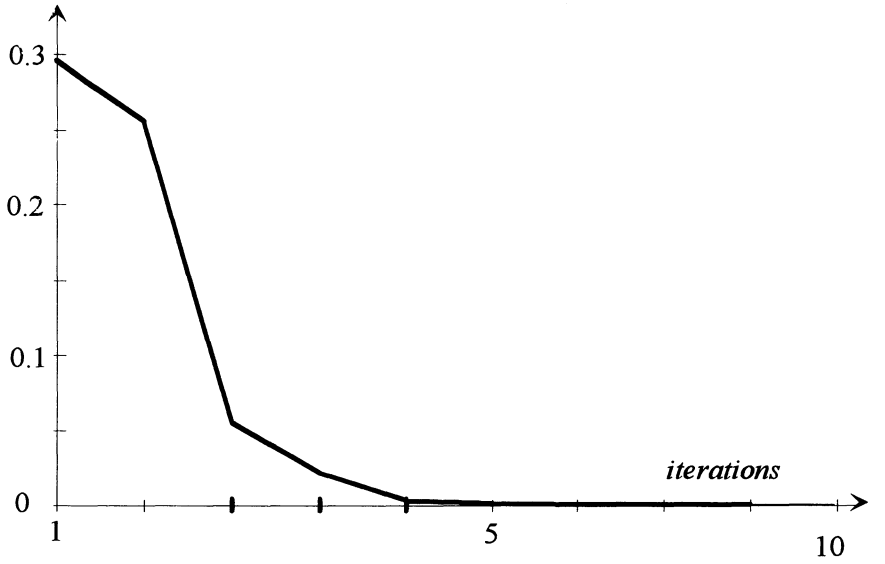

Fig. 1. Relative duality gap

5. SUMMARY AND CONCLUSIONS

A new algorithm for addressing linear programming problems with multiple objective was presented. The algorithm combines the interior-point affine-scaling primal algorithm with the concept of aspiration levels and scalarizing functions. The resulting algorithm requires the decision maker to specify aspiration levels for the various objectives and allows modifying the internal solution trajectory to accommodate these changes in aspirations. Future efforts in this area should address the issue of determining aspiration levels and when, and how, to change them during the search for an optimal solution.

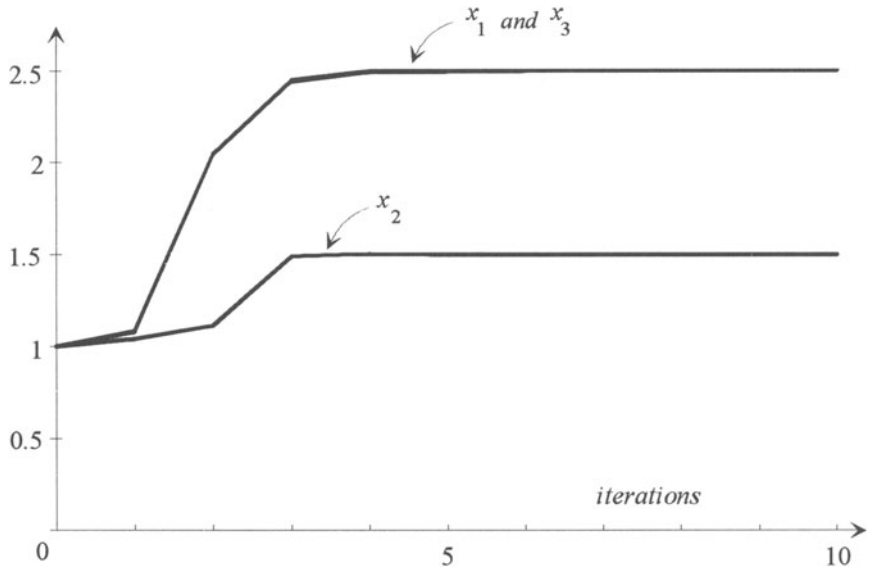

Fig. 2. Solution trajectories for the decision vector's components ($\rho = 0.9$)

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Preemptive Reference Point Method

Włodzimierz Ogryczak

Service de Mathématiques de la Gestion, Université Libre de Bruxelles,
B–1050 Bruxelles, Belgium
on leave from Institute of Informatics, Warsaw University, Warsaw, Poland

Abstract. The reference point method for solving multi–criteria optimization problems is an interactive technique where the decision maker specifies requirements, similar to goal programming, in terms of aspiration levels. Ogryczak [5] showed how the reference point method could be modeled within goal programming methodology provided that the nonnegativity restrictions on weights were dropped. It allows us to consider the reference point approach as an extension of goal programming. However, in most of real–life applications of goal programming the goals are grouped according to the predefined priorities (the so–called preemptive goal programming) whereas in the reference point method all the deviations are considered to be equally important. In this paper we show how the priorities can be incorporated into the reference point method.

1 Introduction

Consider a decision problem defined as an optimization problem with k objective functions. For simplification of the formal presentation we assume, without loss of generality, that all the objective functions are to be minimized. The problem can be formulated then as follows

$$\min \{ \mathbf{F}(\mathbf{x}) \ : \ \mathbf{x} \in Q \} \tag{1}$$

where

$\mathbf{F} = (F_1, \ldots, F_k)$	represents a vector of k objective functions,
Q	denotes the feasible set of the problem,
$\mathbf{x}$	is a vector of decision variables.

Consider further an achievement vector $\mathbf{q} = \mathbf{F}(\mathbf{x})$ which measures achievement of decision $\mathbf{x}$ with respect to the specified set of k objectives $F_1, \ldots, F_k$. It is clear that an achievement vector is better than another if all of its individual achievements are better or at least one individual achievement is better whereas no other one is worse. Such a relation is called domination of achievement vectors. Unfortunately, there usually does not exist an achievement vector that dominates all others with respect to all the criteria. Thus in

terms of strict mathematical relations we cannot distinguish the best achievement vector. The nondominated vectors are incomparable on the basis of the specified set of objective functions. The feasible solutions (decisions) that generate nondominated achievement vectors are called efficient or Pareto–optimal solutions to the multi–criteria problem.

It seems clear that the solution of multi–criteria optimization problems should simply depend on identification of the efficient solutions. However, even finite characteristic of the efficient set for a real-life problem is usually so large that it cannot be considered a solution to the decision problem. So, there arises a need for further analysis, or rather decision support, to help the decision maker (DM) to select one efficient solution for implementation. Of course, the original objective functions do not allow one to select any efficient solution as better than any other one. Therefore this analysis depends usually on additional information about the DM's preferences. The DM, working interactively with a decision support system (DSS), specifies the preferences in terms of some control parameters, and the DSS at each interactive step provides one efficient solution that meets the current preferences. Such a DSS can be used for analysis of decision problems with finite as well as infinite efficient sets. There is important, however, that the control parameters provide the completeness of the control, i.e., that varying the control parameters the DM can identify every nondominated achievement vector.

Goal programming (GP), originally proposed by Charnes & Cooper [2], seems to be a convenient generating technique for a DSS. It is, in fact, commonly used in real–life applications (see [8]). Goal programming requires one to transform objectives into goals by specification of an aspiration level for each objective. An optimal solution is then the one that minimizes deviations from the aspiration levels. Various measures of multidimensional deviations have been proposed. They are expressed as achievement functions. Depending on the type of the achievement function we distinguish (compare [4]): weighted (minsum) GP, fuzzy (minmax) GP, preemptive (lexicographic) GP. If a GP model is used as the basis of a DSS the aspiration levels can be changed during the decision analysis as the DM preferences evolve. One of the most important advantages of the interactive GP approach is that it does not require the DM to be consistent and coherent in the preferences.

Goal programming, unfortunately, does not satisfy the efficiency (Pareto–optimality) principle. Simply, the GP approach does not suggest decisions that optimize the objective functions. It only yields decisions that have outcomes closest to the specified aspiration levels. This weakness of goal programming has led to the development of the so–called quasisatisficing approach which always generates efficient solutions. The quasisatisficing approach also deals with the aspiration levels but they are understood in a different way than in GP approaches. In goal programming the vector of aspiration levels is (weakly) preferred to any other achievement vector whereas in the quasisatisficing approach the aspiration vector is preferred to any other

achievement vector which does not dominate the aspiration vector.

The best formalization of the quasisatisficing approach to multi–criteria optimization was proposed and developed mainly by Wierzbicki [9] as the reference point method. The reference point method is an interactive technique where the DM specifies requirements, as in GP, in terms of aspiration levels. Depending on the specified aspiration levels a scalarizing achievement function is built which, when minimized, generates an efficient solution to the problem. The computed efficient solution is presented to the DM as the current solution allowing comparison with previous solutions and modifications of the aspiration levels if necessary. The scalarizing achievement function not only guarantees efficiency of the solution but also reflects the DM's preferences as specified via the aspiration levels. In building the function it is assumed that the DM prefers outcomes that satisfy all the aspiration levels to any outcome that does not reach one or more of the aspiration levels.

One of the simplest scalarizing functions takes the following form (see [7])

$$s(\mathbf{q}, \mathbf{a}, \lambda) = \max_{1 \le i \le k} \{\lambda_i (q_i - a_i)\} + \varepsilon \sum_{i=1}^{k} \lambda_i (q_i - a_i) \tag{2}$$

where

- $\mathbf{a}$ denotes the vector of aspiration levels,
- λ is a scaling vector, $\lambda_i > 0$,
- ε is an arbitrarily small positive number.

Minimization of the scalarizing achievement function (2) over the attainable set $Y = \{ \mathbf{q} = \mathbf{F}(\mathbf{x}) : \mathbf{x} \in Q \}$ generates an efficient solution. The selection of the solution within the efficient set depends on two vector parameters: an aspiration vector $\mathbf{a}$ and a scaling vector λ. In practical implementations the former is usually designated as a control tool for use by the DM whereas the latter is automatically calculated on the basis of some predecision analysis. The small scalar ε is introduced only to guarantee efficiency in the case of a nonunique optimal solution.

The reference point method although using the same main control parameters (aspiration levels) always generates an efficient solution to the multi–criteria problem whereas goal programming does not. Ogryczak [5] has shown that the implementation techniques of goal programming can be used to model the reference point approach. The proposed reference GP problem takes the following form

$$\text{RGP:} \quad \text{lexmin } \mathbf{g}(\mathbf{d}^-, \mathbf{d}^+) = [g_1(\mathbf{d}^-, \mathbf{d}^+), g_2(\mathbf{d}^-, \mathbf{d}^+)] \tag{3}$$

subject to

$$F_i(\mathbf{x}) + d_i^- - d_i^+ = a_i \quad \text{for } i = 1, 2, \ldots, k \tag{4}$$

$$d_i^- \ge 0, \quad d_i^+ \ge 0 \quad \text{for } i = 1, 2, \ldots, k \tag{5}$$

$$d_i^- d_i^+ = 0 \quad \text{for } i = 1, 2, \ldots, k \tag{6}$$

$$\mathbf{x} \in Q \tag{7}$$

where

$$g_1(\mathbf{d}^-, \mathbf{d}^+) = \max_{i=1,\ldots,k} (-w_i^- d_i^- + w_i^+ d_i^+) \tag{8}$$

$$g_2(\mathbf{d}^-, \mathbf{d}^+) = \sum_{i=1}^{k} (-w_i^- d_i^- + w_i^+ d_i^+) \tag{9}$$

d_i^- and d_i^+ are negative and positive goal deviations, respectively, i.e., nonnegative state variables which measure deviations of the current value of the i-th objective function from the corresponding aspiration level;

w_i^- and w_i^+ are positive weights corresponding to several goal deviations, e.g., $w_i^- = w_i^+ = \lambda_i$ for the exact model of (2).

The main specificity which differentiates the RGP model from the standard GP approaches depends on the use of negative weight coefficients $-w_i^-$ associated with the negative deviations d_i^-. Further, the RGP problem uses both the minmax and the minsum achievement functions. The minmax achievement functions are not very common in lexicographic GP applications but they are the standard GP tool (so–called fuzzy GP, [4]). Moreover, in the case of linear problems, the minmax achievement function does not destroy the linear structure and it can be implemented implicitly in the simplex algorithms [6]. Thus, the achievement function (8) can easily be implemented in the lexicographic GP solution techniques.

As shown by Ogryczak [5], the RGP problem always generates an efficient solution to the original multi–criteria problem (even in the case of nonconvex, e.g. discrete decision problem) satisfying simultaneously the rules of the reference point approach. Namely, whenever a solution with all objectives not worse than the corresponding aspiration levels is attainable, such an efficient solution is provided by optimization of the RGP problem. Moreover, the requirements (6) can be simply omitted in the constraints of the RGP problem provided that the weights satisfy natural for the reference point approach relations

$$w_i^+ > w_i^- > 0 \qquad \text{for } i = 1, 2, \ldots, k \tag{10}$$

Thus the reference point approach may be considered as an extension of goal programming. However, in most real–life applications of goal programming the goals or deviations are grouped according to the predefined priorities (preemptive GP) whereas in the reference point method all the deviations are considered to be equally important. In the next section we show how prioritization of deviations can be incorporated into RGP model of the reference point method. The proposed preemptive reference point method preserves the most important properties of the standard reference point approach.

2 Preemptive Reference Point Method

While incorporating priorities into the reference point method, like in preemptive goal programming, we are interested rather in priorities on the deviation variables than on the original objective functions. That means, we consider the same original multi–criteria optimization problem (1) without any hierarchy of the objective functions. The priorities are considered only as an additional tool for better expression of the DM's preferences during interactive search for an efficient solution.

During the interactive analysis the DM specifies the preferences with vector $\mathbf{a}$ of aspiration levels and two groups of priority sets P_j^+ for $j = 1, 2, \ldots, p_+$ and P_j^- for $j = 1, 2, \ldots, p_-$. The priority sets P_j^+ $(j = 1, 2, \ldots, p_+)$ represent the aspiration levels hierarchy in the sense of importance of achieving outcomes not worse than the corresponding aspiration levels. Similarly, P_j^- $(j = 1, 2, \ldots, p_-)$ represent hierarchy of importance to achieve outcomes better than the corresponding aspiration levels. Both groups of priority sets define partitions of the entire set of objectives. The two partitions may be, in particular, identical. However, for better modeling of real-life preferences it seems to be necessary to allow them to be different, as importance of achieving aspiration levels does not, necessarily, match interests in exceeding the aspiration levels.

Let us summarize the DM's preference model expressed with the aspiration levels and hierarchy sets:

P1. For any individual outcome $F_i(\mathbf{x})$ $(i = 1, 2, \ldots, k)$ less is preferred to more (minimization);

P2. A solution with all individual outcomes $F_i(\mathbf{x})$ equal to the corresponding aspiration levels is preferred to any solution with at least one individual outcome greater than the corresponding aspiration level;

P3. Minimization of any positive deviation d_j^+ is preferred to maximization of each negative deviation d_i^- for $i = 1, 2, \ldots, k$;

P4. If $j < t$ then minimization of positive deviations d_i^+ for $i \in P_j^+$ is preferred to minimization of positive deviations d_i^+ for $i \in P_t^+$;

P5. If $j < t$ then maximization of negative deviations d_i^- for $i \in P_j^-$ is preferred to maximization of negative deviations d_i^- for $i \in P_t^-$;

Property **P1** means that efficient solutions are preferred to nonefficient ones, i.e., the DM's preferences are consistent with the efficiency principle. Property **P2** expresses that the DM prefers outcomes that satisfy all the aspiration levels to any outcome that does not reach one or more of the aspiration levels. In terms of goal deviations (compare (4)–(6)) it means that a solution with positive deviations d_i^+ equal to 0 for all $i = 1, 2, \ldots, k$ is preferred to any solution with positive value of at least one deviation d_i^+. These two properties are crucial for the preference model considered in the reference point approach. Properties **P3**, **P4** and **P5** express the hierarchy of deviations defined with the priority sets.

In order to introduce the hierarchy of deviations into the RGP model we replace the lexicographic achievement function with the following:

$$\text{PRGP:} \quad \text{lexmin } \mathbf{g}(\mathbf{d}^-, \mathbf{d}^+) = [\mathbf{g}^+(\mathbf{d}^+), \mathbf{g}^-(\mathbf{d}^-)] \quad \text{subject to (4), (5) and (7)} \tag{11}$$

where

$$\mathbf{g}^+(\mathbf{d}^+) = [g_{11}^+(\mathbf{d}^+), g_{12}^+(\mathbf{d}^+), \ldots, \mathbf{g}_{p_+1}^+(\mathbf{d}^+), \mathbf{g}_{p_+2}^+(\mathbf{d}^+)] \tag{12}$$

$$\mathbf{g}^-(\mathbf{d}^-) = [g_{11}^-(\mathbf{d}^-), g_{12}^-(\mathbf{d}^-), \ldots, \mathbf{g}_{p_-1}^-(\mathbf{d}^-), \mathbf{g}_{p_-2}^-(\mathbf{d}^-)] \tag{13}$$

$$g_{j1}^+(\mathbf{d}^+) = \max_{i \in P_j^+}(w_i^+ d_i^+) \qquad \text{for } j = 1, 2, \ldots, p_+ \tag{14}$$

$$g_{j2}^+(\mathbf{d}^+) = \sum_{i \in P_j^+} w_i^+ d_i^+ \qquad \text{for } j = 1, 2, \ldots, p_+ \tag{15}$$

$$g_{j1}^-(\mathbf{d}^-) = \max_{i \in P_j^-}(-w_i^- d_i^-) \qquad \text{for } j = 1, 2, \ldots, p_- \tag{16}$$

$$g_{j2}^-(\mathbf{d}^-) = \sum_{i \in P_j^-} -w_i^- d_i^- \qquad \text{for } j = 1, 2, \ldots, p_- \tag{17}$$

Note that constraints of the above PRGP problem do not include the requirements (6) to guarantee proper calculation of goal deviations. The requirements (6) could be simply omitted in the constraints of PRGP problem since all negative deviations d_i^- have assigned lower priorities than any positive deviation d_i^+. This is made precise in Proposition 1.

Proposition 1 *For any aspiration levels a_i and any positive weights w_i^- and w_i^+, any $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ optimal solution to the problem PRGP satisfies requirements (6), i.e., $\bar{d}_i^- \bar{d}_i^+ = 0$ for $i = 1, 2, \ldots, k$.*

Proof. Let $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ be an optimal solution for the problem PRGP. Suppose that $\bar{d}_{i_0}^- \bar{d}_{i_0}^+ > 0$ for some index $1 \leq i_0 \leq k$. Then we can decrease both $\bar{d}_{i_0}^-$ and $\bar{d}_{i_0}^+$ by the same small positive quantity. That means, for small enough positive δ the vector $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^- - \delta \mathbf{e}_0, \bar{\mathbf{d}}^+ - \delta \mathbf{e}_0)$, where $\mathbf{e}_0$ denotes the unit vector corresponding to index i_0, is feasible to the problem PRGP. Due to positive weights w_i^- and w_i^+, the following inequalities are satisfied

$$g_{j1}^+(\bar{\mathbf{d}}^+ - \delta \mathbf{e}_0) \leq g_{j1}^+(\bar{\mathbf{d}}^+) \text{ and } g_{j2}^+(\bar{\mathbf{d}}^+ - \delta \mathbf{e}_0) \leq g_{j2}^+(\bar{\mathbf{d}}^+) \quad \text{for } j = 1, 2, \ldots, p_+$$

Moreover, there exists j_0 such that $i_0 \in P_{j_0}^+$ and thereby $g_{j_0 2}^+(\bar{\mathbf{d}}^+ - \delta \mathbf{e}_0) < g_{j_0 2}^+(\bar{\mathbf{d}}^+)$ which contradicts optimality of $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ for the problem PRGP. Thus $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ must satisfy conditions (6). □

By the definition of the achievement functions (12)–(17), it is lucid that the PRGP problem (11)–(17) complies with the properties **P3**, **P4** and **P5**. However, fulfilling of properties **P1** and **P2** (crucial for the reference point

approach) is not so clear. We will show (Proposition 2) that the lexicographic problem PRGP always generates an efficient solution to the original multi–criteria optimization problem (property **P1**) complying simultaneously with property **P2** (Proposition 3).

Proposition 2 *For any aspiration levels a_i and any positive weights w_i^- and w_i^+, if $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ is an optimal solution to the problem PRGP, then $\bar{\mathbf{x}}$ is an efficient solution to the multi–criteria optimization problem (1).*

Proof. Let $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ be an optimal solution to the problem PRGP. Suppose that $\bar{\mathbf{x}}$ is not efficient to the problem (1). That means, there exists a vector $\mathbf{x} \in Q$ such that $F_i(\mathbf{x}) \leq F_i(\bar{\mathbf{x}})$ for $i = 1, 2, \ldots, k$ and $F_{i_0}(\mathbf{x}) < F_{i_0}(\bar{\mathbf{x}})$ for some index $1 \leq i_0 \leq k$. The deviations $\bar{d}_i^-$ and $\bar{d}_i^+$ satisfy relations: $\bar{d}_i^+ = (F_i(\bar{\mathbf{x}}) - a_i)_+$ and $\bar{d}_i^- = (a_i - F_i(\bar{\mathbf{x}}))_+$, where $(.)_+$ denotes the nonnegative part of a quantity. Let us define similar deviations for the vector $\mathbf{x}$, i.e., $d_i^+ = (F_i(\mathbf{x}) - a_i)_+$ and $d_i^- = (a_i - F_i(\mathbf{x}))_+$ for $i = 1, 2, \ldots, k$. $(\mathbf{x}, \mathbf{d}^-, \mathbf{d}^+)$ is a feasible solution to the problem PRGP and $d_i^+ \leq \bar{d}_i^+$ and $d_i^- \geq \bar{d}_i^-$ for $i = 1, 2, \ldots, k$. Hence, for any positive weights w_i^- and w_i^+ the following inequalities are satisfied

$$g_{j1}^+(\mathbf{d}^+) \leq g_{j1}^+(\bar{\mathbf{d}}^+) \quad \text{and} \quad g_{j2}^+(\mathbf{d}^+) \leq g_{j2}^+(\bar{\mathbf{d}}^+) \quad \text{for} \quad j = 1, 2, \ldots, p_+$$

$$g_{j1}^-(\mathbf{d}^-) \leq g_{j1}^-(\bar{\mathbf{d}}^-) \quad \text{and} \quad g_{j2}^-(\mathbf{d}^-) \leq g_{j2}^-(\bar{\mathbf{d}}^-) \quad \text{for} \quad j = 1, 2, \ldots, p_-$$

Moreover, there exist j_+ such that $i_0 \in P_{j_+}^+$ and j_- such that $i_0 \in P_{j_-}^-$. Hence, for any positive weights w_i^- and w_i^+, $g_{j_+2}^+(\mathbf{d}^+) < g_{j_+2}^+(\bar{\mathbf{d}}^+)$ or $g_{j_-2}^-(\mathbf{d}^-) < g_{j_-2}^-(\bar{\mathbf{d}}^-)$, which contradicts optimality of $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ for the problem PRGP. Thus $\bar{\mathbf{x}}$ must be an efficient solution to the original multi–criteria optimization problem (1). □

Proposition 3 *For any aspiration levels a_i and any positive weights w_i^- and w_i^+, if $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ is an optimal solution to the problem PRGP, then any deviation $\bar{d}_i^+$ is positive only if there does not exist any vector $\mathbf{x} \in Q$ such that $F_i(\mathbf{x}) \leq a_i$ for $i = 1, 2, \ldots, k$.*

Proof. Let $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ be an optimal solution to the problem PRGP. Suppose that $\bar{d}_{i_0}^+ > 0$ (i.e., $F_{i_0}(\bar{\mathbf{x}}) > a_{i_0}$) for some index $1 \leq i_0 \leq k$ and there exists a vector $\mathbf{x} \in Q$ such that $F_i(\mathbf{x}) \leq a_i$ for $i = 1, 2, \ldots, k$. Let us define deviations for the vector $\mathbf{x}$: $d_i^+ = (F_i(\mathbf{x}) - a_i)_+ = 0$ and $d_i^- = (a_i - F_i(\mathbf{x}))_+ \geq 0$ for $i = 1, 2, \ldots, k$. $(\mathbf{x}, \mathbf{d}^-, \mathbf{d}^+)$ is a feasible solution for the problem PRGP and for any positive weights w_i^- and w_i^+ the following inequalities are satisfied

$$g_{j1}^+(\mathbf{d}^+) = 0 \leq g_{j1}^+(\bar{\mathbf{d}}^+) \quad \text{and} \quad g_{j2}^+(\mathbf{d}^+) = 0 \leq g_{j2}^+(\bar{\mathbf{d}}^+) \quad \text{for} \quad j = 1, 2, \ldots, p_+$$

Moreover, there exists j_0 such that $i_0 \in P_{j_0}^+$ and thereby $g_{j_0 1}^+(\mathbf{d}^+) = 0 < w_{i_0}^+ \bar{d}_{i_0}^+ \leq g_{j_0 1}^+(\bar{\mathbf{d}}^+)$, which contradicts optimality of $(\bar{\mathbf{x}}, \bar{\mathbf{d}}^-, \bar{\mathbf{d}}^+)$ for the problem PRGP. Thus cannot exist any vector $\mathbf{x} \in Q$ such that $F_i(\mathbf{x}) \leq a_i$ for $i = 1, 2, \ldots, k$. □

In order to show that the PRGP model provides us with a complete parameterization of the efficient set, we show in the following proposition that for each efficient solution $\bar{\mathbf{x}}$ there exists an aspiration vector for which $\bar{\mathbf{x}}$ with the corresponding values of the deviation variables is an optimal solution of the PRGP problem.

Proposition 4 *For any positive weights w_i^- and w_i^+, if $\bar{\mathbf{x}} \in Q$ is an efficient solution of the multi-criteria problem (1), then $(\bar{\mathbf{x}}, \mathbf{0}, \mathbf{0})$ is an optimal solution of the corresponding PRGP problem with aspiration vector $\mathbf{a} = \mathbf{F}(\bar{\mathbf{x}})$.*

Proof. Note that $(\bar{\mathbf{x}}, \mathbf{0}, \mathbf{0})$ is a feasible solution of the PRGP problem with $\mathbf{a} = \mathbf{F}(\bar{\mathbf{x}})$. Suppose that $(\bar{\mathbf{x}}, \mathbf{0}, \mathbf{0})$ is not optimal for PRGP. For any positive weights w_i^- and w_i^+, $\mathbf{g}^+(\mathbf{d}^+) \geqq \mathbf{0}$ and $\mathbf{g}^-(\mathbf{d}^-) \leqq \mathbf{0}$, whereas $\mathbf{g}^+(\mathbf{0}) = \mathbf{0}$ and $\mathbf{g}^-(\mathbf{0}) = \mathbf{0}$. So, there exists a feasible solution $\mathbf{x} \in Q$ such that

$$\mathbf{g}^+((\mathbf{F}(\mathbf{x}) - \mathbf{F}(\bar{\mathbf{x}}))_+) = \mathbf{0} \quad \text{and} \quad \mathbf{g}^-((\mathbf{F}(\bar{\mathbf{x}}) - \mathbf{F}(\mathbf{x}))_+) <_{lex} \mathbf{0}$$

Hence $\mathbf{F}(\mathbf{x}) \leq \mathbf{F}(\bar{\mathbf{x}})$ which contradicts efficiency of $\bar{\mathbf{x}}$ for the multi-criteria problem (1). □

Note that neither proposition assumes convexity of the feasible set Q. Thus the preemptive reference point method can be applied not only for linear problems but also for integer ones where goal programming may fail to generate efficient solutions (see [3]).

The PRGP problem takes a simpler form when all the priority sets are single element sets. Note that in such a case

$$g_{j1}^+(\mathbf{d}^+) = g_{j2}^+(\mathbf{d}^+) = w_j^+ d_j^+ \qquad \text{for } j = 1, 2, \ldots, k$$
$$g_{j1}^-(\mathbf{d}^-) = g_{j2}^-(\mathbf{d}^-) = -w_j^- d_j^- \qquad \text{for } j = 1, 2, \ldots, k$$

Thus vector functions $\mathbf{g}^+(\mathbf{d}^+)$ and $\mathbf{g}^-(\mathbf{d}^-)$ defined by (12)–(17) can be replaced then with the following simpler formulas

$$\begin{aligned}
\mathbf{g}^+(\mathbf{d}^+) &= [g_1^+(\mathbf{d}^+), g_2^+(\mathbf{d}^+), \ldots, g_k^+(\mathbf{d}^+)] \\
\mathbf{g}^-(\mathbf{d}^-) &= [g_1^-(\mathbf{d}^-), g_2^-(\mathbf{d}^-), \ldots, g_k^-(\mathbf{d}^-)] \\
g_j^+(\mathbf{d}^+) &= d_j^+ \qquad \text{for } j = 1, 2, \ldots, k \\
g_j^-(\mathbf{d}^-) &= -d_j^- \qquad \text{for } j = 1, 2, \ldots, k
\end{aligned}$$

As a special case of PRGP one may consider the problem with all deviations d_i^+ belonging to the same priority set $P_1^+ = \{1, 2, \ldots, k\}$ and all deviations d_i^- belonging to the same priority set $P_1^- = \{1, 2, \ldots, k\}$. Achievement functions for such a problem takes the form

$$\mathbf{g}^+(\mathbf{d}^+) = [g_1^+(\mathbf{d}^+), g_2^+(\mathbf{d}^+)] \quad , \quad \mathbf{g}^-(\mathbf{d}^-) = [g_1^-(\mathbf{d}^-), g_2^-(\mathbf{d}^-)] \tag{18}$$

$$g_1^+(\mathbf{d}^+) = \max_{i=1,\ldots,k}(w_i^+ d_i^+) \quad , \quad g_2^+(\mathbf{d}^+) = \sum_{i=1}^{k} w_i^+ d_i^+ \tag{19}$$

$$g_1^-(\mathbf{d}^-) = \max_{i=1,\ldots,k}(-w_i^- d_i^-) \quad , \quad g_2^-(\mathbf{d}^-) = \sum_{i=1}^{k} -w_i^- d_i^- \tag{20}$$

Problem PRGP with achievement functions (18)–(20) defines the reference GP model without priorities for the deviations. However, it differs from the RGP problem due to the form and properties. While in the RGP model weights w_i^- and w_i^+ must satisfy (10), PRGP without priorities requires only positivity of the weights. It is due to property **P3** built in the PRGP model. This property is equivalent to the requirement that minimization of any individual outcome greater than its aspiration level is preferred to minimization of any individual outcome which is less or equal to the corresponding aspiration level. It is a stronger requirement than property **P2** and it is not always satisfied by the standard reference point method as well as by the RGP model (3)–(9). For instance, in the standard reference point method based on the scalarizing achievement function (2) with all $\lambda_i = 1$ or in the RGP problem with $w_1^+ = w_2^+ = w_3^+$ and $0.1w_3^+ < w_3^- < w_3^+$, achievement vector $(a_1 + 1, a_2 + 1, a_3 - 10)$ is preferred to $(a_1 + 1, a_2, a_3)$. Preferring of vector $(a_1 + 1, a_2, a_3)$ seems to be better consistent with the basic quasisatisficing rule that the DM concentrates on improvement of these individual outcomes which do not reach their aspiration levels. In the PRGP model (without priorities) vector $(a_1 + 1, a_2, a_3)$ is preferred to $(a_1 + 1, a_2 + 1, a_3 - 10)$ for any positive weights w_i^- and w_i^+. Thus, with the PRGP model we not only allow the DM to introduce hierarchy of aspiration levels but even without such a hierarchy we refine the standard reference point method with better modeling of the quasisatisficing approach.

3 Illustrative Example

In this section we discuss the PRGP model for a sample decision problem. To keep the problem description short and clear, and the model itself easy solvable by graphical analysis we have decided to use rather a textbook example than a real–life decision problem. Our example is based on the goal programming example from the OR/MS textbook [1].

Consider the media mix problem concerned with allocating the advertising budget among various media. For simplicity, we consider only two media: television and radio. Suppose rated exposures (people per month per advertising outlay) per \$ 1 000 of advertising expenditure are 10 000 and 7 500, respectively, for television and radio. Assume the management has set the following goals it wish to achieve, arranged from highest to lowest priority:

1. Avoid expenditures of more than \$ 100 000;
2. Reach at least 750 000 exposures;
3. Avoid expenditures of more than \$ 70 000 for TV advertisements;
4. Maximize number of exposures;
5. Minimize total expenditures;
6. Minimize expenditures for TV advertisements.

One may easily notice that the baseline problem is three-criteria optimization problem where the objective functions are: number of exposures (max-

imized), total expenditures (minimized) and expenditures for TV advertisements (minimized). However, the management has specified its preferences with aspiration levels and priorities.

Introducing two decision variables x_1 and x_2, expressing expenditures for several media in thousands of dollars, one gets the following goal constraints:

$$\begin{array}{rrrrr} x_1+ & x_2+ & d_1^- - & d_1^+ = & 100 \\ 10\,000x_1+ & 7\,500x_2+ & d_2^- - & d_2^+ = & 750\,000 \\ x_1+ & & d_3^- - & d_3^+ = & 70 \\ x_1, x_2, d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+ & & & \geq & 0 \end{array}$$

The management preferences can be expressed with the following PRGP achievement function:

$$\text{lexmin } [d_1^+, d_2^-, d_3^+, -d_2^+, -d_1^-, -d_3^-]$$

Note that d_2^- has got there higher priority than $-d_2^+$ as the corresponding objective is maximized whereas for theoretical considerations in the previous section we assumed all the objective functions to be minimized.

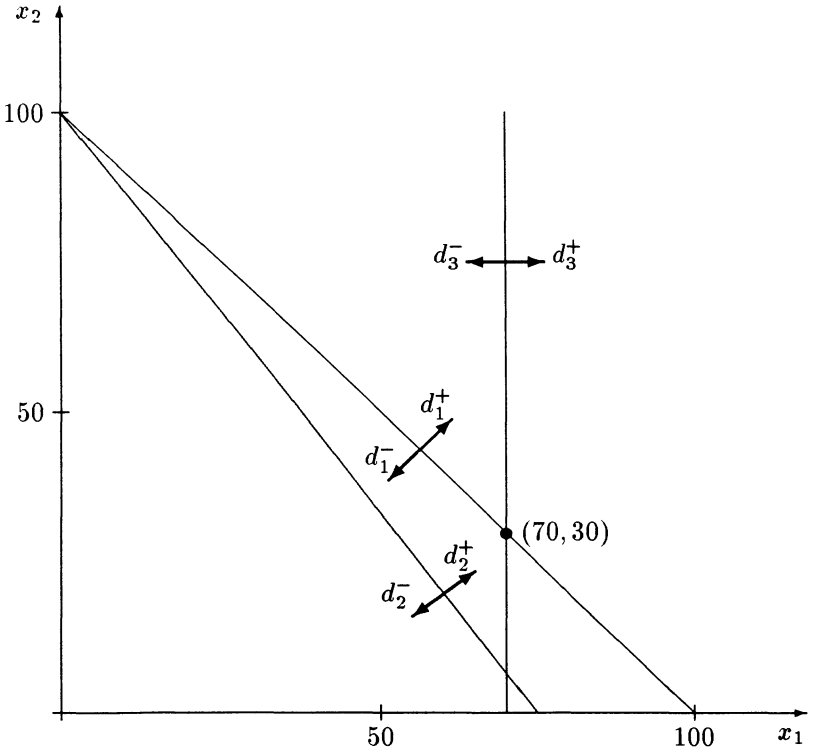

Figure 1: Graphical analysis for the illustrative example

One can easily verify with the graphical analysis (compare Fig. 1) that the above PRGP problem has a unique optimal solution $\mathbf{x} = (70, 30)$ with

outcomes: \$ 100 000 of total expenditures including \$ 70 000 expended on TV advertisements, and 925 000 exposures. The graphical analysis of the problem shows that this solution could not be reached for any achievement function using only positive weights. In fact, while solving the problem

$$\text{lexmin } [w_1^+ d_1^+, w_2^- d_2^-, w_3^+ d_3^+, w_2^+ d_2^+, w_1^- d_1^-, w_3^- d_3^-]$$

for any positive weights w_i^- and w_i^+ one gets a unique optimal solution $\mathbf{x} = (0, 100)$ with outcomes: \$ 100 000 of total expenditures all expended on radio advertisements and 750 000 exposures. This solution definitely seems not to be what the management preferred. Thus the usage of negative weights turns out to be important in this example.

4 Concluding Remarks

The most widely used technique for multi–criteria optimization and usually the only one taught in general OR/MS courses is goal programming. Goal programming, however, does not satisfy the efficiency (Pareto-optimality) principle. Simply, the GP approach does not suggest decisions that optimize the objective functions. It only yields decisions that have outcomes closest to the specified aspiration levels. This weakness of goal programming led to the development of the reference point method which though using the same main control parameters as GP always generates an efficient solution.

Ogryczak [5] showed how the reference point method could be modeled within goal programming methodology provided that the nonnegativity restrictions on weights were dropped. It allows us to consider the reference point approach as an extension of goal programming. However, in most of real–life applications of goal programming the goals are grouped according to the predefined priorities whereas in the reference point method all the deviations are considered to be equally important. In this paper we have shown how the priorities can be incorporated into the reference point method. The proposed preemptive reference point method not only preserves the most important properties of the standard reference point method (such as compliance with the efficiency principle and controllability) but even without the use of priorities it refines the standard reference point method with better modeling of the quasisatisficing approach. Moreover, the preemptive reference point method has been developed using GP implementation techniques. So, it can be considered as an extension of preemptive goal programming. It allows us to extend applications of the powerful reference point approach and to build a unique decision support systems providing the DM with both preemptive GP and reference point approaches.

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Some Optimality Conditions in Multiobjective Programming

Alberto Cambini, Laura Martein and Riccardo Cambini
Department of Statistics and Applied Mathematics - University of Pisa
Via Cosimo Ridolfi, 10 - 56124 Pisa - ITALY - Fax: +39-50-541437
E-mail: acambini@ec.unipi.it, lmartein@ec.unipi.it, cambric@ec.unipi.it

Abstract: One of the aim of this paper is to introduce new classes of vector generalized concave functions and to point out their role in investigating local and global efficiency and in establishing sufficient optimality conditions for a vector optimization problem. Another aim is to stress the role of the Bouligand tangent cone at a point of the feasible region in deriving optimality conditions.

1. Introduction

In these last years several of articles dealing with scalar generalized concavity have appeared in scientific journals and numerous textbooks have specific chapters in this subject. On the contrary the role of vector generalized concavity in multiobjective optimization is not yet sufficiently explored; only occasionally, with the aim to extend to the vector case some properties of scalar generalized concavity, some authors have considered, in the Paretian case, componentwise generalized concavity or, in studying specific topics like as the connectedness of the set of all efficient points, have defined some special classes of vector generalized concave functions with respect to a cone [8,9,10,12,15,16].

For such a reason some classes of generalized concave multiobjective functions with their properties have been recently introduced and studied [2,3,5,6,13].

First of all in this paper, taking into account the results obtained in [5,6], we will introduce some classes of vector generalized concave functions pointing out their role in vector optimization, and successively we will investigate local and global efficiency and we will state several necessary and/or sufficient optimality conditions established by means of the Bouligand tangent cone to the feasible region at a point.

The obtained results generalize and extend the ones given in [2].

2. Some classes of generalized concave multiobjective functions

In finding conditions under which a local maximum point is also global or a necessary optimality condition becomes sufficient too, an important role is played

by the concept of generalized concavity at a point introduced in [11] for a scalar optimization problem.

Let us note that there are different way in generalizing to the vector case the definitions of generalized concave functions given in the scalar case. In this section, following [5,6], we introduce wide classes of vector generalized concave function which are more general than the ones suggested in [2,3].

With this aim, let us consider an open set X of the n-dimensional space $\mathbf{R}^n$, a function $F:X\to\mathbf{R}^s$ and a non trivial cone $U\subset\mathbf{R}^s$ with vertex at the origin $0\in U$ and with non empty interior, i.e. $\text{int}U\neq\emptyset$. Set $U^0=U\setminus\{0\}$.

We recall that a set $S\subset X$ is said to be *star shaped* at $x_0\in S$ if $x\in S$ implies:

$$[x, x_0] = \{tx+(1-t)x_0 : t\in[0,1]\} \subset S.$$

Definition 2.1

Consider the cones $U^*,U^{\#}\in\{U, U^0, \text{int}U\}$.

The function F is said to be *$(U^*,U^{\#})$-quasiconcave* (shortly $(U^*,U^{\#})$-qcv) at x_0 (with respect to the star shaped set S at x_0) if:

$$x\in S,\ x\neq x_0,\ F(x)\in F(x_0)+U^* \ \Rightarrow\ F(x_0+\lambda(x-x_0))\in F(x_0)+U^{\#};\quad \forall\lambda\in(0,1)$$

when $U^*=U^{\#}$ we will refer to a $(U^*,U^{\#})$-quasiconcave function as a *U^*-quasiconcave* function (shortly U^*-qcv).

Let us note that when $U^*=U^{\#}=U$ we obtain the definition of a vector quasiconcave function given by Yahn [8], while when $U^*=U^{\#}=U^0$ we obtain the class of U-semistrictly quasiconcave vector functions introduced by Martein [13].

In the scalar case (s=1) Definition 2.1 reduces to the classical definition of a quasiconcave, strictly quasiconcave, semistrictly quasiconcave function when $U^*=U^{\#}=U=\mathbf{R}_+$, $U^*=\mathbf{R}_+$ $U^{\#}=\mathbf{R}_{++}$, $U^*=U^{\#}=\mathbf{R}_{++}$ respectively, where $\mathbf{R}_+$ is the set of the non negative real numbers and $\mathbf{R}_{++}$ is the set of the positive real numbers.

Definition 2.2

Consider the cones $U^*\in\{U,U^0,\text{int}U\}$, $U^{\#}\in\{U^0,\text{int}U\}$ and let F be directionally differentiable at x_0. F is said to be *$(U^*,U^{\#})$-pseudoconcave* (shortly $(U^*,U^{\#})$-pcv) at x_0 (with respect to the star shaped set S at x_0) if:

$$x\in S,\ x\neq x_0,\ F(x)\in F(x_0)+U^* \ \Rightarrow\ \frac{\partial F}{\partial d}(x_0)\in U^{\#},\ d=\frac{x-x_0}{\|x-x_0\|}$$

When $U^*=U^{\#}=U^0$ we obtain the class of U-weakly pseudoconcave vector functions introduced by Martein [13], while when $U^*=U^0$, $U^{\#}=\text{int}U$, we obtain the class of U-pseudoconcave vector functions introduced by Cambini - Martein [3]. Let us note that in the scalar case these last two definitions collapse to the ordinary definition of a pseudoconcave function while a (U, intU)- pcv collapse to the ordinary definition of a strictly pseudoconcave function.

Relationships among the defined classes of generalized concave functions are studied in [6].

3. Local and global efficiency

Consider the following vector optimization problem:

$$P : U\text{-max } F(x),\ x \in S \subseteq X$$

where X is an open set of $\mathbf{R}^n$, $F : X \to \mathbf{R}^s$, and $U \subset \mathbf{R}^s$ is a non trivial cone with vertex at the origin $0 \in U$ and with nonempty interior. Set $U^0 = U \setminus \{0\}$.

We recall that a feasible point x_0 is said to be:

- *weakly efficient* if:

$$F(x) \in F(x_0) + \text{int}U, \qquad \forall x \in S \tag{3.1a}$$

- *efficient* if:

$$F(x) \in F(x_0) + U^0, \qquad \forall x \in S \tag{3.1b}$$

- *strictly efficient* if:

$$F(x) \in F(x_0) + U, \qquad \forall x \in S, \tag{3.1c}$$

If (2.1a), (2.1b), (2.1c), are verified in $I \cap S$, where I is a suitable neighbourhood of x_0, the feasible point x_0 is said to be a *local weak efficient point*, a *local efficient point* and a *local strict efficient point*, respectively.

Obviously a (local) strict efficient point is also a (local) efficient point, and a (local) efficient point is also a (local) weak efficient point.

Definitions (3.1) can be rewritten in a unified way as follows:
let $U^* \in \{U, U^0, \text{int}U\}$; x_0 is said to be a (local) U^*-efficient point if:

$$F(x) \in F(x_0) + U^*, \qquad \forall x \in S \quad (\forall x \in I \cap S) \tag{3.2}$$

When $U^* = \text{int}U$, $U^* = U^0$, $U^* = U$, (3.2) collapses to definition (3.1a), (3.1b), (3.1.c), respectively.

The classes of generalized concave functions introduced in section 2, allow us to investigate relationships between local and global U^*-efficiency. The following theorem holds:

Theorem 3.1
Consider problem P where S is a star shaped set at x_0 and F is $(U^*, U^\#)$-qcv at x_0. If x_0 is a local $U^\#$-efficient point, then x_0 is also U^*-efficient for P.

Proof.
Ab absurdo suppose that there exists $x^* \in S$ such that $F(x^*) \in F(x_0) + U^*$. Since F is $(U^*, U^\#)$-qcv at x_0, we have $F(x_0 + \lambda(x^* - x_0)) \in F(x_0) + U^\#$ $\forall \lambda \in (0,1)$ and such a relation implies, choosing λ small enough, the non local $U^\#$-efficiency of x_0. ♦

Specifying the cones U^* and $U^\#$ we obtain Theorem 4.1 in [2] and some other different kinds of results. For instance:

- $U^*=U^\#= U^0$: if x_0 is a local efficient point and F is (U^0,U^0)-qcv at x_0, then x_0 is an efficient point for P;
- $U^*=U^\#=$ intU: if x_0 is a local weak efficient point and F is (intU,intU)-qcv at x_0, then x_0 is a weak efficient point for P;
- $U^*=$intU, $U^\#= U^0$: if x_0 is a local weak efficient point and F is (intU,U^0)-qcv at x_0, then x_0 is an efficient point for P;
- $U^*=$intU, $U^\#= U$: if x_0 is a local weak efficient point and F is (intU,U)-qcv at x_0, then x_0 is a strict efficient point for P.

Theorem 3.2

Consider problem P where S is a star shaped set at x_0 and F is (U^*, intU)-pcv at x_0. If x_0 is a local U^*-efficient point, then x_0 is also U^*-efficient for P.

Proof.

Ab absurdo suppose that there exists $x^*\in S$, $x^*\neq x_0$, such that $F(x^*)\in F(x_0)+U^*$. Since F is (U^*, intU)-pcv at x_0, we have

$\frac{\partial F}{\partial d}(x_0)\in \text{intU}$, $d=\frac{x^*-x_0}{\|x^*-x_0\|}$, that is $\lim_{t\to 0+} \frac{F(x_0+td)-F(x_0)}{t} \in \text{intU}$ and this implies the existence of a suitable $\varepsilon>0$, such that $F(x_0+td)-F(x_0) \in \text{intU}$ $\forall t\in(0,\varepsilon)$.

Set $t=\lambda\|x^*-x_0\|$; we have $F(x_0+\lambda(x^*-x_0))\in F(x_0)+\text{intU}$ $\forall\ \lambda\in(0, \frac{\varepsilon}{\|x^*-x_0\|})$ and this contradicts the local U^*-efficiency of x_0. ♦

Let us note that specifying the cone U^* in Theorem 3.2 we obtain different kinds of results.

4. Efficiency along a direction and efficiency

As is known, the property for which a local efficient point with respect to every feasible direction of a star shaped set is also a local efficient point for P does not hold for every function F. For such a reason in this section we investigate by means of vector generalized concavity the relationships between the local U^*-efficiency of x_0 and the local U^*-efficiency of x_0 with respect to all directions starting from x_0.

With this aim we give the following definition:

A point x_0 is said to be a local U^*-efficient point along the direction $d=\frac{x-x_0}{\|x-x_0\|}$, $x\in S$ and with respect to the cone U if there exists $t^*>0$ such that:

$$F(x)\in F(x_0)+U^*,\ \forall x=x_0+td,\ t\in(0,t^*)$$

Let $\mathcal{D}=\{d=\frac{x-x_0}{\|x-x_0\|},\ x\in S\}$ be the set of feasible directions at $x_0\in S$.

The following theorem holds.

Theorem 4.1
Let us consider problem P where S is star shaped at x_0 and F is (U*,U*)-qcv at x_0. Then x_0 is a local U*-efficient point if and only if x_0 is a local U*-efficient for every $d \in \mathcal{D}$.

Proof.
if. If x_0 is a local U*-efficient point obviously it is also local U*-efficient for every $d \in \mathcal{D}$.
only if. Ab absurdo suppose that there exists $x^* \in S$ such that $F(x^*) \in F(x_0)+U^*$. Since F is (U*,U*)-qcv at x_0, we have $F(x_0+\lambda(x^*-x_0)) \in F(x_0)+U^* \quad \forall \lambda \in (0,1)$ and such a relation implies, choosing λ small enough, the non local U*-efficiency of x_0 with respect to the direction $d=\frac{x^*-x_0}{\|x^*-x_0\|} \in \mathcal{D}$. ♦

Let us note, once again that specifying the cone U* in Theorems 4.1, 4.2, we obtain different kinds of results. Some other characterizations can be found in [5].

5. Optimality conditions

In this section and in the following one, we state some necessary and/or sufficient first-order optimality conditions for problem P stated by means of a general approach involving the directions belonging to the Bouligand tangent cone to the feasible region at a point x_0.
With this aim we need of the following definition:

Definition 5.1
Let $G:A \to \mathbf{R}^k$ be a function defined in the open set $A \subseteq \mathbf{R}^n$.
G is said to be *regular directionally differentiable* at $x_0 \in A$ if $\lim_{h_n \to 0} \frac{h_n}{\|h_n\|} = d$ implies:

$$\lim_{h_n \to 0} \frac{G(x_0+h_n)-G(x_0)}{\|h_n\|} = \lim_{t \to 0^+} \frac{G(x_0+td)-G(x_0)}{t} \triangleq \frac{\partial G}{\partial d}(x_0),$$

where $\{h_n\}$ is a sequence of directions converging to 0.

It is known that functions which are directionally differentiable and locally lipschitzian at x_0 or differentiable at x_0 are regular directionally differentiable at x_0.

Consider now the Bouligand tangent cone to the set S at $x_0 \in S$, that is the set:

$$T(S,x_0)=\{v: \exists \{\alpha_n\} \subset \mathbf{R}, \{x_n\} \subset S, \alpha_n \to +\infty, x_n \to x_0 \text{ with } \alpha_n(x_n-x_0) \to v\}.$$

Let us note that $T(S,x_0)=\{0\}$ if and only if x_0 is an isolated point and in such a case x_0 is obviously an efficient point for problem P. For this reason, throughout this paper, it is assumed that $T(S,x_0) \neq \{0\}$.

Now we give some extensions and generalizations of the results stated in [2].

Denote with $J_{F_{x_0}}$ the Jacobian matrix of F at x_0 and with $J_{F_{x_0}}(v)$ the product between $J_{F_{x_0}}$ and the vector v.

The following theorem, whose proof is similar to the one given in [2], states a necessary optimality condition which holds for any kind of local efficient point.

Theorem 5.1
Let x_0 be a local U*-efficient point for P.
i) if F is regular directionally differentiable at x_0, then:

$$\frac{\partial F}{\partial v}(x_0) \in \text{int}U\,, \quad \forall v \in T(S,x_0)\,, \quad v \neq 0. \tag{5.1}$$

ii) if F is directionally differentiable and locally lipschitzian at x_0, then (5.1) holds;
iii) if F is differentiable at x_0, then:

$$J_{F_{x_0}}(v) \in \text{int}U\,, \quad \forall v \in T(S,x_0)\,, \quad v \neq 0. \tag{5.2}$$

Condition (5.1) is a necessary but not sufficient optimality condition; it is easy to verify that it becomes sufficient too for the classes of vector generalized pseudoconcave functions, as it is stated in the following Theorem.

Theorem 5.2
Let us consider problem P where S is locally star shaped at x_0 and F is (U*,intU)-pcv at x_0.
i) if F is regular directionally differentiable at x_0, then x_0 is a local U*-efficient point for P if and only if (5.1) holds;
ii) if F is directionally differentiable and locally lipschitzian at x_0, then x_0 is a local U*-efficient point for P if and only if (5.1) holds;
iii) if F is differentiable at x_0, then x_0 is a local U*-efficient point for P if and only if (5.2) holds.

Remark 5.1
Let $U^+=\{\alpha : \alpha^T u \geq 0\,, \forall u \in U\}$ be the positive polar cone of U.
Let us note that in problem P if the cone U is closed convex and pointed, S is a convex closed set and F is differentiable at x_0, then the Bouligand tangent cone $T(S,x_0)$ becomes a closed convex cone, so that by applying a separation Theorem, relation (5.2) implies the following condition:

$$\exists \alpha \in U^+ \setminus \{0\} \text{ such that } \alpha^T J_{F_{x_0}}(v) \leq 0 \;\; \forall v \in T(S,x_0)\,, \quad v \neq 0$$

This last relation reduces to condition (5.3) when x_0 is an interior point:

$$\exists \alpha \in U^+ \setminus \{0\} \text{ such that } \alpha^T J_{F_{x_0}} = 0 \tag{5.3}$$

Condition (5.3) is a necessary but not sufficient condition for an interior point x_0 to be U*-efficient; it becomes sufficient too under suitable assumption of generalized concavity. To this regards, the following theorem, which generalizes the result given in [2], points out the different roles played by some classes of generalized concave functions.

Theorem 5.3

Let us consider the unconstrained problem P where S is star shaped and F is differentiable at x_0.

i) if F is (U*, intU)-pcv at x_0, then (5.3) becomes a sufficient condition for x_0 to be a local U*-efficient point;

ii) if F is (U*, U^0)-pcv at x_0, then (5.3) becomes a sufficient condition for x_0 to be a local U*-efficient point if $\alpha \in \text{int}U^+$.

Proof.

i) Ab absurdo suppose that there exists $x^* \in S$ such that $F(x^*) \in F(x_0)+U^*$.

Since F is (U*, intU)-pcv at x_0, we have $J_{F_{x_0}}(d) \in \text{int}U$, $d=\dfrac{x^*-x_0}{\|x^*-x_0\|}$, so that $\alpha^T(J_{F_{x_0}}(d))>0$ and this contradicts (5.3).

ii) Ab absurdo suppose that there exists $x^* \in S$ such that $F(x^*) \in F(x_0)+U^*$. Since F is (U*, U^0)-pcv at x_0, we have $J_{F_{x_0}}(d) \in U^0$, $d=\dfrac{x^*-x_0}{\|x^*-x_0\|}$, so that, being $\alpha \in \text{int}U^+$, it results $\alpha^T(J_{F_{x_0}}(d))>0$ and this contradicts (5.3). ◆

In order to state some sufficient optimality conditions, we introduce the cone:

$$K(d,\varepsilon)=\{x \in \mathbf{R}^n: x=\lambda y,\ \lambda \geq 0,\ \|y-d\|<\varepsilon\},$$

where $d \in \mathbf{R}^n$, $\|d\|=1$, and $\varepsilon>0$ is a real number.

The following Theorem states a necessary and sufficient optimality condition with respect to any kind of efficient point.

Theorem 5.4

Let us consider problem P where F is a regular directionally differentiable function at x_0. Then x_0 is a local U*-efficient point for P if and only if the following conditions hold:

i) $\dfrac{\partial F}{\partial d}(x_0) \notin \text{int}U \quad \forall d \in T(S,x_0)$;

ii) for every $d \in T(S,x_0)$ such that $\dfrac{\partial F}{\partial d}(x_0) \in \text{cl}U \backslash \text{int}U$ there exists $\varepsilon>0$ such that x_0 is a local U*-efficient point with respect to the region $S \cap (x_0+K(d,\varepsilon))$.

Proof.

<u>if</u>. i) follows by Theorem 5.1 while ii) follows by noting that $(S \cap (x_0+K(d,\varepsilon))) \subset S$.

<u>only if</u>. Assume that x_0 is not a local U*-efficient point; then there exist a sequence $\{x_k\} \subset S$, $x_k \to x_0$ such that $F(x_k) \in F(x_0)+U^* \ \forall k$ and $\lim\limits_{k \to +\infty} \dfrac{x_k-x_0}{\|x_k-x_0\|}=\tilde{d} \in T(S,x_0)$.

Since $\dfrac{F(x_k)-F(x_0)}{\|x_k-x_0\|} \in U^*$ and F is regular directionally differentiable, taking into account of the assumption $\dfrac{\partial F}{\partial d}(x_0) \notin \text{int}U \quad \forall d \in T(S,x_0)$, we have:

$$\lim_{k \to +\infty} \frac{F(x_k)-F(x_0)}{\|x_k-x_0\|} = \frac{\partial F}{\partial \tilde{d}}(x_0) \in \text{cl}U \backslash \text{int}U.$$

For ii) we have $\tilde{d} \in \text{int}(x_0+K(d,\varepsilon))$ and this implies the existence of n^* such that $x_k \in S \cap (x_0+K(\tilde{d},\varepsilon))$ $\forall k>n^*$, and this contradicts the local U*-efficiency of x_0 with respect to $S \cap (x_0+K(d,\varepsilon))$. ♦

As a direct consequence of the previous Theorem we obtain the following sufficient optimality condition which extends the one given in [2] to any kind of efficient point.

Corollary 5.1
Let us consider problem P where U is a closed cone.

i) if F is regular directionally differentiable at x_0 then (5.4) is a sufficient condition for x_0 to be a local U*-efficient point for P:

$$\frac{\partial F}{\partial v}(x_0) \in U, \quad \forall v \in T(S,x_0), \quad v \neq 0. \tag{5.4}$$

ii) if F is directionally differentiable and locally lipschitzian at x_0 then (5.4) is a sufficient condition for x_0 to be a local U*-efficient point for P;

iii) if F is differentiable at x_0 then (5.5) is a sufficient condition for x_0 to be a local U*-efficient point for P:

$$J_{F_{x_0}}(v) \in U, \quad \forall v \in T(S,x_0), \quad v \neq 0. \tag{5.5}$$

Under suitable assumptions of convexity relation (5.5) can be characterized in the form given in the following Theorem.

Theorem 5.5
Let us consider problem P where U is a closed convex and pointed cone, S is a convex closed set and F is differentiable at x_0. Then condition (5.5) is equivalent to the following condition (5.6):

$$\exists \alpha \in \text{int}U^+ \text{ such that } \alpha^T J_{F_{x_0}}(v) \leq 0 \quad \forall v \in T(S,x_0), \quad v \neq 0 \tag{5.6a}$$

$$J_{F_{x_0}}(v) \neq 0 \quad \forall v \in T(S,x_0), \quad v \neq 0 \tag{5.6b}$$

Proof.
The convexity of S implies that $T(S,x_0)$ is a closed convex cone, so that (5.6a) follows from (5.5) by applying a known separation Theorem; (5.6b) follows directly from (5.5) by noting that $0 \in U$.
Assume now that (5.6) holds. If (5.5) does not hold there exist $v \in T(S,x_0)$, $v \neq 0$ such that $J_{F_{x_0}}(v) \in U$; from (5.6b) we have $J_{F_{x_0}}(v) \in U^0$ so that $\alpha^T J_{F_{x_0}}(v) > 0$ and this contradicts (5.6a). ♦

The proof given in the previous Theorem points out that relation (5.6) implies (5.5) without any requirement of convexity so that, taking into account Corollary 5.1 and Remark 5.2 and since (5.6b) implies the injectivity of $J_{F_{x_0}}$ when $v \in \mathbf{R}^n$, $v \neq 0$, we obtain the following sufficient optimality conditions for any kind of efficient point.

Theorem 5.6

Let us consider problem P where U is a closed cone and F is differentiable at x_0.

i) if condition (5.6) holds then x_0 is a local U*-efficient point for P;

ii) if x_0 is an interior point, $J_{F_{x_0}}$ is injective and $\exists \alpha \in \text{int}U^+$ such that $\alpha^T J_{F_{x_0}} = 0$ then x_0 is a local U*-efficient point for P.

At last we point out the role of vector generalized concavity in stating sufficient optimality conditions of the Kuhn-Tucker type. With this aim consider the vector optimization problem P in the form:

$$P: \text{ U-max } F(x), \quad x \in S=\{x \in X: G(x) \in V\}$$

where $X \subset \mathbf{R}^n$ is an open set, $F: X \to \mathbf{R}^s$, $G: X \to \mathbf{R}^m$ are continuous and differentiables functions, $s \geq 1$, $m \geq 1$, and $U \subset \mathbf{R}^s$, $V \subset \mathbf{R}^m$ are closed, pointed, convex cones with verteces at the origin such that $\text{int}U \neq \emptyset$, $\text{int}V \neq \emptyset$.

Let x_0 be a feasible point and assume that $G(x_0)=0$ (when $V=\mathbf{R}^m_+$, $G(x_0)=0$ means that x_0 is binding at all the constraints so that such an assumption is not restrictive taking into account the continuity of F and G).

It is well known that the U*-efficiency of the point x_0 implies the validity of the following F. John conditions:

$$\exists (\alpha_F, \alpha_G) \neq 0, \ \alpha_F \in U^+, \ \alpha_G \in V^+: \ \alpha_F^T J_{F_{x_0}} + \alpha_G^T J_{G_{x_0}} = 0 \tag{5.7}$$

The following Theorem is the analogous of Theorem 5.6.

Theorem 5.7

If (5.7) holds with $\alpha \in \text{int}U^+$ and $J_{F_{x_0}}$ is injective then x_0 is a local U*-efficient point for P.

Proof.

The non local U*-efficiency of x_0 implies the existence of a sequence $\{x_n\} \subset S$, with $x_n \to x_0$, $\frac{x_n - x_0}{\|x_n - x_0\|} \to v$ such that $F(x_n)-F(x_0) \in U^*$ and $G(x_n)-G(x_0) \in U^*$ since $G(x_0)=0$. Consequently Property 5.1 and the injectivity of $J_{F_{x_0}}$ implies:

$$J_{F_{x_0}}(v) \in U^0, \ J_{G_{x_0}}(v) \in U^*, \text{ so that } \alpha_F^T J_{F_{x_0}}(v) + \alpha_G^T J_{G_{x_0}}(v) > 0,$$

and this contradicts (5.7). ◆

The following theorem generalizes a result given in [2].

Theorem 5.8

Let us consider the vector optimization problem P where S is a star shaped set at x_0 and the functions F and G are differentiable at x_0.

i) if F is (U^*, U^0)-pcv at x_0, G is V-qcv at x_0 and (5.7) holds with $\alpha_F \in \text{int}U^+$, then x_0 is a local U^*-efficient point for P;

ii) if F is $(U^*, \text{int}U)$-pcv at x_0, G is V-qcv at x_0 and (5.7) holds with $\alpha_F \in U^+ \setminus \{0\}$, then x_0 is a local U^*-efficient point for P.

Proof.

i) Suppose that there exists $x^* \in S$ such that $F(x^*) \in F(x_0)+U^*$. Since F is (U^*, U^0)-pcv at x_0 we have $J_{F_{x_0}}(x^*-x_0) \in U^0$; on the other hand if G is V-qcv at x_0 it easy to prove that $J_{G_{x_0}}(x^*-x_0) \in V$ and thus $\alpha_F^T J_{F_{x_0}}(x^*-x_0)>0$, $\alpha_G^T J_{G_{x_0}}(x^*-x_0)\geq 0$ since $\alpha_F \in \text{int}U^+$ and $\alpha_G \in V^+$. Consequently $\alpha_F^T J_{F_{x_0}}(x^*-x_0)+ \alpha_G^T J_{G_{x_0}}(x^*-x_0)>0$ and this contradicts (5.7).

ii) similar to the one given in i). ♦

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Some Remarks on Trade-off Analysis in Multi-objective Programming

Hirotaka Nakayama
Department of Applied Mathematics
Konan University
8-9-1 Okamoto, Higashinada, Kobe 658, JAPAN
Fax: +81-78-452-5507, Tel: +81-78-435-2534

Abstract: In multi-objective programming, the trade-off analysis is very important because it reflects decision makers' value judgment. In many practical applications, people usually take a scalarization of linearly weighted sum of vector-valued objective function without paying much attention to the trade-off analysis. However, the linearly weighted sum causes several difficulties in the trade-off analysis, e.g. "duality gap" for nonconvex problems, and that the weight can not necessarily reflect decision makers' value faithfully.

The author already suggested a few method for making the tade-off analysis easily in his "satsisficing trade-off method", namely the automatic trade-off method and the exact trade-off method. In this paper, some remarks in using them will be given along with some other methods.

1. Introduction

Multi-objective programming problems are formulated as follows:

$$\text{(MOP)} \qquad \text{Minimize} \quad f(x) \equiv (f_1(x),\ f_2(x), \ldots,\ f_r(x))$$

$$\text{over} \quad x \in X.$$

The constraint set X may be given by

$$g_j(x) \leq 0, \qquad j = 1, \ldots, m,$$

and/or a subset of R^n itself. For the problem (MOP), we define Pareto solutions as follows:

Definition 1.1 A solution $\hat{x}$ is said *Pareto optimal*, if there is no better solution $x \in X$ other than $\hat{x}$, namely, if

$$f(x) \lneqq f(\hat{x}) \qquad \text{for any} \quad x \neq \hat{x} \in X.$$

In general, there may be many Pareto solutions. The final decision is made among them taking the total balance over all criteria into account. This is a problem of value judgment of decision maker (in abbreviation, DM). The totally balancing over criteria is usually called *trade-off*. It should be noted that there are a great number of criteria, say, over one hundred in some practical problems such as erection management of cable stayed bridge, and camera lens design. Therefore, it is very important to develop effective methods for helping DM to trade-off easily even in problems with a great number of criteria.

Interactive multi-objective programming search a solution in an interactive way with DM while eliciting information on his/her value judgment. Along this line, several methods have been developed remarkably for the last about fifteen years: Among them, the aspiration level approach is now recognized very effective in practice, because

(i) it does not require any consistency of DM's judgment,

(ii) aspiration levels reflect the wish of DM very well,

and

(iii) aspiration levels play the role of probe better than the weight for objective functions.

In the following, we will discuss the difficulty in weighting method which is commonly used in the traditional goal programming.

2. Why is the Weighting Method Ineffective?

In multi-objective programming problems, the final decision is made on the basis of the value judgment of DM. Hence it is important how we elicit the value judgment of DM. In many practical cases, the vector objective function is scalarized in such a manner that the value judgment of DM can be incorporated.

The most well known scalarization technique is the linearly weighted sum:

$$\sum_{i=1}^{r} w_i f_i(x). \tag{2.1}$$

The value judgment of DM is reflected by the weight. Although this type of scalarization is widely used in many practical problems, there is a serious drawback in it. Namely, it can not provide a solution among sunken parts of Pareto surface due to "*duality gap*" for nonconvex cases. Even for convex cases, for example, in linear cases, even if we want to get a point in the middle of line segment between two vertices, we merely get a vertex of Pareto surface, as long as the well known simplex method is used. This implies that

depending on the structure of problem, the linearly weighted sum can not necessarily provide a solution as DM desires.

In the traditional goal programming (Charnes-Cooper, 1961), some kind of metric function from the goal f^* is used as the one representing the preference of DM. For example, the following is well known:

$$\left(\sum_{i=1}^{r} w_i |f_i(x) - f_i^*|^p\right)^{1/p} \tag{2.2}$$

The preference of DM is reflected by the weight w_i, the value of p, and the value of the goal f_i^*. If the value of p is chosen appropriately, a Pareto solution among a sunken part of Pareto surface can be obtained by minimizing the function (2.2). However, it is usually difficult to pre-determine appropriate values of them. Moreover, the solution minimizing (2.2) can not be better than the goal $\boldsymbol{f}^*$, even though the goal is underestimated.

In addition, one of the most serious drawbacks in the goal programming is that people tend to misunderstand that a desirable solution can be obtained by adjusting the weight. It should be noted that there is no positive correlation between the weight w_i and the value $f(\hat{x})$ corresponding to the resulting solution $\hat{x}$ as will be seen in the following example.

Example 2.1 Let $y_1 = f_1(x)$, $y_2 = f_2(x)$ and $y_3 = f_3(x)$, and let the feasible region in the objective space be given by

$$\{(y_1, y_2, y_3) |\ (y_1 - 1))^2 + (y_2 - 1)^2 + (y_3 - 1)^2 \leq 1\}.$$

Suppose that the goal is $(y_1^*,\ y_2^*,\ y_3^*) = (0,\ 0,\ 0)$.The solution minimizing the metric function (2.2) with $p = 1$ and $w_1 = w_2 = w_3 = 1$ is $(y_1,\ y_2,\ y_3) = (1 - 1/\sqrt{3},\ 1 - 1/\sqrt{3},\ 1 - 1/\sqrt{3})$. Now suppose that DM wants to decrease the value of f_1 a lot more and that of f_2 a little more, and hence modify the weight into $w_1' = 10,\ w_2' = 2,\ w_3' = 1$. The solution associated with the new weight is $(1 - 10/\sqrt{105},\ 1 - 2/\sqrt{105},\ 1 - 1/\sqrt{105})$. Note that the value of f_2 is worse than before despite that DM wants to improve it and hence increase the weight of f_2 up to twice. Someone might think that this is due to the normalization of weight. Therefore, we normalize the weight by $w_1 + w_2 + w_3 = 1$. The original weight normalized in this way is $w_1 = w_2 = w_3 = 1/3$ and the renewed weight by the same normalization is $w_1' = 10/13,\ w_2' = 2/13,\ w_3' = 1/13$. We can observe that w_2' is less than w_2. Now increase the normalized weight w_2 to be greater than $1/3$. To this end, set the unnormalized weight $w_1 = 10,\ w_2 = 7$ and $w_3 = 1$. With this new weight, we have a solution $(1 - 10/\sqrt{150},\ 1 - 7/\sqrt{150},\ 1 - 1/\sqrt{150})$. Despite that the normalized weight $w_2'' = 7/18$ is greater than the original one $(= 1/3)$, the obtained solution is still worse than the previous one.

As is readily seen in the above example, it is usually very difficult to adjust the weight in order to obtain a solution as DM wants. Therefore, it

seems much better to take the aspiration level of DM rather than the weight as the probe. Interactive multi-objective programming techniques based on aspiration levels have been developed so that the drawbacks of the traditional goal programming may be overcome. In the following section, we shall discuss the satisficing trade-off method developed by the author (Nakayama 1984) as one of them.

3. Satisficing Trade-off Method

In the aspiration level approach, the aspiration level at the k-th iteration $\overline{f}^k$ is modified as follows:

$$\overline{f}^{k+1} = T \circ P(\overline{f}^k) \tag{3.1}$$

Here, the operator P selects the Pareto solution nearest in some sense to the given aspiration level $\overline{f}^k$. The operator T is the trade-off operator which changes the k-th aspiration level $\overline{f}^k$ if DM does not compromise with the shown solution $P(\overline{f}^k)$. Of course, since $P(\overline{f}^k)$ is a Pareto solution, there exists no feasible solution which makes all criteria better than $P(\overline{f}^k)$, and thus DM has to trade-off among criteria if he wants to improve some of criteria. Based on this trade-off, a new aspiration level is decided as $T \circ P(\overline{f}^k)$. Similar process is continued until DM obtains an agreeable solution. This idea is implemented in DIDASS (Wierzbicki 1981 and Grauer *et al.* 1984) and the satisficing trade-off method (Nakayama 1984). In particular, the satisficing trade-off method provides several devices which make the trade-off analysis easier by using sensitivity analysis and parametric optimization techniques in traditional mathematical programming.

3.1 On The Opearation P

The operation which gives a Pareto solution $P(\overline{f}^k)$ nearest to $\overline{f}^k$ is performed by some auxiliary scalar optimization. It has been shown in Sawaragi-Nakayama-Tanino (1985) and Wierzbicki (1986) that the only one scalarization technique, which provides any Pareto solution regardless of the structure of problem, is of the Tchebyshev norm type. However, the scalarization function of Tchebyshev norm type yields not only a Pareto solution but also a weak Pareto solution. Since weak Pareto solutions have a possibility that there may be another solution which improves a critera while others being fixed, they are not necessarily "*efficient*" as a solution in decision making. In order to exclude weak Pareto solutions, the following scalarization function of the augmented Tchebyshev type can be used:

$$\max_{1 \le i \le r} w_i(f_i(x) - \overline{f}_i) + \alpha \sum_{i=1}^{r} w_i f_i(x). \tag{3.2}$$

where α is usually set a sufficiently small positive number, say 10^{-6}.

Theorem 3.1 (Nakayama-Tanino 1994) For arbitrary $w \geq 0$ and $\alpha > 0$, $\hat{x} \in X$ minimizing (3.2) is a properly Pareto optimal solution to (MOP). Conversely, if $\hat{x}$ is a properly Pareto optimal solution to (MOP), then there exist $w > 0$, $\alpha > 0$ and $\overline{f}$ such that $\hat{x}$ minimizes (3.2) over X.

The weight w_i is usually given as follows: Let f_i^* be an ideal value which is usually given in such a way that $f_i^* < \text{Min}\{f_i(x) | \ x \in X\}$, and let f_{*i} be a nadir value which is usually given by

$$f_{*i} = \max_{1 \leq j \leq r} f_i(x_j^*) \tag{3.3}$$

where

$$x_j^* = arg \min_{x \in X} f_j(x). \tag{3.4}$$

For this circumstance, we set

$$w_i^k = \frac{1}{\overline{f}_i^k - f_i^*} \tag{3.5}$$

or

$$w_i^{k'} = \frac{1}{f_{*i} - f_i^*}. \tag{3.6}$$

The minimization of (3.2) with (3.5) or (3.6) is usually performed by solving the following equivalent optimization problem, because the original one is not smooth:

(Q) Minimize $z + \alpha \sum_{i=1}^{r} w_i f_i(x)$

subject to

$$w_i^k (f_i(x) - \overline{f}_i^k) \leq z \tag{3.7}$$

$$x \in X.$$

3.2 On The Operation T

In cases DM is not satisfied with the solution for $P(\overline{\boldsymbol{f}}^k)$, he/she is requested to set his/her new aspiration level $\overline{\boldsymbol{f}}^{k+1}$. Let x^k denote the Pareto solution obtained by projection $P(\overline{\boldsymbol{f}}^k)$, and classify the objective functions into the following three groups:

(i) the class of criteria which are to be improved,

(ii) the class of criteria which may be relaxed,
(iii) the class of criteria which are acceptable as they are.

Let the index set of each class be denoted by I_I^k, I_R^k, I_A^k, respectively. Clearly, $\overline{f}_i^{k+1} < f_i(x^k)$ for all $i \in I_I^k$. Usually, for $i \in I_A^k$, we set $\overline{f}_i^{k+1} = f_i(x^k)$. For $i \in I_R^k$, DM has to agree to increase the value of $\overline{f}_i^{k+1}$. It should be noted that an appropriate sacrifice of f_j for $j \in I_R^k$ is needed for attaining the improvement of f_i for $i \in I_I^k$.

Example 3.1 Consider the same problem as in Example 2.1: Let $y_1 = f_1(x)$, $y_2 = f_2(x)$ and $y_3 = f_3(x)$, and let the feasible region in the objective space be given by

$$\{(y_1, y_2, y_3) | \ (y_1 - 1))^2 + (y_2 - 1)^2 + (y_3 - 1)^2 \leq 1\}.$$

Suppose that the ideal point is $(y_1^*, y_2^*, y_3^*) = (0, 0, 0)$, and the nadir point is $(y_{*1}, y_{*2}, y_{*3}) = (1, 1, 1)$. Therefore, using (3.6) we have $w_1 = w_2 = w_3 = 1.0$. Let the first aspiration level be $(\overline{y}_1^1, \overline{y}_2^1, \overline{y}_3^1) = (0.4, 0.4, 0.4)$. Then the solution to (Q) is $(y_1^1, y_2^1, y_3^1) = (0.423, 0.423, 0.423)$. Now suppose that DM wants to decrease the value of f_1 a lot more and that of f_2 a little more, and hence modify the aspiration level into $\overline{y}_1^2 = 0.35$ and $\overline{y}_2^2 = 0.4$. Since the present solution $(y_1^1, y_2^1, y_3^1) = (0.423, 0.423, 0.423)$ is already Pareto optimal, it is impossible to improve all of criteria. Therefore, suppose that DM agrees to relax f_3, and with its new aspiration level of $\overline{y}_3^2 = 0.5$. With this new aspiration level, the solution to (Q) is $(y_1^2, y_2^2, y_3^2) = (0.359, 0.409, 0.509)$. Although the obtained solution does not attain the aspiration level of f_1 and f_2 a little bit, it should be noted that the solution is improved in comparison to the previous one. The reason why the improvement of f_1 and f_2 does not attain the wish of DM is that the amount of relaxation of f_3 is not much enough to compensate for the improvement of f_1 and f_2. In the satisficing trade-off method, DM can find a satisfactory solution easily by making the trade-off analysis deliberately. To this end, it is also possible to use sensitivity analysis in mathematical programming. (Refer to the following automatic trade-off or exact trade-off). We have observed in the previous section that it is difficult to adjust weights for criteria so that we may get a desirable solution in the goal programming. However, the aspiration level can lead DM to his/her desirable solution easily in many practical problems.

3.3 Automatic Trade-off

It is of course possible for DM to answer new aspiration levels of all objective functions. In practical probelms, however, we often encounter cases with a great number of objective functions, for which DM tends to get tired with setting new aspiration levels for all objective functions. Therefore, it is more practical in problems with a great number of objective functions for DM to

answer only his/her improvement rather than both improvement and relaxation. At this stage, we can use the assignment of sacrifice for f_j $(j \in I_R)$ which is automatically set in the equal proportion to $(\lambda_i + \alpha)w_i$, namely, by

$$\Delta f_j = \frac{-1}{N(\lambda_j + \alpha)w_j} \sum_{i \in I_I} (\lambda_i + \alpha) w_i \Delta f_i, \qquad j \in I_R \tag{3.8}$$

where N is the number of elements of the set I_R, and λ is the Lagrange multiplier associated with the constraints in Problem (Q). The reason why (3.8) is available is given by the following:

Theorem 3.2 (Nakayama 1991-b, Nakayama-Tanino 1994) Let $\tilde{x}$ be a solution to (Q) with $X = \{x |\ g_j(x) \leq 0,\ j = 1, \ldots, m\}$ and $\tilde{f} = f(\tilde{x})$. Suppose that the second order sufficient condition holds; namely,

$$u^T \nabla^2 L(\tilde{x}; \lambda, \mu) u > 0$$

for any $u \neq 0$ such that

$$\nabla f_i(\tilde{x}) u = 0, \quad i = 1, \ldots, r-1$$

and

$$\nabla g_j(\tilde{x}) u = 0, \quad j \in J = \{j |\ g_j(\tilde{x}) = 0\}.$$

Suppose also that the vectors $(\nabla f_1(\tilde{x}), -1), \ldots, (\nabla f_r(\tilde{x}), -1)$, $(\nabla g_{k_1}(\tilde{x}), 0)$, $\ldots, (\nabla g_{k_s}(\tilde{x}), 0)$ are linearly independent, where $(k_1, \ldots, k_s)$ is the index set of active constraints of (Q). In addition, suppose that the following strict complementary slackness condition holds:

$$\lambda_i > 0 \quad \text{for any} \quad i \in 1, \ldots, r$$

$$\mu_j > 0 \quad \text{for any} \quad j \in J.$$

Then, with Δf_i $(i = 1, \ldots, r)$ such that $(\tilde{f}_1 + \Delta f_1, \ldots, \tilde{f}_r + \Delta f_r)$ is on the Pareto surface, we have

$$0 = \sum_{i=1}^{r} \lambda_i \Delta f_i + o(\| \Delta f \|)$$

Therefore, under some appropriate condition, $((\lambda_1 + \alpha)w_1, \ldots, (\lambda_r + \alpha)w_r)$ is the normal vector of the tangent hyperplane of the Pareto surface. In particular, in multi-objective linear programming problems, the simplex multipliers corresponding to a nondegenerated solution is the feasible trade-off vector of Pareto surface (Nakayama 1992).

By using the above automatic trade-off method, the burden of DM can be decreased so much in cases that there are a large number of criteria. Of course, if DM does not agree with this quota Δf_j laid down automatically,

he/she can modify it in a manual way.

Example 3.2 Consider the same problem as in Example 3.1. Suppose that DM has the solution $(y_1^1,\ y_2^1,\ y_3^1) = (0.423,\ 0.423,\ 0.423)$ associated with his first aspiration level $(\overline{y}_1^1,\ \overline{y}_2^1,\ \overline{y}_3^1) = (0.4,\ 0.4,\ 0.4)$. The Lagrange multipliers at this solution is $\lambda_1 = \lambda_2 = \lambda_3 = 0.333$. Now suppose that DM modifies the aspiration level into $\overline{y}_1^2 = 0.35$ and $\overline{y}_2^2 = 0.4$. For the amount of improvement of $|\Delta f_1| = 0.073$ and $|\Delta f_2| = 0.023$, the amount of relaxation of f_3 on the basis of automatic trade-off is $|\Delta f_3| = 0.095$. In other words, the new aspiration level of f_3 should be 0.52. If DM agrees with this trade-off, he/she will have the new Pareto solution $(y_1^2,\ y_2^2,\ y_3^2) = (0.354,\ 0.404,\ 0.524)$ to the problem (Q) corresponding to the new aspiration level $(\overline{y}_1^2,\ \overline{y}_2^2,\ \overline{y}_3^2) = (0.35,\ 0.4,\ 0.52)$. It should be noted that the obtained solution is much closer to DM's wish rather than the one in Example 3.1.

Example 3.3 Consider the following multiple objective linear programming problem:

$$\begin{aligned} f_1 &= -2x_1 - x_2 + 25 \rightarrow Min \\ f_2 &= x_1 - 2x_2 + 18 \rightarrow Min \end{aligned}$$

subject to

$$\begin{aligned} -x_1 + 3x_2 &\leq 21 \\ x_1 + 3x_2 &\leq 27 \\ 4x_1 + 3x_2 &\leq 45 \\ 3x_1 + x_2 &\leq 30 \\ x_1,\ x_2 &\geq 0 \end{aligned}$$

Suppose that the ideal point $\boldsymbol{f}^* = (4,\ 4)$ and the nadir point $\boldsymbol{f}_* = (18,\ 21)$ by using the pay-off matrix based on minimization of each objective function separately. Lettiing the first aspiration level be $\overline{\boldsymbol{f}}^1 = (15,\ 9)$, we have the first Pareto solution (11.632, 4.910) by solving the auxiliary min-max problem (Q). This Pareto point in the objective function space is the intercept of the line parallel to the line passing through $\boldsymbol{f}^*$ and $\boldsymbol{f}_*$ with the Pareto surface (curve, in this case). Now we shall consider the following three cases:

<Case 1> Suppose that DM is not satisfied with the obtained Pareto solution, and he/she wants to improve the value of f_2. Let the new aspiration level of f_2 be 4.5. Instead, suppose that DM agrees with some sacrifice of f_1. The new aspiration level of f_1 based on the automatic trade-off is 14.5.

Since the automatic trade-off is based on the linear approximation of Pareto surface at the present point, the new aspiration level obtained by the automatic trade-off is itself Pareto optimal in this case as shown in Fig. 3.2.

<Case 2> Suppose that DM wants to improve f_1 rather than f_2 at the moment when the first Pareto solution (11.632, 4.910) is obtained. Let the new asipration level of f_1 that DM desires be 9.0. Then the new aspiration level by the automatic trade-off is $\bar{\boldsymbol{f}}^2 = (9.0,\ 5.286)$, and is not Pareto optimal. Solving the auxiliary min-max problem (Q) with the new aspiration level, in this case, we have the new Pareto solution (9.774, 6.226). Since the improvement which DM desires is not so large after solving the min-max problem (Q) with an aspiration level in many practical cases, the new aspiration level produced by automatic trade-off based on the linear approximation of Pareto surface is close to the Pareto surface. Therefore, the satisficing trade-off method using the automatic trade-off yields the desirable solution usually only in a few iterations.

<Case 3> Suppose that DM wants to make f_1 less than 9.0 absolutely at the moment when the first Pareto solution (11.632, 4.910) is obtained. In this case, we have to treat f_1 as the constraint

$$f_1(x) \leq 9.0.$$

As will be shown in the subsection 3.5 below, the interchange between objectives and constraints can be made so easily in the formulation of auxiliary min-max problem. (We can treat the criteria as DM wishes between objectives and constraints by adjusting one parameter β in the min-max problem.)

3.4 Exact Trade-off

In linear or quadratic cases, we can evaluate the exact trade-off in an extended form of the automatic trade-off stated above. This implies that we can calculate exactly the amount of relaxation such that the new aspiration level is on the Pareto surface (Nakayama, 1992). The main idea in it is that the parametric optimization technique is used in stead of the simple sensitivity analysis. Using this technique, a new Pareto solution can be obtained without solving the auxiliary sclarized optimization problem again. This implies that we can obtain the new solution very quickly. Therefore, using some graphic presentation as computer outputs, DM can see the trade-off among criteria in a dynamic way, e.g. as an animation. This makes DM's judgement easier.

3.5 Interchange between Objectives and Constraints

In the formulation of the auxiliary scalarized optimization prolem (Q), change

the right hand side of the equation (3.7) into $\beta_i z$, namely

$$w_i(f_i(x) - \overline{f}_i) \leq \beta_i z. \tag{3.9}$$

As is readily seen, if $\beta_i = 1$, then the function f_i is considered to be an objective function, for which the aspiration level $\overline{f}_i$ is not necessarily attained, but the level of f_i should be better as much as possible. On the other hand, if $\beta_i = 0$, then f_i is considered to be a constraint function, for which the aspiration level $\overline{f}_i$ should be guaranteed. In many practical problems, there is almost no cases in which we consider the role of objective and constraint fixed from the beginning, but usually we want to interchange them depending on the situation. Using the formula (3.9), this can be done very easily (Korhonen 1987). In addition, if the value of β_i is set in the middle of 0 and 1, f_i can play a role in the middle of objective and constraint which is neither a complete objective nor a complete constraint (Kamenoi *et al.* 1992). This is also very effective in many practical problems.

3.6 Remarks on Trade-off for Objective Functions with 0-Sensitivity

Since α is sufficiently small like 10^{-6} and $\lambda_1 + \ldots \lambda_r = 1$, we can consider in many cases

$$(\lambda_i + \alpha)w_i \simeq \lambda_i w_i.$$

When $\lambda_j = 0$ and f_j is not to be improved, we set $\Delta f_j = 0$ in the automatic trade-off. Therefore, unless we select at least one f_j with $\lambda_j \neq 0$ as an objective function to be relaxed, we can not attain the improvement that DM wishes.

Since $(\lambda_i + \alpha)w_i$ (or approximately, $\lambda_i w_i$) can be regarded to provide the sensitivity information in the trade-off, $\lambda_i = 0$ means that the objective function f_i does not contribute to the trade-off among the objective functions. In other words, since the trade-off is the negative correlation among objective functions, $\lambda_i = 0$ means that f_i has the positive correlation with some other objective functions. Therefore, if all objective functions to be relaxed, f_j $(j \in I_R)$, have $\lambda_j = 0$ $(j \in I_R)$, then they can not compensate for the improvement which DM wishes, because they are affected positively by some of objective functions to be improved.

Example 3.4 (Steuer 1990) Consider the following problem:

$$\text{Minimize} \quad (f_1,\ f_2,\ f_3) = (x_1,\ x_2,\ x_3)$$

subject to

$$\begin{aligned} x_1 + x_2 + x_3 &\geq 1 \\ x_1 \qquad - x_3 &\geq 0 \\ x_1,\ x_2,\ x_3 &\geq 0 \end{aligned}$$

The ideal point is (0, 0, 0) and the nadir point is (1.0, 1.0, 0.5). For the first aspiration level (0.2, 0.2, 0.4), we have a Pareto value (0.333, 0.333, 0.333) and the corresponding simplex multiplier $(\lambda_1, \lambda_2, \lambda_3) = (2/3, 1/3, 0)$. Suppose that DM wants to improve f_1 and f_2, and sets their new aspiration levels 0.2 and 0.3, respectively. Since the relaxation $\Delta f_3 = 0$ by the automatic trade-off, the new aspiration level becomes (0.2, 0.3, 0.333). Associated with the new aspiration level, we have the Pareto value (0.3, 0.4, 0.3), in which neither f_2 is improved nor f_3 is relaxed. This is because that the objective functions f_1 and f_3 has a positive correlation along the edge of Pareto surface at the point (0.333, 0.333, 0.333), while f_1 and f_2 have trade-off relation with each other there. As a result, though f_3 was considered to be relaxed, it was affected strongly by f_1 and hence improved. On the other hand, despite that f_3 was considered to be improved, it was relaxed finally. This is due to the fact that the objective function to be relaxed is only f_3 despite that λ_3 is 0, and the fact that we did not consider that f_3 has positive correlation with f_1. This example suggests that we should make the trade-off analysis deliberately seeing the value of simplex multiplier (Lagrange multiplier, in nonlinear cases). For further discussion on the trade-off analysis using information of directional derivatives of Pareto surface, see Henig *et al.* (1992) and Granat (1993). Like this, the satisficing trade-off method makes the DM's trade-off analysis easier by utilizing the information of sensitivity.

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Eight Concepts of Optimality

Milan Zeleny
Graduate School of Business Administration,
Fordham University at Lincoln Center,
New York, New York 10023, U.S.A.
Fax. 212-765-5573; e-mail: zeleny@mary.fordham.edu

Abstract: The notion of optimality and the processes of optimization are clearly pivotal in the areas of economics, engineering, management and business. These are mostly "constrained" problems where search for optimality relies on some form of evaluating of tradeoffs. There can be not tradeoffs along a single dimension: optimality is therefore a fundamentally multicriterion concept. Here we develop the notion of optimum as a well-defined *balance among multiple criteria*. We end up with the *eight* different, separate and mutually irreducible optimality concepts where the traditional single-objective optimality is only a special case.

1. INTRODUCTION

Any criterion (measure, yardstick) or attribute is characterized by the most preferred score (or range of scores) by a decision agent in a given context. This contextually most preferred score is, *if feasible*, clearly optimal.

There is no difficulty with this and no need for further elaboration. We can usually recognize the optimal amount of sugar in our coffee, optimal temperature of our body, optimal amount of water we drink daily, or optimal amount of cash we carry - as long as the preferred scores are feasible.

The difficulty arises when our most preferred scores are infeasible, i. e., if there are explicit or implicit constraints on our criteria which prevent the achievement of the "most preferred".

When the optimum is infeasible, it can be approached only through maximization or minimization subject to constraints. This is virtually always the case in economics, business or management. When the constraints are fixed and there is only a single criterion the situation becomes trivial.

If we relax at least some constraints or consider multiple criteria then the situation turns multidimensional: tradeoffs have emerged, criteria scores have to be balanced and mechanistic maximization or minimization does not suffice.

When there is only a single criterion - no matter how comprehensive or complex - selected to describe reality, its maximization or minimization with respect to *constraints* is sufficient. When there are multiple criteria, much richer forms of optimality and optimization need to be explored.

2. EIGHT CONCEPTS OF OPTIMALITY

What is a priori given, fixed or determined cannot be subjected to subsequent optimization and it does not have to be optimized.

What is not given remains to be selected, chosen or identified and it is therefore, by definition, subject to optimization. Consequently, different optimality concepts can be derived from the distinctions between what is given and what is yet to be (optimally) determined.

We have chosen here to present each of the eight optimality concepts through (a) small real-life example and (b) through graphical representation of two-dimensional linear programming (LP) problem.

2.1. Single-objective "optimality"

This refers to conventional maximization (or "optimization"). Although not strictly a "tradeoff balancing" problem, it should be included for the sake of completeness as a special case of *bona fide* optimization.

To maximize a single criterion, it is entirely sufficient to perform technical measurement and search processes. Once X and f were formulated or specified, the "optimum" (e. g., maximum) is found by computation, not via decision or conflict dissolution processes. Search for optimality is simulated by *scalarization*, assigning each alternative a number (scalar) and then identifying the highest-numbered alternative.

(a) Example: From a list of five places (X), find the one that is the cheapest (Min f) for vacations.

(b) LP problem:

$$\text{Max } f = cx$$
$$\text{s. t. } Ax \leq b,\ x \geq 0,$$

where $c \in \mathcal{R}^n$ and $A \in \mathcal{R}^{mxn}$ are coefficient vector and matrix of dimensions 1xn and mxn respectively, $b \in \mathcal{R}^m$ is mx1 dimensional vector of *given* resources (constraint levels) and $x \in \mathcal{R}^n$ is nx1 vector of decision variables (solutions). This problem in two dimensions is graphically represented in Figure 1.

2.2. Multiobjective optimality

Optimization is not a simple maximizing and must involve balancing and harmonizing *multiple criteria*. In reality, humans continually resolve conflicts among multiple criteria which are competing for their attention and assignation of importance. This corresponds to the *vector optimization* problem of Max $f_1(x)$, Max $f_2(x)$, ... and Max $f_k(x)$ simultaneously and subject to $x \in X$.

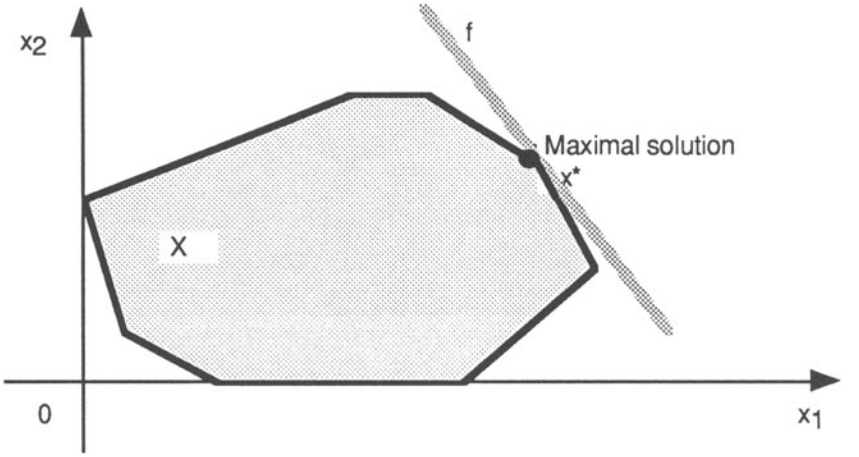

Figure 1. Single-objective "optimality"

Such parallel maximization of individual functions should remain non-scalarized, separate and independent, i. e., not subject to aggregation, like forming and maximizing a superfunction $U(f_1(x), f_2(x), ..., f_k(x))$. Such aggregation U effectively reduces multiobjective optimization to a single-objective maximization. Multiple criteria, if they are to remain meaningful and functional, should be optimized (or balanced) in the *vector sense*, i. e., non-scalarized and in mutual competition with each other.

(a) Example: From a list of five places (X), select the one that is the cheapest (Min f_1) and safest (Max f_2) for vacations.

(b) LP problem:

$$\text{Max } F = Cx$$
$$\text{s. t. } Ax \leq b,\ x \geq 0,$$

where $C \in \mathcal{R}^{kxn}$ coefficient matrix of dimensions kxn. The solution set X* is a *set of nondominated solutions*. The situation is portrayed in Figure 2.

2.3. Optimal system design: single criterion

Instead of optimizing a given system X with respect to selected criteria,

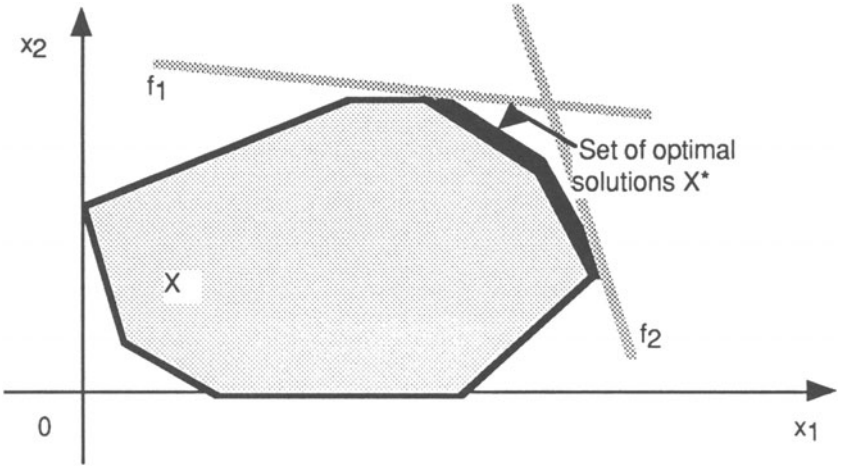

Figure 2. Multiobjective optimality

humans often seek to form or construct an optimal system X of decision alternatives, designed with respect to given criteria. Single-criterion design is analogous to single-criterion "optimization," producing the best (optimal) set of alternatives X at which a given, single objective function f(x) is maximized subject to the cost of design (affordability).

(a) Example: Design an affordable list of places (X) which would assure the cheapest (Min f) vacations.

(b) LP problem:

$$\text{Max } f = cx$$
$$\text{s. t. } Ax \leq b,\ x \geq 0,$$
$$pb \leq B$$

where $p \in \mathcal{R}^m$ is 1xm vector of unit prices of m resources and B is total available budget (or cost).

Solving the above problem requires finding the optimal allocation of budget B so that the resulting (purchased) portfolio of resources b assures the feasibility of x^* while $f(x^*) = \text{Max } f(x)$. The resulting X^* (the intersection of active constraints) is reduced to a single point: maximal solution x^*. This situation is represented in Figure 3.

2.4. Optimal system design: multiple criteria

This optimality form refers to the best system design with respect to multiple criteria.

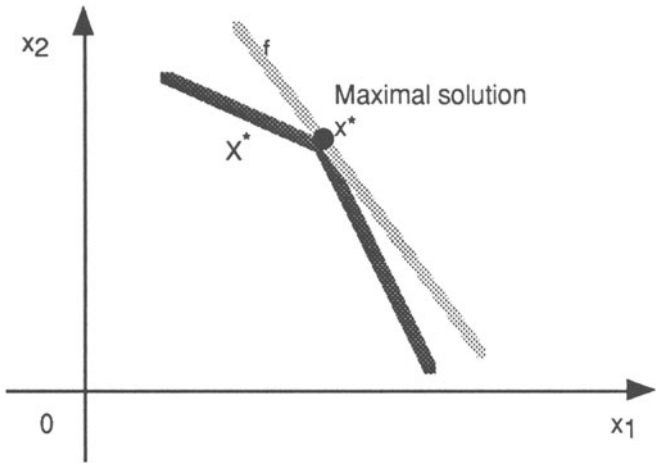

Figure 3. Optimal system design: single criterion

(a) Example: Design an affordable list of places (X), which assures the cheapest (Min f_1) and the safest (Max f_2) for vacations.

(b) LP problem:

$$\text{Max } F = Cx$$
$$\text{s. t. } Ax \leq b,\ x \geq 0,$$
$$pb \leq B$$

Solving this problem implies finding the optimal allocation of budget B so that the resulting (purchased) portfolio b of resources assures the feasibility of x^* and $F(x^*)$ at cost B and remains "as close as possible" to metaoptimum design F^* at cost B^*, $B^* \geq B$.

Linear inequalities become linear equations and the resulting X^* (their intersection) is reduced to a single point: maximal solution x^* at cost B. This situation is displayed in Figure 4.

Instead of the set of nondominated solutions, we now have a set (or a family) of optimal *system designs*, characterized by the equivalent cost B and the differential importance of objective functions f_1 and f_2.

2.5. Optimal valuation: single criterion

All of the so far considered optimization forms assume that decision criteria are given a priori. Yet, in human decision-making processes, different criteria are continually tried and applied, some are discarded, new ones added, until a

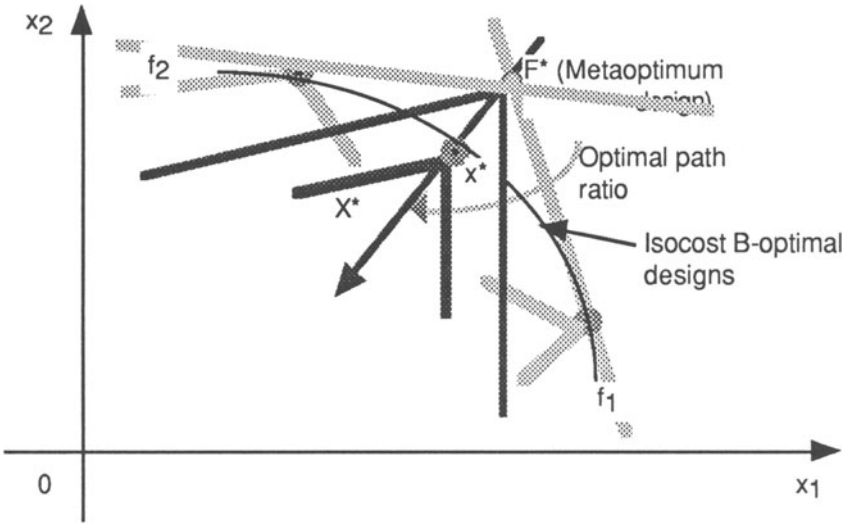

Figure 4. Optimal system design: multiple criteria

proper (balanced) mix (or portfolio) of both quantitative and qualitative criteria is achieved. There is nothing more wasteful than engaging perfectly good means X towards unworthy, ineffective or only arbitrarily determined criteria (goals and objectives).

If the set of alternatives X is given and fixed a priori, we speak of a problem of *valuation*: How should we order the alternatives? According to criterion f_1 or f_2 or f_3? Which criterion best captures our values and purposes?

(a) Example: Select a single criterion (f), perhaps from a list (f_1, f_2, f_3, ...), like entertainment, education, privacy, cost, etc., which would best evaluate a given, affordable list of places (X) in order to attain most satisfactory or fulfilling (through Max or Min f) vacations.

(b) LP problem:

Find f such that

$$\text{Max } f = cx$$
$$\text{s. t. } Ax \leq b,\ x \geq 0,$$

provides the best valuation of X with respect to a given complex of values.

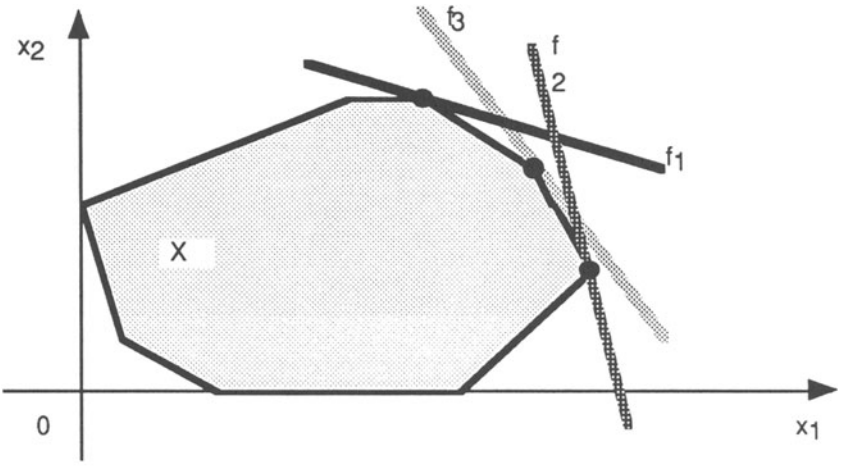

Figure 5. Single-objective valuation

2.6. Optimal valuation: multiple criteria

If the set of alternatives X is given and fixed a priori, but a set of multiple criteria is still to be selected for the evaluation and ordering of X, we speak of a problem of multiple-criteria *valuation*: Which set of criteria best captures our complex of values and purposes? Is it (f_1 and f_2)? Or (f_3 and f_4)? Or perhaps (f_1 and f_2 and f_3)? Or some other combination?

(a) Example: Select a set of criteria, perhaps a combination from a list (f_1, f_2, f_3, f_4, ...), like entertainment, education, privacy, cost, etc., which would best evaluate a given, affordable list of places (X) in order to attain most satisfactory or fulfilling (through Max or Min f) vacations.

(b) LP problem: Find F such that

$$\text{Max } F = Cx$$
$$\text{s. t. } Ax \leq b,\ x \geq 0,$$

provides the best valuation of X with respect to a given complex of values.

2.7. Optimal pattern matching: single criterion

Here we optimize both X and f simultaneously. There is a problem formulation representing an "optimal pattern" of interaction between alternatives and criteria. It is this optimal, ideal or balanced problem formulation or pattern that is to be approximated or *matched* by decision makers.

(a) Example: Select a criterion (f), like entertainment, education, privacy, cost, etc., *and* design an affordable list of places (X), which assures the most

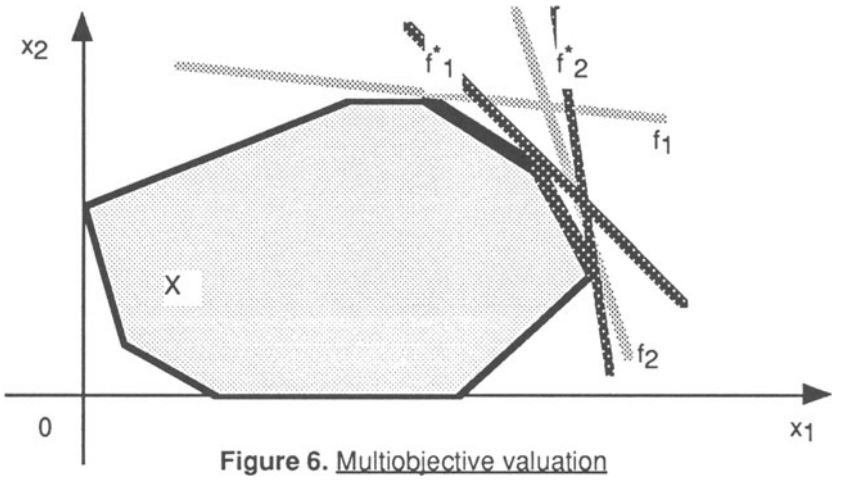

Figure 6. Multiobjective valuation

satisfactory or fulfilling (through Max or Min f) vacations.

(b) LP problem:

Find f and b such that

$$\text{Max } f = cx$$
$$\text{s. t. } Ax \leq b,\ x \geq 0,$$

matches as closely as possible optimal but infeasible (or unaffordable) pattern f* and b*, at $pb \leq B$.

Solving the above problem means establishing the optimal pattern (f*, b*, x*) and its budgetary level B*, then finding the optimal allocation of actual budget B so that the resulting (purchased) portfolio of resources b, chosen objective function f and implied solution x assures that pattern (f, b, x) matches optimal pattern (f*, b*, x*) as closely as possible. This situation is graphically represented in Figure 7.

2.8. Optimal pattern matching: multiple criteria

Pattern matching with multiple criteria is more involved and the most complex optimality concept examined so far. In all "matching" optimality concepts there is a need to evaluate the closeness (resemblance or match) of a proposed problem formulation (single- or multi-criterion) to the optimal problem formulation (or pattern).

(a) Example: Choose the necessary criteria (f_1, f_2, ..., f_k), like entertainment, education, privacy, cost, etc., *and* design an affordable list of

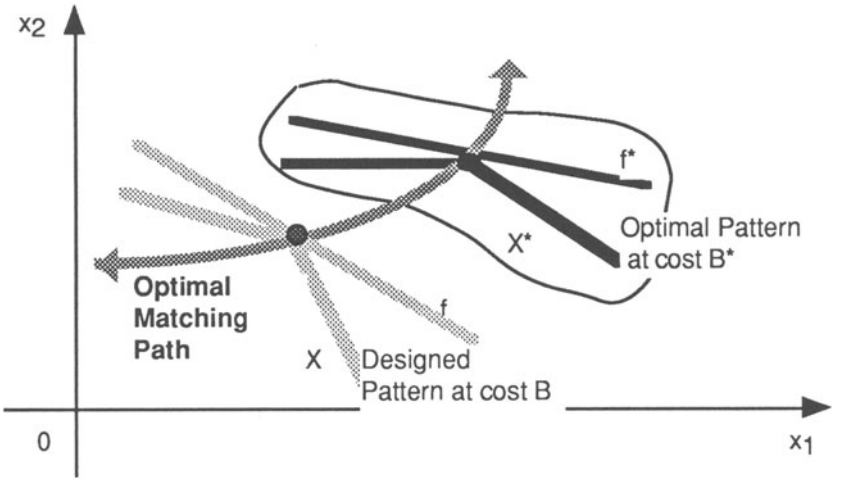

Figure 7. Optimal pattern matching: single criterion

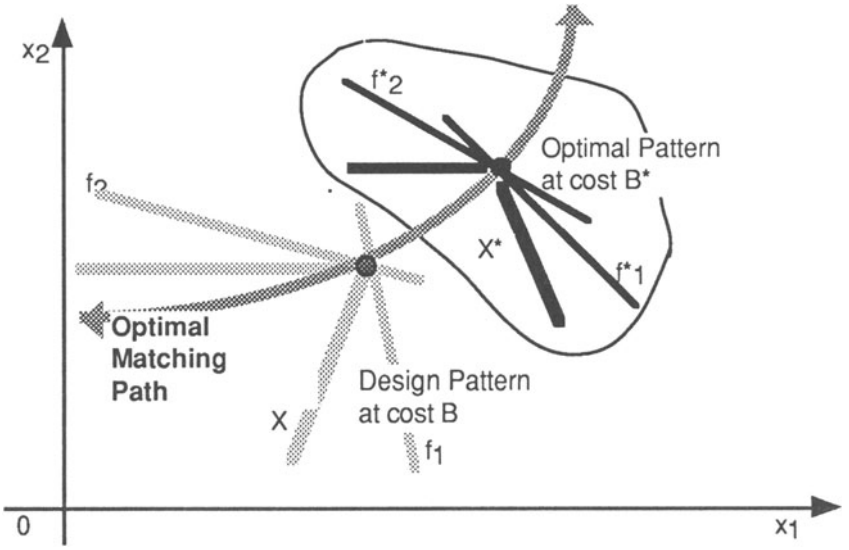

Figure 8. Optimal pattern matching: multiple criteria

places (X), which assure the most satisfying or fulfilling (through Max f_1 and Min f_2) vacations.

(b) LP problem:

Find F and b such that

$$\text{Max } F = Cx$$
$$\text{s. t. } Ax \leq b,\ x \geq 0,$$

matches as closely as possible optimal but infeasible (unaffordable) pattern F^* and b^*, at $pb \leq B$.

Solving the above problem means establishing the optimal pattern (F^*, b^*, x^*) at its budgetary level B^*, then finding the optimal allocation of actual budget B so that the resulting (purchased) portfolio of resources b and chosen set of objective functions F imply solution x, assuring that pattern (F, b, x) matches optimal pattern (F^*, b^*, x^*) as closely as possible. This situation is graphically represented in Figure 8.

3. CONCLUSION

In the invited essay "Towards the Tradeoffs-Free Optimality in MCDM," included in this volume, there is a summary (Table 1, p. 592) of the eight fundamental concepts of optimality and their classification. It is obvious that the distance from the upper left-hand to the lower right-hand corner remains as large as the gap between reality and its model.

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IV

Uncertainty in Multiobjective Programming

An invited leading paper by Slowinski presents an introduction to the subject of fuzzy multiobjective linear programming. However, the scope of this chapter also includes stochastic multiobjective programming. The paper by Slowinski presents an overview of interactive methods for solving three classes of fuzzy multiobjective linear programming problems: flexible programming, multiobjective linear programming with fuzzy coefficients and flexible multiobjective linear programming with fuzzy coefficients. Special attention is paid to interpretation of fuzzy goals and fuzzy constraints, to interpretation of fuzzy relations and to the character of interaction with the decision maker.

Shi and Liu extend their previous work on potential solutions to fuzzy potential solutions for a given fuzzy multi-criteria and multi-constraint level linear programming problem, based on the decision makers' goal-seeking and compromise behaviour. The multi-criteria and multi-constraint level linear programming problem is treated as a problem with fuzzy optimal objectives.

Abdelaziz, Lang and Nadeau examine a set of strongly efficient solutions (the unanimous solutions), assuming that the problem's randomness can be described by discrete scenarios with known probabilities and that decision makers' preferences can be represented by a class of utility functions. Inclusion relations between this and other classes of efficient solutions previously studied are stated.

The paper by Sakawa and Yano deals with large-scale multiobjective nonlinear programming problems with a block angular structure in a fuzzy environment. By incorporating the fuzzy goals of the decision maker for the objective functions and adopting the add-operator for aggregating them, a fuzzy decomposition method by right-hand-side allocation is proposed.

Interactive Fuzzy Multiobjective Programming

Roman Słowiński

Institute of Computing Science
Poznań University of Technology
60-965 Poznań, Poland
and
Institute of Theoretical and Applied Informatics
Polish Academy of Sciences
44-100 Gliwice, Poland

Abstract: The paper presents an overview of interactive methods for solving three classes of fuzzy multiobjective linear programming problems: flexible programming, multiobjective linear programming with fuzzy coefficients and flexible multiobjective linear programming with fuzzy coefficients. Special attention is paid to interpretation of fuzzy goals and fuzzy constraints, to interpretation of fuzzy relations and to the character of interaction with the decision maker.

1. Introduction

Mathematical programming and, particularly, linear programming (LP) is today one of the most frequently applied OR techniques in real-world problems. This is due to the powerful simplex method able to handle thousands of variables and constraints. Because of this popularity, many efforts have been devoted to generalizations introducing to the LP models some new realistic aspects of decision problems while preserving the applicability of the simplex method. Probably the most striking example of this generalization is the Fuzzy Multiobjective Linear Programming (FMOLP) which extends the LP model in two important aspects:

- multiple objective functions representing multiple points of view used for evaluation of feasible solutions,
- uncertainty and imprecision inherent to information used in the modelling and solving stage.

A general model of the FMOLP problem can be presented as the following system:

$$[\tilde{c}_1 x, \tilde{c}_2 x, \ldots, \tilde{c}_k x] \rightarrow \tilde{min} \tag{1}$$

$$\text{subject to } \tilde{a}_i x \mathrel{\tilde{\leq}} b_i \qquad i=1,\ldots,m \tag{2}$$

$$x \geq 0 \tag{3}$$

where $\tilde{c}_l = [\tilde{c}_{l1},\ldots,\tilde{c}_{ln}]$ (l=1,...,k), $x = [x_1,\ldots,x_n]^T$, $\tilde{a}_i = [\tilde{a}_{i1},\ldots,\tilde{a}_{in}]$ (i=1,...,m). The coefficients with the sign of wave are fuzzy numbers, i.e. convex continuous fuzzy subsets of the real line. The wave over *min* and relation $\leq$ means "fuzzification" of their meaning. Conditions (2) and (3) define a set of feasible solutions (decisions) X. An additional information completing (1) is a set of fuzzy aspiration levels on particular objectives, thought of as goals, denoted by $\tilde{g}_1,\ldots,\tilde{g}_k$.

There are important special cases of the above problem which gave birth to three classes of problems:

- flexible programming,
- multiobjective linear programming (MOLP) with fuzzy coefficients,
- flexible MOLP with fuzzy coefficients.

In flexible programming, coefficients are crisp but there is a fuzzified relation $\tilde{\leq}$ between objective functions and the goals, and between left- and right-hand sides of the constraints. It means that the goals and constraints are fuzzy ("soft") and the key question is the degree of satisfaction. In MOLP with fuzzy coefficients all the coefficients are, in general, fuzzy numbers and the key question is a representation of relation $\leq$ between fuzzy left- and right-hand sides of the constraints. Flexible MOLP with fuzzy coefficients concerns the most general form of (1)-(3) and combines the two key questions of the previous problems.

The two first problems deal with different kinds of uncertainty in the information. In flexible programming, the uncertainty follows from vagueness of preferential information concerning satisfaction of constraints and achievement of goals. In MOLP with fuzzy coefficients, the uncertainty follows from imprecision of data used for model building.

The aim of this paper is not to make another survey (cf. [11, 12, 14, 21, 25]) - it would need much more place than available. Instead, we wish to make a tutorial characterization of the three classes of problems and solution methods. It is done in the three subsequent sections. They are followed by a final section with concluding remarks.

2. Flexible Programming

In 1970, Bellman and Zadeh [1] defined the concept of *fuzzy decision* as an intersection of *fuzzy goals* and *fuzzy constraints*. A fuzzy goal corresponding to objective $c_l x$ is defined as a fuzzy set in X; its membership function $\mu_l: X \rightarrow [0,1]$ characterizes the decision maker's aspiration of making $c_l x$ "essentially smaller or equal to g_l". A fuzzy constraint corresponding to $a_i x \tilde{\leq} b_i$ is also defined as a fuzzy set in X; its membership function $\mu_i: X \rightarrow [0,1]$ characterizes the degree of satisfaction of the i-th constraint.

An aspiration level g_l and a tolerance margin $d_l \geq 0$ have to be assumed for the l-th goal (l=1,...,k) when assessing the membership function $\mu_l(\boldsymbol{x})$ as:

$$\mu_l(\boldsymbol{x}) = \begin{cases} 1 & \text{for } \boldsymbol{c}_l\boldsymbol{x} \leq g_l \\ \text{strictly decreasing from 1 to 0} & \text{for } g_l < \boldsymbol{c}_l\boldsymbol{x} < g_l + d_l \\ 0 & \text{for } \boldsymbol{c}_l\boldsymbol{x} \geq g_l + d_l \end{cases} \tag{4}$$

Then, a Pareto optimal solution based on inequality relationships between the membership functions and not between the objective functions is introduced, called an M-Pareto optimal solution [25].

Definition (M-Pareto optimal solution) Solution $\boldsymbol{x}^*$ is said to be M-Pareto optimal if and only if there does not exist another $\boldsymbol{x} \in X$ such that $\mu_l(\boldsymbol{x}) \geq \mu_l(\boldsymbol{x}^*)$, l=1,...,k, with strict inequality holding for at least one l.

In order to define the membership function $\mu_i(\boldsymbol{x})$ for the i-th constraint, one has to know the tolerance margin $d_i \geq 0$ for the right-hand side b_i (i=1,...,m):

$$\mu_i(\boldsymbol{x}) = \begin{cases} 1 & \text{for } \boldsymbol{a}_i\boldsymbol{x} \leq b_i \\ \text{strictly decreasing from 1 to 0} & \text{for } b_i < \boldsymbol{a}_i\boldsymbol{x} < b_i + d_i \\ 0 & \text{for } \boldsymbol{a}_i\boldsymbol{x} \geq b_i + d_i \end{cases} \tag{5}$$

The set of fuzzy decisions can be characterized by an aggregation of the component membership functions, e.g. by the conjunctive minimum operator, what results in the membership function:

$$\mu_D(\boldsymbol{x}) = \min_{l,i} \{\mu_l(\boldsymbol{x}), \mu_i(\boldsymbol{x})\} \tag{6}$$

Then, the problem of finding the best decision (solution) boils down to the following optimization problem:

$$\begin{aligned} &\mu_D(\boldsymbol{x}) \rightarrow max \\ &\text{subject to } \boldsymbol{x} \geq 0 \end{aligned} \tag{7}$$

The value of the aggregated function $\mu_D(\boldsymbol{x})$ can be interpreted as the overall degree of satisfaction of the decision maker with k fuzzy goals and m fuzzy constraints.

In case of minimum operator (6), problem (7) becomes:

$$\begin{aligned} &v \rightarrow max \\ &\text{subject to } v \leq \mu_l(\boldsymbol{x}) \qquad l=1,...,k \\ &\qquad\qquad\quad v \leq \mu_i(\boldsymbol{x}) \qquad i=1,...,m \\ &\qquad\qquad\quad \boldsymbol{x} \geq 0 \end{aligned} \tag{8}$$

In 1978, Zimmermann [34] applied for the first time linear membership functions (4), (5) in problem (8) thus getting an ordinary LP problem. He also proposed to use the product operator instead of minimum, however, then (7) becomes nonlinear even if linear membership functions are used.

A comprehensive review of various propositions for modelling the functions $\mu_D(x)$ can be found in [35].

Several interactive methods have also been proposed for flexible programming (see [25]). In these methods, the decision maker determines membership functions for fuzzy goals and then specifies reference levels for the membership functions, denoted by $\bar{\mu}_l$ (l=1,...,k). Assuming some minimum levels for membership functions of fuzzy constraints, denoted by t_i (i=1,...,m), one gets the following optimization problem:

$$\begin{aligned} &\max_l \{\bar{\mu}_l - \mu_l(x)\} \to min \\ &\text{subject to} \quad \mu_i(x) \geq t_i \qquad i=1,...,m \\ &\qquad\qquad x \geq 0 \end{aligned} \tag{9}$$

which is equivalent to:

$$\begin{aligned} &v \to min \\ &\text{subject to} \quad v \geq \bar{\mu}_l - \mu_l(x) \qquad l=1,...,k \\ &\qquad\qquad \mu_i(x) \geq t_i \qquad\quad i=1,...,m \\ &\qquad\qquad x \geq 0 \end{aligned} \tag{10}$$

Again, problem (10) becomes an ordinary LP problem when all membership functions are linear.

This approach is interactive in the sense that the reference levels can be changed from one iteration to another, as well as the membership functions of fuzzy goals (see [10]).

3. MOLP with Fuzzy Coefficients

All fuzzy coefficients of the FMOLP problem are given in a convenient form of L-R fuzzy numbers [9]. An L-R fuzzy number $\tilde{a} = \left(a^L, a^R, \alpha^L, \alpha^R\right)_{LR}$ is defined by the membership function:

$$\mu_{\tilde{a}}(r) = \begin{cases} L((a^L - r)/\alpha^L) & \text{for} \quad r \leq a^L \\ 1 & \text{for} \quad a^L \leq r \leq a^R \\ R((r - a^R)/\alpha^R) & \text{for} \quad r \geq a^R \end{cases}$$

where L and R are symmetric bell-shaped reference functions which are strictly decreasing in [0,1] and such that $L(0)=R(0)=1,\ L(1)=R(1)=0;\ \left[a^L, a^R\right]$ is an interval of the most possible values, and α^L and α^R are nonnegative left and right "spreads" of $\tilde{a}$, respectively.

According to Rommelfanger [20], the precise form of a fuzzy number can be described by an expert only rarely. Therefore, as a practical way of getting a suitable membership functions of fuzzy coefficients, he has proposed that the expert begins with the specification of some prominent membership levels α, e.g.

$\alpha=1$: $\mu_{\tilde{a}}(r)=1$ means that value r certainly belongs to the set of possible values,

$\alpha=\lambda$: $\mu_{\tilde{a}}(r)\geq\lambda$ means that the expert estimates that value r with $\mu_{\tilde{a}}(r)\geq\lambda$ has a good chance of belonging to the set of possible values,

$\alpha=\varepsilon$: $\mu_{\tilde{a}}(r)<\varepsilon$ means that value r with $\mu_{\tilde{a}}(r)<\varepsilon$ has only a very little chance of belonging to the set of possible values, i.e. the expert is willing to neglect the corresponding values of r with $\mu_{\tilde{a}}(r)<\varepsilon$.

For example, it is reasonable to assume that $\lambda=0.6$, $\varepsilon=0.1$.

For the sake of clarity, let us assume that the reference functions of all fuzzy coefficients are of two kinds only: L and R. It should be specified, moreover, that all arithmetic operations on fuzzy numbers taking place in (1), (2) are extended operations in the sense of Zadeh's extension principle [33]:

$$f_{\tilde{a}*\tilde{b}}(r)=\sup_{r=y*z} T(f_{\tilde{a}}(y), f_{\tilde{b}}(z)),\quad r\in\Re \tag{11}$$

where $*$ is a real operation $*:\ \Re\times\Re\rightarrow\Re$ and $T:\ [0,1]\times[0,1]\rightarrow[0,1]$ is any given t-norm.

For any $\boldsymbol{x}\geq 0$, the left-hand side of the i-th constraint and the value of the l-th objective function can be summarized to the following fuzzy numbers:

$$\tilde{\boldsymbol{a}}_i\boldsymbol{x}=\left(\boldsymbol{a}_i^L\boldsymbol{x}, \boldsymbol{a}_i^R\boldsymbol{x}, \alpha_i^L\boldsymbol{x}, \alpha_i^R\boldsymbol{x}\right)_{LR}\quad i=1,\ldots,m$$

$$\tilde{\boldsymbol{c}}_l\boldsymbol{x}=\left(\boldsymbol{c}_l^L\boldsymbol{x}, \boldsymbol{c}_l^R\boldsymbol{x}, \gamma_l^L\boldsymbol{x}, \gamma_l^R\boldsymbol{x}\right)_{LR}\quad l=1,\ldots,k$$

In the literature, the *min* t-norm is generally applied. Then,

$$\boldsymbol{a}_i^L\boldsymbol{x}=\sum_{j=1}^{n} a_{ij}^L x_j,\quad \boldsymbol{c}_l^L\boldsymbol{x}=\sum_{j=1}^{n} c_{lj}^L x_j \tag{12}$$

and other characteristic parameters of $\tilde{\boldsymbol{a}}_i\boldsymbol{x}$ and $\tilde{\boldsymbol{c}}_l\boldsymbol{x}$ are calculated analogously. Obviously, the spreads of these fuzzy numbers extend when number and size of variables increase. For getting a more realistic extended addition of the left-hand

sides of fuzzy constraints and of fuzzy objectives, Rommelfanger and Keresztfalvi [23] recommend the use of Yager's parametrized *t*-norm:

$$T_p(t_1,\ldots,t_s)=max\left\{0,\ 1-\left(\sum_{i=1}^{s}(1-t_i)^p\right)^{1/p}\right\} \tag{13}$$

$$t_1,\ldots,t_s \in [0,1],\quad p>0$$

Then, $a_i^L x$, $a_i^R x$, $c_l^L x$, $c_l^R x$ are calculated according to (12), however, the spreads $\alpha_i^L x$, $\alpha_i^R x$, $\gamma_l^L x$, $\gamma_l^R x$ are calculated according to a new, less cumulative formula:

$$\alpha_i^L x=\left(\sum_{j=1}^{n}\left(\alpha_{ij}^L x_j\right)^q\right)^{1/q} \quad \text{where} \quad q=\frac{p}{p-1}\geq 1.$$

Coming back to the MOLP problem with fuzzy coefficients, we have to answer the question how to interprete the relation between fuzzy left- and right-hand side of the constraints. If constraints (2) were transformed to equality constraints (by addition of slack variables on the left) then the equality relation could be interpreted in terms of a weak inclusion of fuzzy sets [8, 15]:

$$\tilde{a}_i x \subseteq \tilde{b}_i \quad i=1,\ldots,m \tag{14}$$

It says that the region of possible values of the left-hand side should be contained in the tolerance region of the right-hand side. The LP problem with constraints (14) is called *robust programming problem.*

Each constraint (14) is then reduced to four deterministic constraints:

$$\begin{aligned} &a_i^L x \geq b_i^L, \quad a_i^R x \leq b_i^R \\ &a_i^L x-\alpha_i^L x \geq b_i^L-\beta_i^L \\ &a_i^R x+\alpha_i^R x \leq b_i^R+\beta_i^R, \quad \text{for } i=1,\ldots,m \end{aligned} \tag{15}$$

where $\tilde{b}_i=\left(b_i^L,b_i^R,\beta_i^L,\beta_i^R\right)_{LL}$ or $\tilde{b}_i=\left(b_i^L,b_i^R,\beta_i^L,\beta_i^R\right)_{RR}$, $i=1,\ldots,m.$

In order to transform fuzzy objectives into deterministic ones, one can consider a "mean" value $\tilde{c}_l x$ at some level $\xi \in [0,1]$, $l=1,\ldots,k$. The "mean" can be understood [6] as a combination of the most possible values $c_l^L x$ and $c_l^R x$, and of the least and the greatest (extreme) values at possibility level ξ. Thus, the objectives (1) become:

$$[z_1(x),z_2(x),\ldots,z_k(x)]\rightarrow min \tag{16}$$

where $z_l(x) = w_1 c_l^L - w_2 \gamma_l^L x\, L^{-1}(\xi) + w_3 c_l^R x + w_4 \gamma_l^R x\, R^{-1}(\xi)$, $l = 1,\ldots,k$; w_1, w_2, w_3, w_4 are non-negative weights, e.g. $w_1 = w_3 = 0.3$, $w_2 = w_4 = 0.2$. The deterministic objectives (16) are linear even if reference functions L and R are nonlinear.There are also approaches proposing a substitution of each objective by several deterministic objectives corresponding to extreme values of few ξ-level sets [7, 22].

In consequence, the MOLP problem with fuzzy coefficients is transformed to an associate deterministic MOLP problem of the form (16), (15), (3) which should, preferably, be solved by one of existing interactive procedures (see e.g. [30]).

4. Flexible MOLP with Fuzzy Coefficients

This problem is a combination of problems previously described. In addition to fuzzy coefficients in the objective functions and on the both sides of the constraints, an important feature is the degree of satisfaction of fuzzy constraints and fuzzy goals.

In most of existing approaches, fuzzy constraints (2) are replaced by one or two deterministic linear constraints. To give an idea of these surrogates, let us present them in common terms from the most pessimistic to the most optimistic attitude [24]:

- $a_i^R x + \alpha_i^R x\, R^{-1}(\rho) \le b_i^R, \quad \rho \in [0,1]$, [2, 29]
- $a_i^R x + \alpha_i^R x\, R^{-1}(\sigma) \le b_i^R + \beta_i^R\, R^{-1}(\sigma), \quad \sigma \in [0,1]$, [3]
- $\begin{cases} a_i^R x \le b_i^R \\ a_i^R x + \alpha_i^R x\, R^{-1}(\varepsilon) \le b_i^R + \beta_i^R\, R^{-1}(\varepsilon), \quad \varepsilon \in [0,1], \end{cases}$ [16, 19, 32]
- $\begin{cases} a_i^L - b_i^R \le L^{-1}(\tau)\left(\alpha_i^L x + \beta_i^R\right), \quad \tau \in (0,1] \\ a_i^R x + \alpha_i^R x\, R^{-1}(\eta) \le b_i^R + \beta_i^R\, L^{-1}(\eta), \quad \eta \in [0,1], \end{cases}$ [6, 27, 28]
- $\begin{cases} a_i^R x \le b_i^R + \delta\beta_i^R, \quad \delta + \varepsilon \in [0,1],\ \delta \ge 0,\ \varepsilon \ge 0 \\ a_i^R x + (1-\varepsilon-\delta)\alpha_i^R x \le b_i^R + (1-\varepsilon)\beta_i^R, \end{cases}$ [17]
- $a_i^L x - \alpha_i^L x\, L^{-1}(\alpha) \le b_i^R + \beta_i^R\, R^{-1}(\alpha), \quad \alpha \in [0,1]$, [13, 26]

In all these approaches, the parameters $\alpha, \delta, \varepsilon, \eta, \tau, \rho, \sigma$ can be used by the decision maker to control the degree of satisfaction of fuzzy constraints in an interactive way.

Another interpretation of fuzzy constraints has been given in [18]. The *i*-th fuzzy constraint is replaced by the pessimistic constraint proposed in [27] and by a new objective:

$$a_i^R x + \alpha^{R\varepsilon} x \le b_i^R + \beta_i^{R\varepsilon} \tag{17}$$

$$\mu_i(x) \to max \tag{18}$$

where membership function $\mu_i(x)$ is defined according to (5).

If fuzzy goals are specified as *L-R* fuzzy numbers $\tilde{g}_l = (g_l, 0, v_l)_{LL}$ (l=1,...,k), then the satisfying condition

$$\tilde{c}_l x \tilde{\le} \tilde{g}_l, \quad l = 1,\dots,k \tag{19}$$

can be treated as an additional fuzzy constraint. In accordance to the chosen interpretation of the inequality relation, (19) can be substituted by one or two crisp inequalities listed above or by (17) and (18). Another proposal has been made in [27, 28]; the associate deterministic objectives represent the levels of intersection of left reference functions of $\tilde{c}_l x$ with right reference functions of g_l (l=1,...,k):

$$L\left(\left(c_l^L x - g_l\right) / \left(\gamma_l^L x + v_l\right)\right) \to max \qquad l = 1,\dots,k \tag{20}$$

In the case of linear reference functions L, functions (20) become linear fractional:

$$\left(c_l^L x - g_l\right) / \left(\gamma_l^L x + v_l\right) \to min \qquad l = 1,\dots,k \tag{21}$$

The resulting associate deterministic multiobjective programming problem is then solved using an interactive procedure, e.g. the sampling procedure described in [6, 28]. In each calculation step of this procedure, a sample of efficient points is generated and then shown to the decision maker who is asked to select the one that best fits his/her preferences. If the selected point is not the final compromise, it becomes a central point of an efficient region which is sampled in the next calculation step. In this way, the sampled part of the efficient border is successively reduced until the most satisfactory efficient point is reached.

The interaction with the decision maker takes place at two levels: first when fixing the parameters controlling the satisfaction (safety) of fuzzy constraints and then in the course of the guided generation and evaluation of the efficient points of the associate deterministic problem.

An important feature of the software implementing a fuzzy multiobjective programming method is presentation of candidate solutions in the interactive process. In the software called FLIP [5], the solutions of the associate deterministic problem are shown not only numerically, but also graphically, in terms of mutual positions of fuzzy numbers corresponding to original objectives

and aspiration levels on the one hand, and to left- and right-hand sides of original constraints, on the other hand. In this way, the decision maker gets quite a complete idea about the quality of each proposed solution. The quality is evaluated taking into account the following characteristics:

- scores of fuzzy objectives in relation to the goals,
- dispersion of values of the fuzzy objectives due to uncertainty,
- safety of the solution or, using a complementary term, the risk of violation of the constraints.

So, the definition of the best compromise involves not only the scores on particular objectives but also the safety of the corresponding solution. It is possible due to *visual interaction.*

5. Conclusions

Fuzzy multiobjective programming methods have often been proposed in view of specific applications (see, e.g., [4, 12, 27, 31]). It means that the many proposals described in this paper are based on different assumptions which are verified in different practical situations. The choice of a procedure for an actual decision problem should take into account these assumptions. In any case, the interactive process should enable the best use of the decision maker's knowledge of the problem.

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A Fuzzy Potential Solution Approach to Multi-Criteria and Multi-Constraint Level Linear Programming Problems

Yong Shi
Department of Information Systems and Quantitative Analysis
College of Business Administration
University of Nebraska at Omaha
Omaha, NE 68182, USA

Yi-Hsin Liu
Department of Mathematics
College of Arts and Sciences
University of Nebraska at Omaha
Omaha, NE 68182, USA

Abstract: Based on the decision makers' goal-seeking and compromise behavior, this paper extends the potential solutions to fuzzy potential solutions for a given fuzzy multi-criteria and multi-constraint level (MC^2) linear programming problem. We treat the MC^2 problem as a problem with fuzzy optimal objectives. Then, we use the MC^2-simplex method to locate two sets of potential solutions for a maximum MC^2 problem and a minimum MC^2 problem, respectively. With these two sets of potential solutions, we reform the ranges of the decision parameters so that each reformed range results in both the upper and lower bounds of the objective payoff over the changes of decision parameters within the range. A membership function is constructed for each of the ranges. For every reformed range, a fuzzy potential solution is determined in terms of the membership function and a primal potential basis. We also propose a heuristic algorithm to locate all fuzzy potential solutions for all possible changes of the decision parameters.

1. INTRODUCTION

A model of multi-criteria and multi-constraint level (MC^2) linear programming was originally developed by Seiford and Yu [13]. This model is an extension of the traditional linear programming (see Dantzig [4], Charnes and Cooper [1]) and the multi-criteria (MC) linear programming (see Zeleny [23], Gal [5], Goicoechea *et al* [6], Yu [21], Steuer [19]). Since the traditional linear programming problem is formulated with only a single criterion (objective) and a single constraint (resource availability) level (right hand side), it has limitations in dealing with real world problems which involve multiple conflicting criteria. The MC linear programming can be used to handle some problems with multiple conflicting criteria. However, it cannot be used to deal with problems with both multiple conflicting criteria and multiple constraint levels. For example, the problem of selecting optimal linear production system designs is a decision problem with multiple conflicting criteria and multiple constraint levels. The multiple conflicting criteria are maximizing the total sales of selected products, total profit, and customer satisfaction, while the multiple constraint levels are determined by a group of decision makers, such as the

vice president for finance, the vice president for production, and the vice president for marketing. Such a problem cannot be solved by either linear programming or MC linear programming. It, however, can be formulated as an MC^2 linear programming problem and solved by an MC^2-simplex method (see Lee *et al* [10], Shi and Yu [16] and Shi and Yu [17]).

The model of MC^2 linear programming is mainly supported by two fold backgrounds. First, from the linear system structure's point of view, the criteria and constraints may be "interchangeable" (Yu [21]). Thus, like multiple criteria, multiple constraint levels can be considered. Second, from the application's point of view, it is more realistic to consider multiple resource availability levels (discrete right hand sides) than a single resource availability level in isolation. The philosophy behind this perspective is that the availability of the resources can fluctuate depending on decision situation forces, such as the levels of desirability believed by the different decision makers (see [10] for the further detailed discussion).

According to [13], the basic method for solving a given MC^2 problem is to identify a set of potential solutions by using an MC^2-simplex method. Each potential solution optimizes the MC^2 problem under a certain range of decision parameters, namely the criteria weight vector and the constraint level weight vector. Given any values of decision parameters, there is a potential solution optimizing the MC^2 problem. In other words, the set of potential solutions can potentially optimize the MC^2 problem for all possible changes of decision parameters.

It has been recognized that in many decision problems, instead of finding the exact "optimal solution" (a goal value), decision makers often approach a "satisficing solution" between an upper and a lower aspiration levels that represent by the upper and lower bounds of acceptability for objective payoffs, respectively (For references, see Charnes and Cooper [1], Lee [11], Yu [21], Shi and Yu [15]). This behavior, which has an important and pervasive impact on human decision making (see Lindsay and Norman [12]), is called the decision makers' goal-seeking and compromise behavior. Zimmermann [24] expressed it as the basis of his pioneering work on fuzzy linear programming.

In this paper, considering the decision makers' goal-seeking and compromise behavior, we extend the potential solutions to fuzzy potential solutions for a given fuzzy MC^2 problem. We first treat the MC^2 problem as a problem with fuzzy optimal objectives. Next, we use the MC^2-simplex method to locate two sets of potential solutions for a maximum MC^2 problem and a minimum MC^2 problem, respectively. With these two sets of potential solutions, we reform the ranges of the decision parameters so that each reformed range results in both the upper and lower bounds of the objective payoff over the changes of decision parameters within the range. Then, we construct a membership function for each of the ranges. Finally, for every reformed range, a fuzzy potential solution is determined in terms of the membership function and a primal potential basis. Based on this procedure, we propose a heuristic algorithm to locate all fuzzy potential solutions for all possible changes of the decision parameters.

This paper proceeds as follows. In Section 2, we briefly sketch the notation and definitions of the MC^2 linear programming problem and the MC^2-simplex method [13] to facilitate the discussion for a fuzzy approach. In Section 3, we describe the fuzzy approach to constructing a membership function of the MC^2 problem for a reformed range of the decision parameters. In Section 4, we develop the algorithm for locating fuzzy potential solutions. A numerical example is used to

illustrate the algorithm. In Section 5, we conclude the paper with some further research problems.

2. A MODEL MC^2 LINEAR PROGRAMMING PROBLEMS

In this section, we sketch the solution concepts of MC^2 linear programming in [13] and Chapter 8 of [21].

An MC^2 linear programming problem can be formulated as

$$\begin{aligned} &\text{Max } \lambda^t Cx \\ &\text{s. t. } Ax = D\gamma \\ &\qquad x \geq 0, \end{aligned} \tag{1}$$

where $C \in \mathbf{R}^{qxn}$, $A \in \mathbf{R}^{man}$, and $D \in \mathbf{R}^{mxp}$ are matrices of dimensions qxn, mxn, and mxp, respectively; and $\lambda > 0$ is called the q-dimensional *criteria weight vector* and $\gamma > 0$ is the p-dimensional *constraint level weight vector.*

Remark 2.1. Problem (1) has q criteria (objectives) and p constraint levels. If $D\gamma$ in Problem (1) is replaced by an m-dimensional vector $d \in \mathbf{R}^m$, then (1) is reduced to an MC linear programming problem; in addition, if $\lambda^t C$ in (1) is replaced by a n-dimensional vector $c \in \mathbf{R}^n$, then (1) is reduced to a linear programming problem. The details for the theoretical connection of MC^2 linear programming and MC linear programming can be found in Gyetvan and Shi [7].

In Problem (1), if values of vectors (γ, λ) are known ahead of decision time, we can select the best combination of k variables from possible n variables as the optimal solution by employing linear programming techniques. However, when (γ, λ) cannot be known ahead of time, the linear programming approach would not be effective because there are infinitely many possible combinations of (γ, λ). Assuming (γ, λ) are unknown ahead of time, we describe an MC^2-method for effectively and systematically locating a set of potential solutions over all possible changes of (γ, λ).

Given basic variables $\{x_{j_1}, \ldots, x_{j_m}\}$ for Problem (1), we denote the index set of the basic variables by $J = \{j_1, \ldots, j_m\}$. Without confusion, J is also called a *basis* for Problem (1). Because a potential basis J depends on (γ, λ), we have:

Definition 2.1. A basis J is a primal potential basis for the MC^2 problem (1) iff there exist some $\gamma > 0$ and $\lambda = 0$ such that J is an optimal basis for Problem (1).

Definition 2.2. A basis J is a dual potential basis for the MC^2 problem (1) iff there exist some $\lambda > 0$ and $\gamma = 0$ such that J is an optimal basis for Problem (1).

Definition 2.3. A basis J is a potential basis for the MC^2 problem (1) iff there exist a $\gamma > 0$ and $\lambda > 0$ such that J is an optimal basis for Problem (1).

Remark 2.2. From Definitions 2.1-2.3, we say that a basis J is a potential basis iff J is both a primal and a dual potential basis.

Let x(J) be the basic variables corresponding to a potential basis J. We call x(J) the potential solution of Problem (1). To find the all potential solutions for Problem (1), we need to identify the corresponding set of potential bases for Problem (1). Given a basis J with its basic variables x(J), we define the *associated basis matrix* B_J as the submatrix of A in (1) with column index of J (i.e., column j of A is in B_J iff $j \in J$), and the *associated objective function coefficient* C_J as the submatrix of C in (1) with column index of J. Let x(J') be the nonbasic variables corresponding to given x(J). Then we rearrange the index, if necessary, and decompose A into $[B_J, R]$, where R is the submatrix of A associated with x(J') and C into $[C_J, C_{J'}]$, where $C_{J'}$ is the submatrix of C associated with x(J'). The initial simplex tableau of Problem (1) can be:

x(J)	x(J')	RHS	
B_J	R	$D\gamma$	(2)
$-\lambda^t C_J$	$-\lambda^t C_{J'}$	0	(3)

In the above tableau, the first and second block of columns are the coefficients associated with the basic and nonbasic variables, respectively. Row block (2) represents constraints while row block (3) is the objective function.

Tableau (2)-(3) can be rewritten as:

x(J)	x(J')	RHS	
I_m	$B_J^{-1}R$	$B_J^{-1} D \gamma$	(4)
0	$\lambda^t [C_J B_J^{-1} R - C_{J'}]$	$\lambda^t C_J B_J^{-1} D \gamma$	(5)

where I_m is an mxm identity matrix.

Note that row block (4) is the result of premultiplying (2) by B_J^{-1}. Row block (5) is obtained by first premultiplying (4) by $\lambda^t C_J$ and then adding the result to row block (3). Tableau (4)-(5) implies that $x(J, \gamma) = B_J^{-1} D\gamma$ is a basic solution associated with (J, γ) and its objective value is given by $\lambda^t C_J B_J^{-1} D\gamma$ when (γ, λ) are specified.

By dropping (γ, λ) from Tableau (4)-(5), we obtain an MC^2-simplex tableau with a basis B_J as follows:

x(J)	x(J')	RHS	
I_m	$B_J^{-1}R$	$B_J^{-1}D$	(6)
0	$C_J B_J^{-1} R - C_{J'}$	$C_J B_J^{-1} D$	(7)

Remark 2.3. If the matrix D in (6)-(7) is replaced by a vector $d \in \mathbf{R}^m$, then the MC^2-simplex tableau is reduced to an MC-simplex tableau (Yu and Zeleny [22]); if the matrix C in (6)-(7) is further replaced by a vector $c \in \mathbf{R}^n$, then the MC-simplex tableau is reduced to a traditional simplex tableau (Charnes and Cooper [1], Dantzig [4]).

Definition 2.4. Given a basis J for Problem (1), define its corresponding
primal weight set by
$\Gamma(J) = \{\gamma > 0 \mid B_J^{-1} D \gamma \geq 0\}$; and
dual weight set by
$\Lambda(J) = \{\lambda > 0 \mid \lambda^t [C_J B_J^{-1} R - C_{J'}] \geq 0\}$.

Remark 2.4. Given a basis J for a minimum MC^2 problem (see Problem (9)), its primal weight set can be defined as same as $\Gamma(J)$ in Definition 2.4, but its dual weight set is defined by $\Lambda^*(J) = \{\lambda > 0 \mid \lambda^t [C_J B_J^{-1} R - C_{J'}] \leq 0\}$.

The following theorem is based on Seiford and Yu [13] and Chapter 8 of Yu [21].

Theorem 2.1.
Comparing (4)-(5) *with* (6)-(7), *we see that given* J:
(i) J *is a primal potential basis iff* $\Gamma(J) \neq \emptyset$;
(ii) J *is a dual potential basis iff* $\Lambda(J) \neq \emptyset$;
(iii) J *is a potential basis iff* $\Gamma(J) \times \Lambda(J) \neq \emptyset$;
(iv) *The resulting solution* $x(J, \gamma) = B_J^{-1} D \gamma \geq 0$ *iff* $\gamma \in \Gamma(J)$;
(v) *The solution* $x(J, \gamma)$ *is potential iff* $\gamma \in \Gamma(J)$ *and* $\lambda \in \Lambda(J)$;

By using the MC^2-simplex method discussed as above, we can find all potential solutions by locating a set of potential bases, denoted by $\mathcal{J} = \{J_1, \ldots, J_r\}$ for a given MC^2 problem (1). Let $\Gamma \times \Lambda$ be the Cartesian product space of weight vectors (γ, λ). Then, we have:

Definition 2.5. The space $\Gamma x \Lambda$ is said to be *covered* by $\mathfrak{J}$ *if* $(\Gamma(J_1)x\Lambda(J_1)) \cup \ldots \cup (\Gamma(J_r)x\Lambda(J_r)) \supset \Gamma x \Lambda$.

From Definitions 2.4 - 2.5 and Theorem 2.1, for every $(\gamma, \lambda) \in \Gamma x \Lambda$, if there is a J_i of $\mathfrak{J}$ such that $(\gamma, \lambda) \in \Gamma(J_i)x\Lambda(J_i)$, then $\Gamma x \Lambda$ is covered by $\mathfrak{J}$. We call such a $\mathfrak{J}$ the set of *all* potential bases over all possible changes of (γ, λ) for the MC^2 problem (1).

3. A FUZZY APPROACH TO MC^2 PROBLEM

Given an MC^2 problem (1), we now assume that decision makers have a goal-seeking and compromise behavior for a satisficing solution (see Section 2). In this section, we represent this satisficing solution by a fuzzy potential solution with a satisficing level between an upper aspiration bound and a lower aspiration bound of the objective payoff. To describe such a fuzzy potential solution, we can first rewrite Problem (1) as the following fuzzy-objective MC^2 problem:

$$\begin{aligned} &\text{Fuzzy-Max } \lambda^t Cx \\ &\text{s. t. } Ax = D\gamma \\ &\qquad x \geq 0. \end{aligned} \tag{8}$$

Then, we split it into two crisp optimization problems: one is Problem (1) itself and another is given by

$$\begin{aligned} &\text{Min } \lambda^t Cx \\ &\text{s. t. } Ax = D\gamma \\ &\qquad x \geq 0. \end{aligned} \tag{9}$$

We note that Problem (9) is a minimum MC^2 problem, while Problem (1) is a maximum MC^2 problem. By using the MC^2-simplex method (see Section 2), we can identify two sets of all potential bases over all possible changes of (γ, λ) for Problems (1) and (9), respectively. Let $\mathfrak{U}^+ = \{U_1, \ldots, U_r\}$ be the set of all potential bases over all possible changes of (γ, λ) for Problem (1) satisfying $(\Gamma(U_i)x\Lambda(U_i)) \cap (\Gamma(U_g)x\Lambda(U_g)) = \varnothing$ for $i \neq g$, $i, g = 1, \ldots, r$, and $\mathfrak{L}^- = \{L_1, \ldots, L_k\}$ be the set of all potential solutions over all possible changes of (γ, λ) for Problem (9) satisfying $(\Gamma(L_j)x\Lambda(L_j)) \cap (\Gamma(L_h)x\Lambda(L_h)) = \varnothing$ for $j \neq h$, $j, h = 1, \ldots, k$. Then, given a problem (8), we may view U_i, $i = 1, \ldots, r$, as the "upper" potential basis for $\Gamma(U_i)x\Lambda(U_i)$ and L_j, $j = 1, \ldots, k$, as the "lower" potential basis for $\Gamma(L_j)x\Lambda(L_j)$. This means that for each $(\gamma, \lambda) \in \Gamma(U_i)x\Lambda(U_i)$, $x(U_i)$ maximizes Problem (1) and for each $(\gamma, \lambda) \in \Gamma(L_j)x\Lambda(L_j)$, $x(L_j)$ minimizes Problem (9).

Given $\mathfrak{U}^+ = \{U_1, \ldots, U_r\}$ and $\mathfrak{L}^- = \{L_1, \ldots, L_k\}$, by Definition 2.5, we see that the space $\Gamma x \Lambda$ is covered by both $\mathfrak{U}^+$ and $\mathfrak{L}^-$. To establish the upper and lower aspiration bounds of the objective payoff for a certain range of (γ, λ), we can reform the (γ, λ) ranges determined by $\mathfrak{U}^+$ and $\mathfrak{L}^-$ by taking *intersections* over the weight sets of elements of $\mathfrak{U}^+$ and $\mathfrak{L}^-$. Let $R(i, j) = (\Gamma(U_i) x \Lambda(U_i)) \cap (\Gamma(L_j) x \Lambda(L_j)) \neq \varnothing$, $i = 1, \ldots, r$; $j = 1, \ldots, k$, be reformed (γ, λ) ranges and $\mathfrak{R} = \cup\{R(i, j) \mid \text{for all } (i, j)\}$ be a reformed (γ, λ) space. Then, we have:

Theorem 3.1.

(i) $\Gamma(U_i) x \Lambda(U_i) \supset R(i, j)$ *and* $\Gamma(L_j) x \Lambda(L_j) \supset R(i, j)$, *for all* $i = 1, \ldots, r$; $j = 1, \ldots, k$.

(ii) $R(i, j) \cap R(g, h) = \varnothing$ for $i \neq g$, $j \neq h$, $i, g = 1, \ldots, r$; $j, h = 1, \ldots, k$.

(iii) $\mathfrak{R} \supset \Gamma x \Lambda$.

The proof of this theorem can be found in Shi and Liu [14].

Remark 3.1. Since $\Gamma x \Lambda$ is the space of all (γ, λ), we see that $\Gamma x \Lambda \supset \mathfrak{R}$. By (iii) of Theorem 3.1, $\Gamma x \Lambda = \mathfrak{R}$.

Theorem 3.1 shows characteristics of the reformed (γ, λ) ranges on $\mathfrak{R}$. (i) of Theorem 3.1 says that such a range on $\mathfrak{R}$ is contained in both a (γ, λ) range determined an element of $\mathfrak{U}^+$ and a (γ, λ) range determined an element of $\mathfrak{L}^-$. This guarantees that given a range $R(i, j)$, there is a corresponding upper potential solution from an element of $\mathfrak{U}^+$ and a corresponding lower potential solution from an element of $\mathfrak{L}^-$ (see (i) of Theorem 3.2). (ii) of Theorem 3.1 says that any two reformed (γ, λ) ranges on $\mathfrak{R}$ are not overlapped, which allows us to construct a fuzzy membership function for each reformed (γ, λ) range on $\mathfrak{R}$ without worrying about overlapping situations. (iii) of Theorem 3.1 states that the resulting $\mathfrak{R}$ is a coverage of the space $\Gamma x \Lambda$. This leads us to find all fuzzy potential solutions for all possible changes of (γ, λ). Thus, from Theorem 3.1, we have:

Theorem 3.2.

(i) *For a given* $R(i, j)$ *on* $\mathfrak{R}$, $x(U_i)$ *is its upper potential solution and* $x(L_j)$ *is its lower potential solution.*

(ii) *For any* $(\gamma, \lambda) \in \Gamma x \Lambda$, *there is a corresponding maximum solution resulting from an element of* $\mathfrak{U}^+$ *and a corresponding minimum solution resulting from an element of* $\mathfrak{L}^-$.

The proof of Theorem 3.2 can be found in [14].

Now we discuss how to construct a fuzzy membership function for each reformed (γ, λ) range on $\mathfrak{R}$. Given a (γ, λ) range $R(i, j)$ on $\mathfrak{R}$, both its upper potential solution $x(U_i)$ and its lower potential solution $x(L_j)$ are specified. That is, let B_{U_i} be

the basis for $x(U_i)$ and the corresponding objective payoff be $P(U_i; \gamma, \lambda) = \lambda^t C_{U_i} B_{U_i}^{-1} D\gamma$; and let B_{L_j} be the basis for $x(L_j)$ and the corresponding objective payoff be $P(L_j; \gamma, \lambda) = \lambda^t C_{L_j} B_{L_j}^{-1} D\gamma$. Then, whenever any $(\gamma, \lambda) \in R(i, j)$ are specified, $P(U_i; \gamma, \lambda)$ is the upper aspiration bound with respect to U_i, while $P(L_j; \gamma, \lambda)$ is the lower aspiration bound with respect to L_j. By applying the fuzzy approach methodologies in linear programming or MC linear programming (see Zimmermann [24] and [25]), a linear fuzzy membership function of any R(i, j) on $\mathfrak{S}$ for Problem (8) can be constructed as

$$\mu_{R(i,j)}(x) = \begin{cases} 1, & \lambda^t Cx \geq P(U_i; \gamma, \lambda) \\ \dfrac{\lambda^t Cx - P(L_j; \gamma, \lambda)}{P(U_i; \gamma, \lambda) - P(L_j; \gamma, \lambda)}, & P(L_j; \gamma, \lambda) < \lambda^t Cx < P(U_i; \gamma, \lambda) \\ 0, & P(L_j; \gamma, \lambda) \geq \lambda^t Cx \end{cases} \tag{10}$$

From (10), we see that if $\lambda^t Cx = P(U_i; \gamma, \lambda)$, then $\mu_{R(i,j)}(x) = 1$ and $x(U_i; \gamma, \lambda)$ is the maximum solution for Problem (1); if $\lambda^t Cx = P(L_j; \gamma, \lambda)$, then $\mu_{R(i,j)}(x) = 0$ and $x(L_j; \gamma, \lambda)$ is the minimum solution for Problem (9). The fuzzy potential solutions can be defined in terms of the membership function (10).

Definition 3.1. Let K be a primal potential basis of Problem (8) with the basic variables x(K) and the corresponding primal weight set $\Gamma(K)$. x(K) is said to be a *fuzzy potential solution* of R(i, j) for Problem (8) if $\Gamma(K) \supset \Gamma(U_i) \cap \Gamma(L_j)$ and x(K) has a membership function (10). In this case, K is called a *fuzzy potential basis* of R(i, j).

Remark 3.2. By Definition 3.1, a primal potential basis K of Problem (8) could be constructed as a fuzzy potential basis for several reformed (γ, λ) ranges on $\mathfrak{S}$.

Definition 3.2. Let $(x(K); \mu_{R(i,j)}(x))$ be a fuzzy potential solution of R(i, j) for Problem (8). The set $\mathfrak{F} = \{(x(K); \mu_{R(i,j)}(x)) \mid \text{for all } R(i, j) \text{ and } K\}$ is called a set of *all* fuzzy potential solutions for all possible changes of $(\gamma, \lambda) \in \Gamma x \Lambda$.

Remark 3.3. Given $(\gamma^0, \lambda^0) \in R(i, j)$, the solution $(x(K; \gamma^0, \lambda^0); \mu_{R(i,j)}(x))$ is a fuzzy optimal solution for the linear programming with (γ^0, λ^0) resulting from Problem (8).

Based on the above discussion, the relationship between fuzzy potential solutions and potential solutions is summarized as follows.

Theorem 3.3.
(i) *Given a fuzzy potential solution* $(x(K); \mu_{R(i,j)}(x))$, *there exist an upper and a lower potential solutions for* R(i, j).
(ii) *Conversely, with an upper and a lower potential solution of* R(i, j), *a fuzzy potential solution* $(x(K); \mu_{R(i,j)}(x))$ *can be constructed.*

The proof of Theorem 3.3 can also be found in Shi and Liu [14]. (i) of Theorem 3.3 basically shows that both the upper and lower potential solutions are extreme cases of a fuzzy potential solution for a R(i, j) on $\mathfrak{D}$. (ii) of Theorem 3.3 allows us to construct a fuzzy potential solution from given upper and lower potential solutions for a R(i, j).

4. CONSTRUCTING FUZZY POTENTIAL SOLUTIONS

By integrating the theoretical results from Sections 2 and 3, in this section, we propose and demonstrate a heuristic algorithm for effectively and systematically constructing all fuzzy potential solutions for all possible changes of (γ, λ) of the fuzzy-objective MC^2 problem (8).

Algorithm 4.1.

Step 1. Given a fuzzy-objective MC^2 problem (8), identify the set of all primal potential (feasible) bases, denoted by $\mathfrak{K} = \{K_1, \ldots, K_z\}$, by using the MC^2-simplex method. Note that (i) of Definition 2.4 and (i) of Theorem 2.1 can be used to find k.

Step 2. Break Problem (8) into the maximum problem (1) and minimum problem (9). Then use the MC^2-simplex method to obtain $\mathfrak{U}^+ = \{U_1, \ldots, U_r\}$ for Problem (1) and $\mathfrak{L}^- = \{L_1, \ldots, L_k\}$ for Problem (9). Since the constraints of Problems (1), (8) and (9) are identical, $\mathfrak{K} \supset \mathfrak{U}^+$ and $\mathfrak{K} \supset \mathfrak{L}^-$. Thus, $\mathfrak{U}^+$ and $\mathfrak{L}^-$ can be readily found from $\mathfrak{K}$ by applying (ii) of Definition 2.4, Remark 2.4, and (ii) and (iii) of Theorem 2.1.

Step 3. Based on $\mathfrak{U}^+$ and $\mathfrak{L}^-$, form the (γ, λ) range $R(i, j) = (\Gamma(U_i) \times \Lambda(U_i)) \cap (\Gamma(L_j) \times \Lambda(L_j)) \neq \emptyset$, $i = 1, \ldots, r$; $j = 1, \ldots, k$, and the (γ, λ) space $\mathfrak{D} = \cup\{R(i, j) \mid \text{for all } (i, j)\}$.

Step 4. For each R(i, j) on $\mathfrak{D}$, construct the fuzzy membership function (10).

Step 5. For each R(i, j) on $\mathfrak{D}$, identify the corresponding fuzzy potential solution with the membership function (10). This can be readily done by checking $K \in \mathfrak{K} \setminus (\mathfrak{U}^+ \cup \mathfrak{L}^-)$ for $\Gamma(K) \supset \Gamma(U_i) \cap \Gamma(L_j)$. By Definition 3.1, such a K is a fuzzy

potential solution with a membership function for R(i, j). Note that the fuzzy potential solution may be not unique for the R(i, j).

Step 6. The algorithm terminates when the set $\mathfrak{F}$ of Definition 3.2 is identified.

The above algorithm is implemented by using computer software of Chien, Shi and Yu [3]. A numerical example that is used to demonstrate Algorithm 4.1 for constructing all fuzzy potential solutions for all possible changes of (γ, λ) of Problem (8) can be found in Shi and Liu [14].

5. CONCLUSIONS

Given a fuzzy linear programming problem with multi-criteria and multi-constraint level (MC^2), we have described how to use a fuzzy approach and an MC^2-simplex method to construct all fuzzy potential solutions for all possible changes of the decision parameters. The basic idea is that we first split the fuzzy MC^2 problem into a maximum MC^2 problem and a minimum MC^2 problem. Next, we apply the MC^2-simplex method of [13] to identify the set of upper potential solutions for the maximum problem and the set of lower potential solutions for the minimum problem. Each upper (lower) potential solution maximizes (minimizes) the problem under a certain range of decision parameters, such as the criteria weight vector and the constraint level weight vector. Then, we construct a membership function of the fuzzy problem for a certain range of the decision parameters. The fuzzy potential solution for the range is characterized by the membership function and the primal potential basis. We have also developed a heuristic computer algorithm to locate all fuzzy potential solutions for all possible changes of the decision parameters. There are a number of research problems remaining to be explored. We outline them as follows:

In addition to the membership function constructed in form (10), we can construct a class of membership functions of fuzzy goals assigned to objective functions for the fuzzy MC^2 problem (Chanas [2]). Alternatively, an nonlinear membership function can be constructed through a piecewise linear approximation (Yang and Ignizio [20]). Another interesting theoretical research problem is that based on the framework of the fuzzy MC^2 model described in this paper, we can study the duality and complementary slackness of fuzzy MC^2 problems (Zimmermann [25]).

Perhaps, the most important remaining research problems are how to apply the fuzzy MC^2 model to formulate and solve the real world problems. We note that the previous studies on the crisp MC^2 models have been successfully applied in linear production system designs (Lee *et al* [10], Shi and Yu [16], Shi and Yu [17] and Shi *et al* [18]). Comparing the crisp MC^2 models and the fuzzy MC^2 model, the latter has the better representation on the decision makers' goal-seeking and compromise behavior than the former. Thus, we may apply the fuzzy MC^2 model to select fuzzy optimal production systems. In problems of building telecommunication network systems (Lee *et al* [8]), given a set of candidate cities, we want to select some subsets of candidate cities as hub city designs for the telecommunication system. The selected hub city design (a subset of candidate cities) maximizes the multiple criteria:

population, economy, education, health care, and transportation and to satisfy the location constraints with multiple resource availability levels. By applying the fuzzy MC^2 model, we can construct fuzzy hub city designs to reflect policy (decision) makers' goal-seeking and compromise behavior. Similarly, the fuzzy MC^2 model can be applied to the problem of allocating data files over a wide area network (Lee *et al* [9]) for choosing fuzzy data file allocation designs.

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Distributional Unanimity in Multiobjective Stochastic Linear Programming

F. Ben Abdelaziz[1], P. Lang[2], R. Nadeau[2]

[1] Institut Supérieur de Gestion, rue de la Liberté, Université de Tunis III, Le Bardo 2000, Tunis, Tunisia.

[2] Faculté des Sciences de l'Administration, Université Laval, Québec, P.Q., Canada G1K 7P4.

Abstract: Several notions of efficiency are conceivable for the multiobjective stochastic linear programming problem. In this paper, assuming that the problem's randomness can be described by discrete scenarios with known probabilities and that decision makers' preferences, although unknown, can be represented by a class of utility functions, we examine a set of strongly efficient solutions, the unanimous solutions. We state inclusion relations between this and other classes of efficient solutions (admissible and advocated solutions) previously studied. Under plausible assumptions about decision makers' risk attitudes, we examine how candidates for unanimity can be generated and then tested.

1. Introduction

This paper deals with a particular class of efficient solutions for a Multiobjective Stochastic Linear Programming (MSLP) problem under a finite set of scenarios. Specifically, the problem under consideration has the form:

$$\text{"Maximize"} \ \{C \cdot x \mid x \in X\} \qquad (1)$$

where $X = \{x \in \mathbb{R}^n \mid A \cdot x \leq b\}$ is a deterministic set, which we assume bounded and non-empty, and C is a random $m \times n$ matrix with probability distribution:

$$\text{Prob}\{C = C^s\} = p_s > 0, \quad s \in \mathcal{S} = \{1,...,S\}.$$

We note $\mathcal{I} = \{1,...,m\}$ the set of criteria indices. Contexts for which (1) provides an adequate representation are discussed in Ben Abdelaziz *et al.* (henceforth BLN), 1993.

Efficient solutions to (1) are solutions $x \in X$ which are in some sense non-dominated. Any specific characterization of efficiency thus depends on what kinds of comparisons between pairs of solutions are seen as legitimate; the nature of such comparisons is affected in particular by *(i)* what we can assume about *preference structures* (over the random outcomes $\{C \cdot x \mid x \in X\}$) of the decision maker(s) involved, and *(ii)* the nature of decision makers' future expectations, particularly the definiteness of the probabilistic scenario description. Several such cases of "states of information" about "decision situations" involving problem (1) are investigated in BLN 1993, 1994, 1995.

In this paper we assume that the probability distribution $\{p_s \mid s \in \mathcal{S}\}$ is well specified and that the decision maker(s)' preference structure(s) satisfy the axioms underlying the existence of a cardinal multiattribute utility function; we further assume that this utility function, although unknown, belongs to some subclass $\mathbb{U}$ of the monotone real functions on $\mathbb{R}^m$. In this setting, extending earlier work on efficiency in MSLP (BLN 1993, 1995), we examine a class of efficient solutions of particular importance for both single and multiple decision makers situations, the class of unanimous solutions. To motivate the study of unanimous solutions, section 2 summarizes their relationships with other prominent classes of efficient solutions (as defined in BLN 1993), and section 3 reviews main results (BLN 1995) concerning the identification of so-called "advocated" solutions when certain risk attitudes of the decision-maker can be assumed; as a by-product, this identification enables us to detect the non-existence of unanimous solutions, or alternatively to generate candidates for unanimity. Under the same risk attitude assumptions, section 4 then gives further characterizations of unanimous solutions, and provides a simple test of whether a given feasible solution to (1) is unanimous, thus complementing the results in BLN 1995.

2. Distributionally efficient and unanimous solutions

Let $\mathbb{U}_1$ denote the class of non-decreasing functions on $\mathbb{R}^m$. We assume that the decision maker (DM) has a utility function u, of which it is only known that it belongs to some class $\mathbb{U} \subseteq \mathbb{U}_1$. An expected utility maximizer DM will evaluate decisions $x \in \mathbb{X}$ in terms of the random vectors of outcomes $\{\mathbf{C}\cdot x \mid x \in \mathbb{X}\}$. Since it is not known which utility function in $\mathbb{U}$ is relevant, each element $u \in \mathbb{U}$ can be considered as a potential evaluation viewpoint, or the viewpoint of a particular DM in the "class" $\mathbb{U}$. By analogy with deterministic Pareto efficiency, one may view a decision as dominated if some alternative decision is preferred under all viewpoints, as undominated otherwise.

Definition 1: Let $x, y \in \mathbb{X}$. Solution y $\mathbb{U}$-*dominates* solution x if
$E[u(\mathbf{C}\cdot y)] \geq E[u(\mathbf{C}\cdot x)]$, $\forall\ u \in \mathbb{U}$, and
$\exists\ u_o \in \mathbb{U}$ such that $E[u_o(\mathbf{C}\cdot y))] > E[u_o(\mathbf{C}\cdot x)]$.
The set of $\mathbb{U}$-*admissible* solutions, noted $\mathcal{A}(\mathbb{U})$, is the set of solutions $x \in \mathbb{X}$ which are not $\mathbb{U}$-dominated by any $y \in \mathbb{X}$.

Historically, considerable importance has been given in the multiattribute utility literature to the subclasses of $\mathbb{U}_1$ exhibiting utility independence, which are additively or multiplicatively separable (Keeney and Raiffa, 1976). For these as well as for the general class $\mathbb{U}_1$, $\mathbb{U}$-dominance has been characterized via comparisons of distributions of particular functionals on the space of outcomes (Lehmann 1955, Levhari *et al.* 1975, Mosler 1984). For this reason, and because $\mathbb{U}$-dominance in general is predicated on comparing random vectors, we interchangeably use the term "distributional" efficiency (e.g.

admissibility) and $\mathbb{U}$-efficiency.

The concept of $\mathbb{U}$-admissibility is quite lenient since it requires *unanimity for rejection* in order to eliminate some solution x from consideration: that is, all viewpoints u represented in the class $\mathbb{U}$ must agree to prefer some other (fixed) decision y. $\mathbb{U}$-admissibility can thus be viewed as a somewhat "weak" form of efficiency. By contrast, a strong form of efficiency is obtained if we require *unanimity of preference*: if there exists a solution which is rated at least as well as any other under all utility functions in $\mathbb{U}$, clearly this solution deserves priority consideration.

Definition 2: The set of $\mathbb{U}$-*unanimous* solutions, noted $\mathcal{U}(\mathbb{U})$, is:
$\mathcal{U}(\mathbb{U}) = \{x \in \mathbb{X} \mid \forall\, u \in \mathbb{U}, \forall\, y \in \mathbb{X}, E[u(\mathbf{C}\cdot x)] \geq E[u(\mathbf{C}\cdot y)]\}$.

Clearly, if a unanimous solution exists, it is "optimal" in the sense of being (weakly) prefered over other solutions by all DMs with utility functions in $\mathbb{U}$. The existence of unanimous solutions generally depends on the shape of the feasible set $\mathbb{X}$ and the class $\mathbb{U}$ of utility functions under consideration. For instance, if $\mathbb{U}$ contains all functions v_i: $\mathbb{R}^m \to \mathbb{R}$ of the form: $v_i(z) \equiv z_i$, $i \in \mathcal{I}$, any solution $x \in \mathbb{X}$ such that $C_i^s x \geq C_i^s y$, $\forall\, y \in \mathbb{X}$, $s \in \mathcal{S}$, $i \in \mathcal{I}$, is in $\mathcal{U}(\mathbb{U})$ (BLN 1993). Furthermore, unanimity is all the more stringent a requirement as more points of view are represented in $\mathbb{U}$. Thus, a large class $\mathbb{U}$, and/or conflicting objectives are likely to make the set $\mathcal{U}(\mathbb{U})$ vacuous. If, however, a unanimous solution exists, then all non-unanimous solutions are $\mathbb{U}$-dominated.

Proposition 1 (BLN 1995):
(i) $\mathbb{U} \subseteq \mathbb{U}' \Rightarrow \mathcal{U}(\mathbb{U}') \subseteq \mathcal{U}(\mathbb{U})$
(ii) $\mathcal{U}(\mathbb{U}) \neq \varnothing \Rightarrow \mathcal{U}(\mathbb{U}) = \mathcal{A}(\mathbb{U})$
(iii) $\mathcal{U}(\mathbb{U}) = \varnothing \Leftrightarrow \exists\, x^1, x^2 \in \mathcal{A}(\mathbb{U})$ and $u \in \mathbb{U}$ such that $E[u(\mathbf{C}\cdot x^1)] \neq E[u(\mathbf{C}\cdot x^2)]$.

The class of unanimous solutions is of interest whether in a single or a multiple DM situations. However some of its properties can be inferred from the study of a larger class of efficient solutions, which are particularly pertinent in contexts of a single DM. In such contexts, not all admissible solutions are necessarily of equal relevance: since it is assumed that *some* point of view $u \in \mathbb{U}$ does indeed represent the DM's preference structure, only the solutions which are optimal (in an expected utility sense) under some such utility function are of interest. Solutions which are preferred by at least *one* decision maker in the class $\mathbb{U}$ will be called "advocated".

Definition 3:
(i) The set of *weakly* $\mathbb{U}$-*advocated* solutions is:
$\mathcal{P}_w(\mathbb{U}) = \{x \in \mathbb{X} \mid \exists\, u \in \mathbb{U}$ such that $\forall\, y \in \mathbb{X}, E[u(\mathbf{C}x)] \geq E[u(\mathbf{C}y)]\}$.
(ii) The set of *distinctly* $\mathbb{U}$-*advocated* solutions is:
$\mathcal{P}_d(\mathbb{U}) = \{x \in \mathbb{X} \mid \exists\, u \in \mathbb{U}$ such that $\forall\, y \in \mathbb{X}\backslash\{x\}, E[u(\mathbf{C}x)] > E[u(\mathbf{C}y)]\}$.
(iii) The set of $\mathbb{U}$-*advocated* solutions is: $\mathcal{P}(\mathbb{U}) = \mathcal{P}_d(\mathbb{U}) \cup \mathcal{U}(\mathbb{U})$.

If a solution x is $\mathbb{U}$-advocated, then it is optimal under some utility function $u \in \mathbb{U}$. Note that being weakly advocated does not guarantee admissibility; thus the set $\mathscr{P}(\mathbb{U})$ is arguably of main interest in a single DM context. It is clear that the elements of this set are admissible.

Proposition 2 (BLN 1995): $\mathscr{U}(\mathbb{U}) \subseteq \mathscr{P}(\mathbb{U}) \subseteq \mathscr{A}(\mathbb{U})$.

In the next two sections, we turn to the question of identifying unanimous solutions under additional assumptions on the risk attitudes of the DM(s), which lead us to consider the class $\mathbb{U}_{ac}$ of additive concave non-decreasing utility functions. It will turn out that finding advocated solutions is a convenient starting point for identifying unanimous ones.

3. Generating $\mathbb{U}_{ac}$- advocated and potentially unanimous solutions

Beyond the intrinsic interest one may find in the concepts of admissible, advocated and unanimous solutions, the question arises as to what extent these classes can be operationally identified. This question of identification has to our knowlege no general answer as it depends significantly on the class $\mathbb{U}$ of utility functions that is assumed. In this and the next section, we examine how unanimous solutions can be identified under plausible assumptions about the DM's risk attitudes.

Multiattribute risk attitudes can be measured along two dimensions. One can be called *scale risk aversion* since it defines a risk-averse DM as one who prefers the mathematical expectation of a lottery to the lottery itself; scale risk aversion is equivalent to concavity of the utility function (Stiglitz 1969, Kihlstrom and Mirman 1974, Russell and Seo 1978, Mosler 1984, Scarsini 1988). The second view of risk aversion, which could be called *scope risk aversion*, considers the relative composition of a multivariate lottery: a scope-risk averse DM prefers diversified to all-or-nothing lotteries (Meyer 1972, Richard 1975, Mosler 1984, Scarsini 1985, 1988). Scope risk aversion (proneness) thus indicates a form of substitutability (complementarity) across attributes. The three classes of utility functions exhibiting utility independence (additive and multiplicative of two types) imply scope risk neutrality, aversion, and proneness respectively (Mosler 1984). We will henceforth consider the class $\mathbb{U}_{ac}$ of non-decreasing additive concave functions, that is, preference structures exhibiting utility independence, scope risk neutrality, and (weak) scale risk aversion. The latter assumption is plausible in many decision contexts; on the other hand, scope risk neutrality is a standard assumption in many applications of multiattribute utility theory (e.g. Dyer and Miles 1977, Keeney and Lilien 1987, Merkhofer and Keeney 1987).

The class $\mathbb{U}_{ac}$ includes all functions which are sums of m univariate non-

decreasing concave functions. Since $\mathbb{X}$ is compact, and since utility functions are defined up to a positive affine transformation, we can restrict this class to the functions in $\mathbb{U}_{ac}$ which are normalized between 0 and 1. Let $\mathbf{a}$ and $\mathbf{b} \in \mathbb{R}^m$ be the "anti-ideal" and the "ideal" points in objective space defined by:

$\mathbf{a}_i = \min \{ C_i^s x \mid s \in \mathcal{S}, x \in \mathbb{X} \}, \quad i \in \mathcal{I}$, and

$\mathbf{b}_i = \max \{ C_i^s x \mid s \in \mathcal{S}, x \in \mathbb{X} \}, \quad i \in \mathcal{I}$.

For any non-empty compact interval $[a,b] \subset \mathbb{R}$, let $\mathbb{C}[a,b]$ be the class of non-decreasing, concave, continuous functions u: $[a,b] \to \mathbb{R}$ which are normalized as: $u(a) = 0$, $u(b) = 1$. Then:

$$\mathbb{U}_{ac} = \{u:[\mathbf{a},\mathbf{b}]\to[0,1] \mid u(z) \equiv \textstyle\sum_{i\in\mathcal{I}} \theta_i u_i(z_i),\ u_i \in \mathbb{C}[\mathbf{a}_i,\mathbf{b}_i]\ \forall\, i \in \mathcal{I},\ \theta \in \Theta\}$$

where $\Theta = \{\theta \in \mathbb{R}^m \mid \theta > \mathbf{0},\ \sum_{i\in\mathcal{I}} \theta_i = 1\}$.

We now briefly present a mathematical formulation that enables us to generate $\mathbb{U}_{ac}$-advocated solutions as well as to test whether a given feasible solution of (1) is $\mathbb{U}_{ac}$-advocated; as a by-product, we will obtain candidate solutions for unanimity or conclude to their non-existence. The details of the first results in this section are given in BLN 1995.

It is shown in BLN 1995 that $\mathbb{U}_{ac}$-advocated solutions can be detected via a sublass of $\mathbb{U}_{ac}$, namely the concave functions which are weighted sums of piecewise affine univariate functions. Accordingly, we consider for each $i \in \mathcal{I}$ a "support" $\mathbb{S}_i = \{z_{it} \mid 1 \le t \le T_i\}$, where $\mathbf{a}_i = z_{io} < z_{i1} < \ldots < z_{i,T_i} = \mathbf{b}_i$, representing a set of points at which changes of slopes are allowed for the ith piecewise affine component. The class of weighted sums of such components is noted $A(\mathbb{S})$, where $\mathbb{S} = \prod_{i\in\mathcal{I}} \mathbb{S}_i$ (as presented in detail in BLN 1995). We say that a solution $x \in \mathbb{X}$ is *represented* by the support $\mathbb{S}$ if $C^s x \in \mathbb{S}$, $\forall\, s \in \mathcal{S}$. Furthermore let $\mathcal{K} = \{(t,i) \mid 1 \le t \le T_i,\ i \in \mathcal{I}\}$, and

$$\mathbb{W} = \left\{ \phi \in \prod_{i\in\mathcal{I}} \mathbb{R}_+^{T_i} \mid \sum_{t=1}^{T_i} \phi_{it} > 0\ \forall\, i \in \mathcal{I},\ \sum_{(t,i)\in\mathcal{K}} \phi_{it} = 1 \right\}.$$

The following linear program in (v, w, x), parametrized over $\phi \in \mathbb{W}$, provides an implicit representation of all piecewise linear functions with support $\mathbb{S}$ and, for any $\phi \in \mathbb{W}$, enables us to identify advocated solutions:

$$\text{Max:} \quad \sum_{(t,i)\in\mathcal{K}} \phi_{it} v_{it} \qquad (2.1)$$

$$\text{S.t.:} \quad v_{it} - \sum_{s=1}^{S} p_s \frac{1}{z_{it} - \mathbf{a}_i} \cdot w_{its} \le \frac{\mathbf{a}_i}{z_{it} - \mathbf{a}_i} \qquad (t,i) \in \mathcal{K} \qquad (2.2)$$

$$w_{its} - C_i^s x \le 0 \qquad (t,i) \in \mathcal{K},\ s \in \mathcal{S} \qquad (2.3)$$

$$w_{its} \le z_{it} \qquad (t,i) \in \mathcal{K},\ s \in \mathcal{S} \qquad (2.4)$$

$$Ax \le b \qquad (2.5)$$

Proposition 3 (BLN 1995):

(i) There is a one-to-one correspondance: $\mathbb{W} \leftrightarrow A(\mathbb{S})$ such that x^* is a (unique)

optimal solution of (2) under some $\phi \in \mathbb{W}$ if and only if x^* is a (unique) optimal solution of: Max $\{E[u(Cx)] \mid x \in \mathbb{X}\}$ for the corresponding $u \in A(\mathbb{S})$.

(ii) Given an arbitrary support $\mathbb{S}$, any optimal solution x^* of (2) with $\phi \in \mathbb{W}$ is weakly $\mathbb{U}_{ac}$-advocated. If in addition it is unique, then it is distinctly $\mathbb{U}_{ac}$-advocated.

(iii) If solution $x^* \in \mathbb{X}$ is represented by the support $\mathbb{S}$, and if it is $\mathbb{U}_{ac}$-advocated, there exists a $\phi \in \mathbb{W}$ such that x^* is an optimal solution of (2). In particular, if x^* is distinctly $\mathbb{U}_{ac}$-advocated, there exists a $\phi \in \mathbb{W}$ such that x^* is a unique optimal solution of (2).

According to part *(i)* of proposition 3, the parametric program (2) provides an implicit representation of all piecewise affine functions of $\mathbb{U}_{ac}$ defined on the support $\mathbb{S}$. Part *(ii)* indicates that, based on an arbitrary support $\mathbb{S}$, advocated solutions (which will turn out to be weak or distinctive) can be generated by parametrizing (2). However the set of advocated solutions generated via (2) does depend on the chosen support $\mathbb{S}$, which may put forward some solutions in preference to others. Part *(iii)* enables us to test the advocated status of a given solution.

These results have direct implications for $\mathbb{U}_{ac}$-unanimity since a unanimous solution must necessarily be at least weakly $A(\mathbb{S})$-advocated whatever the support $\mathbb{S}$. Furthermore if a distinctly advocated solution is identified, then (by definition) no other solution can be unanimous. With the help of program (2), one may thus either conclude that no unanimous solution exists or obtain a set of "candidate" unanimous solutions. This set may consist either of a single distinctly advocated solution or of only weakly advocated solutions. For the latter case, proposition 4 below provides a further screening mechanism, looking at images of points $x \in \mathbb{X}$ in the objective space of (2). Given a support $\mathbb{S}$, let

$$\mathbb{V}(\mathbb{S}) = \{v \in \prod_{i \in \mathcal{I}} \mathbb{R}_+^{T_i} \mid (v,w,x) \text{ satisfies (2.2)-(2.5)}\}$$

be the set of v values attainable in (2), and let $\mathbb{E}(\mathbb{S})$ be the efficient frontier of $\mathbb{V}(\mathbb{S})$; that is, $v^* \in \mathbb{E}(\mathbb{S})$ if and only if:

$v^* \in \mathbb{V}(\mathbb{S})$ and

v^* is an optimal solution to: Max $\{\sum_{(t,i) \in \mathcal{K}} \phi_{it} v_{it} \mid v \in \mathbb{V}(\mathbb{S})\}$

for some $\phi \in \mathbb{W}^o = \{\phi \in \mathbb{W} \mid \phi_{it} > 0 \ \forall \ (t,i) \in \mathcal{K}\}$.

Finally, let

$$\mathbb{X}^*(v) = \{x \in \mathbb{X} \mid \exists \ w \text{ such that } (v,w,x) \text{ satisfies (2.2)-(2.5)}\}$$

be the set of solutions $x \in \mathbb{X}$ which enable us to attain the value v in (2); in particular, $v \in \mathbb{E}(\mathbb{S})$ and $x \in \mathbb{X}^*(v)$ if and only if (v,x) is optimal for (2).

Proposition 4: If $\mathcal{U}(\mathbb{U}_{ac}) \neq \varnothing$, then $\mathbb{E}(\mathbb{S})$ consists of a single point v^* and $\mathcal{U}(A(\mathbb{S})) = \mathbb{X}^*(v^*)$.

Proof: Since the feasible set of program (2) is a non-empty polytope (bounded polyhedron), its projection $\mathbb{V}(\mathbb{S})$ is also a non-empty polytope. Then its efficient frontier $\mathbb{E}(\mathbb{S})$ is a non-empty connected finite union of efficient

faces, each of which is a nonempty polytope; hence each face $\mathcal{F}$ of $\mathbb{E}(\mathbb{S})$ has at least one extreme point, and is the convex hull of its extreme points. If $\mathcal{F}$ contains two extreme points v^1 and v^2, there exist two weightings ϕ^1 and $\phi^2 \in \mathbb{W}^o$ such that

$$\sum_{(t,i)\in\mathcal{K}} \phi^1_{it} v^1_{it} > \sum_{(t,i)\in\mathcal{K}} \phi^1_{it} v_{it} \quad \forall\, v \in \mathbb{V}(\mathbb{S}) \setminus \{v^1\} \quad \text{and}$$

$$\sum_{(t,i)\in\mathcal{K}} \phi^2_{it} v^2_{it} > \sum_{(t,i)\in\mathcal{K}} \phi^2_{it} v_{it} \quad \forall\, v \in \mathbb{V}(\mathbb{S}) \setminus \{v^2\}$$

hence, by proposition 3*(i)*, there exist two functions u^1 and $u^2 \in A(\mathbb{S})$ and two points $x^1 \in \mathbb{X}^*(v^1)$ and $x^2 \in \mathbb{X}^*(v^2)$ such that

$E[u^1(Cx^1)] > E[u^1(Cy)] \quad \forall\, y \in \mathbb{X} \setminus \mathbb{X}^*(v^1)$ and

$E[u^2(Cx^2)] > E[u^2(Cy)] \quad \forall\, y \in \mathbb{X} \setminus \mathbb{X}^*(v^2)$

so that $\mathcal{U}(A(\mathbb{S})) = \varnothing$ and, since $A(\mathbb{S}) \subseteq \mathbb{U}_{ac}$, $\mathcal{U}(\mathbb{U}_{ac}) = \varnothing$ by proposition 1 *(i)*. Therefore, $\mathcal{U}(\mathbb{U}_{ac}) \neq \varnothing$ implies that any face of $\mathbb{E}(\mathbb{S})$ consists of a single point. Since $\mathbb{E}(\mathbb{S})$ is connected and contains a finite number of faces, it must itself consist of a single point v^*. Then

$x^* \in \mathbb{X}^*(v^*) \Leftrightarrow (x^*,v^*)$ is an optimal solution to (2) for all $\phi \in \mathbb{W}^o$

$\Leftrightarrow (x^*,v^*)$ is an optimal solution to (2) for all $\phi \in \mathbb{W}$

$\Leftrightarrow x^*$ is an optimal solution to: Max $\{E[u(Cx)] \mid x \in \mathbb{X}\}$ for all $u \in A(\mathbb{S})$ (by proposition 3 *(i)*)

$\Leftrightarrow x^* \in \mathcal{U}(A(\mathbb{S}))$. □

By proposition 4, it is sufficient to obtain two weigtings ϕ^1 and ϕ^2 such that the corresponding optimal values v^1 and v^2 in (2) differ to conclude that problem (1) has no $\mathbb{U}_{ac}$-unanimous solution. In the contrary case, the existence of a $\mathbb{U}_{ac}$-unanimous solution is not assured since v-unicity may depend on the chosen support $\mathbb{S}$. However, given any set of supports $\{\mathbb{S}_k\}$, each of which yields v-unicity, it follows from proposition 1 *(i)* that $\mathcal{U}(\mathbb{U}_{ac}) \subseteq \bigcap_k \mathcal{U}(A(\mathbb{S}_k))$. Thus varying or enriching the support will eventually help narrow down the set of solutions which are candidates for $\mathbb{U}_{ac}$-unanimity. In the next section, we show how such candidate solutions can be further screened and tested for unanimity.

4. Testing for $\mathbb{U}_{ac}$-admissibility and unanimity

Whereas the previous results are convenient for generating and screening candidate unanimous solutions, the screening is incomplete because it relies on finite supports. In this section, we use extensions of classical second degree stochastic dominance (Fishburn and Vickson 1978) to test whether a solution is $\mathbb{U}_{ac}$-admissible and $\mathbb{U}_{ac}$-unanimous. Testing admissibility may be of interest in its own right, but it also provides a further screening of unanimous solutions since, by proposition 1*(ii,iii)*, any unanimous solution is necessarily advocated, whereas the existence of two admissible solutions with distinct expected utilities implies that the unanimous set is empty. As a first step, we establish stochastic dominance conditions for the class of utility functions $\mathbb{U}_{ac}$ in proposition 5, and specialize the results to the case of discrete scenarios

in proposition 6. Let us define:

$$H_i(z,x) = \int_{\mathbf{a}_i}^{z} \text{Prob}\{C_i x \le t\}dt \quad z \in [\mathbf{a}_i, \mathbf{b}_i],\ i \in \mathcal{I}.$$

Proposition 5:

(i) Solution $x \in \mathbb{X}$ is $\mathbb{U}_{ac}$-dominated by solution $y \in \mathbb{X}$ if and only if:

$\forall\ i \in \mathcal{I},\ \forall\ z \in [\mathbf{a}_i, \mathbf{b}_i]$: $H_i(z,y) \le H_i(z,x)$ and

$\exists\ i_o \in \mathcal{I},\ z_o \in [\mathbf{a}_i, \mathbf{b}_i]$: $H_{i_o}(z_o,y) < H_{i_o}(z_o,x)$

(ii) Solution $x \in \mathbb{X}$ is $\mathbb{U}_{ac}$-unanimous if and only if:

$\forall\ i \in \mathcal{I},\ \forall\ z \in [\mathbf{a}_i, \mathbf{b}_i],\ \forall\ y \in \mathbb{X}$: $H_i(z,x) \le H_i(z,y)$.

Proof: (i) y $\mathbb{U}_{ac}$-dominates x

$\Leftrightarrow\ \forall\ u \in \mathbb{U}_{ac}$: $E[u(Cy)] \ge E[u(Cx)]$ and

$\exists\ u_o \in \mathbb{U}_{ac}$: $E[u_o(Cy)] > E[u_o(Cx)]$

$\Leftrightarrow\ \forall\ \theta \in \Theta,\ \{u_i \in \mathbb{C}[\mathbf{a}_i,\mathbf{b}_i] \mid i \in \mathcal{I}\}$: $\sum_{i\in\mathcal{I}} \theta_i E[u_i(C_i y)] \ge \sum_{i\in\mathcal{I}} \theta_i E[u_i(C_i x)]$ and

$\exists\ \theta_o \in \Theta,\ \{v_i \in \mathbb{C}[\mathbf{a}_i,\mathbf{b}_i] \mid i \in \mathcal{I}\}$: $\sum_{i\in\mathcal{I}} \theta_{oi} E[v_i(C_i y)] > \sum_{i\in\mathcal{I}} \theta_{oi} E[v_i(C_i x)]$

$\Leftrightarrow\ \forall\ i \in \mathcal{I},\ u_i \in \mathbb{C}[\mathbf{a}_i,\mathbf{b}_i]$: $E[u_i(C_i y)] \ge E[u_i(C_i x)]$ and

$\exists\ i_o \in \mathcal{I},\ v_{i_o} \in \mathbb{C}[\mathbf{a}_{i_o},\mathbf{b}_{i_o}]$: $E[v_{i_o}(C_{i_o} y)] > E[v_{i_o}(C_{i_o} x)]$

$\Leftrightarrow\ \forall\ i \in \mathcal{I},\ z \in [\mathbf{a}_i,\mathbf{b}_i]$: $H_i(z,y) \le H_i(z,x)$ and

$\exists\ i_o \in \mathcal{I},\ z_o \in [\mathbf{a}_{i_o},\mathbf{b}_{i_o}]$: $H_{i_o}(z_o,y) < H_{i_o}(z_o,x)$

the last equivalence being shown in Fishburn and Vickson (1978).

(ii) Similarly, x is $\mathbb{U}_{ac}$-unanimous

$\Leftrightarrow\ \forall\ u \in \mathbb{U}_{ac},\ \forall\ y \in \mathbb{X}$: $E[u(Cx)] \ge E[u(Cy)]$

$\Leftrightarrow\ \forall\ \theta \in \Theta,\ \forall\ \{u_i \in \mathbb{C}[\mathbf{a}_i,\mathbf{b}_i] \mid i \in \mathcal{I}\},\ \forall\ y \in \mathbb{X}$:

$\sum_{i\in\mathcal{I}} \theta_i E[u_i(C_i x)] \ge \sum_{i\in\mathcal{I}} \theta_i E[u_i(C_i y)]$

$\Leftrightarrow\ \forall\ i \in \mathcal{I},\ \forall\ u_i \in \mathbb{C}[\mathbf{a}_i,\mathbf{b}_i],\ \forall\ y \in \mathbb{X}$: $E[u_i(C_i x)] \ge E[u_i(C_i y)]$

$\Leftrightarrow\ \forall\ i \in \mathcal{I},\ \forall\ z\ [\mathbf{a}_i, \mathbf{b}_i],\ \forall\ y \in \mathbb{X}$: $H_i(z,x) \le H_i(z,y)$. □

In the case of discrete scenarios with which we are concerned:

$$H_i(z,x) = \sum_{s\in\mathcal{S}} p_s (z - C_i^s x)^+, \quad i \in \mathcal{I},$$

(with the usual notation $(z - C_i^s x)^+ = \max\{z - C_i^s x, 0\}$), so that H_i is a convex, piecewise affine function of (z,x), and non-decreasing in z. The conditions of proposition 5 can then be further simplified. We assume in the following that the scenario indices have been renumbered so that $C_i^{t-1} x \le C_i^t x$, $\forall$ t. Let $\mathcal{Z} = \mathcal{S} \cup \{S+1\}$,

$$z_{it} = C_i^t x,\ \forall\ t \in \mathcal{S}, \quad z_{io} = \mathbf{a}_i, \quad z_{i,S+1} = \mathbf{b}_i, \quad \forall\ i \in \mathcal{I},$$
$$\mu_{it} = (z_{i,t-1} + z_{it})/2,\ \forall\ t \in \mathcal{Z},\ i \in \mathcal{I}.$$

Proposition 6: Solution $x \in \mathbb{X}$ is $\mathbb{U}_{ac}$-admissible if there is no $y \in \mathbb{X}$ such that

$\forall\ i \in \mathcal{I},\ \forall\ t \in \mathcal{Z}$: $H_i(z_{it},y) \le H_i(z_{it},x)$ and

$\sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{Z}} \big(H_i(\mu_{it},y) - H_i(\mu_{it},x)\big) < 0.$

Proof: From proposition 5, x is dominated by $y \in \mathbb{X}$ if and only if $H_i(z,y) \le$

$H_i(z,x)$ $\forall$ $i \in \mathcal{I}$, $\forall$ $z \in [a_i, b_i]$, with a strict inequality for some i_o and z_o. Since $H_i(z,y)$ is convex and piecewise linear in z with breakpoints (points of discontinuity of its slope) at $\{z_{it} \mid t \in \mathcal{Z} \cup \{0\}\}$, and since $H_i(0,y) = H_i(0,x) = 0$, by lemma 1 (given in the appendix), this condition is equivalent to:

$\forall\, i \in \mathcal{I}, \forall\, t \in \mathcal{Z}$: $H_i(z_{it},y) \leq H_i(z_{it},x)$ and

$\exists\, i_o \in \mathcal{I}, t_o \in \mathcal{Z}$: $H_{i_o}(\mu_{i_o,t_o},y) < H_{i_o}(\mu_{i_o,t_o},x)$,

or equivalently

$\forall\, i \in \mathcal{I}, \forall\, t \in \mathcal{Z}$: $H_i(z_{it},y) \leq H_i(z_{it},x)$ and

$\sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{Z}} \left(H_i(\mu_{it},y) - H_i(\mu_{it},x)\right) < 0$. □

On the basis of proposition 6, it is possible to ascertain whether a given point $x \in \mathbb{X}$ is $\mathbb{U}_{ac}$-admissible. This is achieved via the linear program below with decision variables (v,w,y) and optimal value $\delta(x)$:

$$\delta(x) = \text{Min} \sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{Z}} \sum_{s\in\mathcal{S}} p_s v_{its} \tag{3.1}$$

$$\text{S.t.: } Ay \leq b \tag{3.2}$$

$$\sum_{s\in\mathcal{S}} p_s w_{its} \leq H_i(z_{it},x) \qquad i \in \mathcal{I}, t \in \mathcal{Z} \tag{3.3}$$

$$w_{its} + C_i^s y \geq z_{it} \qquad i \in \mathcal{I}, t \in \mathcal{Z}, s \in \mathcal{S} \tag{3.4}$$

$$v_{its} + C_i^s y \geq \mu_{it} \qquad i \in \mathcal{I}, t \in \mathcal{Z}, s \in \mathcal{S} \tag{3.5}$$

$$v_{its}, w_{its} \geq 0 \qquad i \in \mathcal{I}, t \in \mathcal{Z}, s \in \mathcal{S} \tag{3.6}$$

Proposition 7: x is $\mathbb{U}_{ac}$-admissible if and only if $\delta(x) = \sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{Z}} H_i(\mu_{it},x)$.

Proof: Constraints (3.4) and (3.6) imply: $w_{its} \geq (z_{it} - C_i^s y)^+$. Since the w_{its} variables only reappear in constraints (3.3), which, if anything, operate towards minimizing them, these w_{its} variables can, for any given y, be chosen as: $w_{its} = (z_{it} - C_i^s y)^+$. Therefore constraints (3.3) enforce (and are in fact equivalent to):

$H_i(z_{it},x) \geq \sum_{s\in\mathcal{S}} p_s (z_{it} - C_i^s y)^+ = H_i(z_{it},y)$.

Furthermore, from constraints (3.5) and (3.6), we have: $v_{its} \geq (\mu_{it} - C_i^s y)^+$, and, since the v_{its} variables do not appear in other constraints, minimization (3.1) will insure that this holds with equality, so that $\sum_{s\in\mathcal{S}} p_s v_{its} = \sum_{s\in\mathcal{S}} p_s (\mu_{it} - C_i^s y)^+ = H_i(\mu_{it},y)$. Thus, $\delta(x) = \text{Min}\, \{\sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{Z}} H_i(\mu_{it},y) \mid y \in \mathbb{X}, H_i(z_{it},y) \leq H_i(z_{it},x)\ \forall\, i \in \mathcal{I}, t \in \mathcal{Z}\}$. Note that program (3) is always feasible, since it is possible to choose y = x; consequently, $\delta(x) \leq \sum_{i\in\mathcal{I}} \sum_{t\in\mathcal{Z}} H_i(\mu_{it},x)$. By proposition 6, x is $\mathbb{U}_{ac}$-dominated if and only if a strict inequality is obtained. □

Finally, we provide a mathematical programming formulation which enables us to ascertain whether a given point $x \in \mathbb{X}$ is $\mathbb{U}_{ac}$-unanimous. Let $\Delta_i = b_i - a_i$, i

$\in \mathcal{I}$. The formulation below, with decision variables (z, y, **v**, **w**, α) and optimal value $d_i(x)$, seeks for some $i \in \mathcal{I}$ a value $(z,y) \in [\mathbf{a}_i, \mathbf{b}_i] \times \mathbb{X}$ which maximizes $H_i(z,x) - H_i(z,y)$:

$$d_i(x) = \text{Max} \quad \sum_{s=1}^{S} p_s v_s - \sum_{s=1}^{S} p_s w_s \tag{4.1}$$

$$\text{S.t.:} \quad \mathbf{A}y \leq \mathbf{b} \tag{4.2}$$

$$\mathbf{a}_i \leq z \leq \mathbf{b}_i \tag{4.3}$$

$$v_s - z + \Delta_i \alpha_s \leq \Delta_i - z_{is} \qquad s \in \mathcal{S} \tag{4.4}$$

$$v_s - \Delta_i \alpha_s \leq 0 \qquad s \in \mathcal{S} \tag{4.5}$$

$$z - C_i^s y - w_s \leq 0 \qquad s \in \mathcal{S} \tag{4.6}$$

$$v_s, w_s \geq 0 \qquad s \in \mathcal{S} \tag{4.7}$$

$$\alpha_s \in \{0,1\} \qquad s \in \mathcal{S} \tag{4.8}$$

Proposition 8: $d_i(x) = \text{Max}\ \{H_i(z,x) - H_i(z,y) \mid (z,y) \in [\mathbf{a}_i,\mathbf{b}_i]\times\mathbb{X}\}$. Solution x is $\mathbb{U}_{ac}$-unanimous if and only if $d_i(x) = 0$, $\forall\ i \in \mathcal{I}$.

Proof: For any given z and y, it follows from (4.6-4.7) that $w_s \geq (z - C_i^s y)^+$. Since the w_s variables do not appear elsewhere in the constraints, minimization will ensure equality, namely:

$$\sum_{s\in\mathcal{S}} p_s w_s = H_i(z,y).$$

Turning now to the v_s variables two cases are possible for any given z and y:

- If $z < z_{is}$, constraints (4.4), (4.7) and (4,8) together force $\alpha_s = 0$, and this in turn implies, through constraint (4.5), $v_s = 0$.
- If $z \geq z_{is}$, it is feasible under constraints (4.4), (4.5), (4.7) and (4.8) to choose $\alpha_s = 1$, and $v_s = z - z_{is}$. Since these variables do not appear in other constraints, maximization ensures that this solution will be preferable to the alternative $\alpha_s = 0$, $v_s = 0$.

Hence again at optimality, for any given z and y:

$$\sum_{s=1}^{S} p_s v_s = H_i(z,x).$$

Consequently, $d_i(x) = \text{Max}\ \{H_i(z,x) - H_i(z,y) \mid (z,y) \in [\mathbf{a}_i,\mathbf{b}_i]\times\mathbb{X}\}$. Note that program (4) is always feasible, since it is possible to choose y = x. Hence $d_i(x) \geq 0$, $\forall\ i \in \mathcal{I}$. By proposition 6*(ii)*, x is $\mathbb{U}_{ac}$-unanimous if and only if $d_i(x) = 0\ \forall\ i \in \mathcal{I}$. □

5. Conclusion

In contrast to previously studied notions of admissibility and advocacy, the concept of unanimity examined here is quite demanding. Thus, in situations of conflicting objectives or diverse potential preference structures (utility functions), it is quite possible that no unanimous solution will exist. However, if unanimous solutions are present, it is imperative to detect at least some of them since all non-unanimous alternatives will then be inferior. In

this paper, we reviewed major inclusion relations between unanimous, admissible and advocated classes of solutions. We further indicated how, under plausible assumptions about DM risk attitudes, candidate solutions for unanimity can be generated and screened, and we provided complete tests of admissibility as well as unanimity. An interesting but open research question would be to relax the stringent requirements for unanimity in a way that would keep this notion relevant for multiple as well as single DM situations.

Appendix

Lemma 1: Let **[a, b]** be a finite non-empty real interval, f: **[a, b]** $\to \mathbb{R}$ be convex, and g: **[a, b]** $\to \mathbb{R}$ be piecewise affine with breakpoints $\{z_t \mid 0 \le t \le T\}$ where

$\mathbf{a} = z_0 < z_1 < \ldots < z_T = \mathbf{b}$

(that is, $g((1-\alpha)z_{t-1}+\alpha z_t) = (1-\alpha)g(z_{t-1}) + \alpha g(z_t)$, $\forall\ \alpha\in[0,1]$, $\forall\ t\in\{1,\ldots,T\}$).

Let $\mu_t = (z_{t-1}+z_t)/2$, $t \in \{1,\ldots,T\}$. Then

(i) $f \le g \Leftrightarrow f(z_t) \le g(z_t)\ \forall\ t \in \{0,\ldots,T\}$

(ii) if $f \le g$, then $f \ne g \Leftrightarrow \exists\ t \in \{1,\ldots,T\}$ such that $f(\mu_t) < g(\mu_t)$.

Proof: *(i)* the implication ($\Rightarrow$) is obvious since $z_t \in$ **[a, b]** $\forall$ t. Conversely, for any $z;\in$ **[a, b]**, we have $z = (1-\alpha)z_{t-1} + \alpha z_t$ for some $\alpha \in [0,1]$ and $t \in \{1,\ldots,T\}$. Then by the convexity and linearity assumptions,

$f(z) \le (1-\alpha)f(z_{t-1}) + \alpha f(z_t) \le (1-\alpha)g(z_{t-1}) + \alpha g(z_t) = g(z)$.

(ii) The implication ($\Leftarrow$) is obvious since $\mu_t \in$ **[a, b]** $\forall$ t. We show the reverse implication ($\Rightarrow$) by contradiction. For any $t \in \{1,\ldots,T\}$, let $z_{t-1} \le z' < \mu_t < z'' \le z_t$ and let $\alpha = (\mu_t - z')/(z'' - z') \in (0,1)$. By convexity of f and linearity of g on the interval $[z_{t-1},z_t]$,

$f(\mu_t) \le (1-\alpha)f(z') + \alpha f(z'')$ and $g(\mu_t) = (1-\alpha)g(z') + \alpha g(z'')$.

Hence, $f(\mu_t) = g(\mu_t)$ would imply: $(1-\alpha)[f(z') - g(z')] + \alpha[f(z'') - g(z'')] \ge 0$. Given that $f \le g$, this would only be possible if $f(z') = g(z')$ and $f(z'') = g(z'')$. □

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A Fuzzy Decomposition Method by Right-Hand-Side Allocation for Large-Scale Multiobjective Nonlinear Programming Problems

Masatoshi Sakawa[1] **and Hitoshi Yano**[2]

[1] Department of Industrial and Systems Engineering, Faculty of Engineering, Hiroshima University, Higashi-Hiroshima, 739 Japan
[2] Nagoya Municipal Women's College, Nagoya, 464, Japan

Abstract: This paper deals with large-scale multiobjective nonlinear programming problems with the block angular structure in a fuzzy environment. By incorporating the fuzzy goals of the decision maker for the objective functions and adopting the add-operator for aggregating them, a fuzzy decomposition method by right-hand-side allocation is proposed. An illustrated numerical example is provided to indicate the feasibility of the proposed method.

1. Introduction

In the past decade, for solving large-scale nonlinear programming problems (LS-NLPs) with the block angular structure, which well reflect real-world complicated decision situations, several kinds of decomposition methods have been proposed by exploiting their special structures [2,3].

In our previous paper [6], by incorporating multiple conflicting objectives together with the fuzzy goals of the decision maker (DM) [5] into the LS-NLPs, we formulated multiobjective nonlinear programming problems (LS-MONLPs) with the block angular structure in a fuzzy environment and proposed a fuzzy dual decomposition method for deriving the compromise solution for the DM to the LS-MONLPs. In the proposed method, however, the coupling constraints are not satisfied at any cycle until the convergence is achieved (for this reason, this approach is called a infeasible decomposition [2]). In contrast with such a infeasible decomposition approach, a decomposition approach by right-hand-side allocation (which is called a feasible decomposition) [2,3] has also been proposed to LS-NLPs, in which a generating point at any cycle is guaranteed to be feasible.

In this paper, to decrease such drawbacks of the infeasible decomposition approach, we propose a fuzzy decomposition method for LS-MONLPs with the block angular structure by right-hand-side allocation in a fuzzy environment. In the proposed method, assuming that the DM has a fuzzy goal for each of the objective functions in the LS-MONLP and adopting the add-operator [7] to aggregate the membership functions representing the fuzzy goals of the DM, a large-scale master problem is formulated for obtaining

the compromise solution for the DM to the LS-MONLP. Then, the master problem is decomposed into several small-size subproblems by introducing allocation vectors (these can be interpreted as the allocated resources for each subproblems), and the compromise solution for the DM to the LS-MONLP is obtained by the coordination between the master problem and the decomposed subproblems through the allocation vectors. Based on the proposed algorithm, computer program is developed in FORTRAN and an illustrated numerical example is demonstrated to clarify the feasibility of the proposed method.

2. A Fuzzy Decomposition Method by Right-hand-side Allocation

Consider a large-scale multiobjective nonlinear programming problem (LS-MONLP) with the following block angular structure:

$$\begin{array}{lllllll} \min & f_1(x) & \triangleq & f_{11}(x_1) & +f_{12}(x_2) & +\cdots & +f_{1p}(x_p) \\ \min & f_2(x) & \triangleq & f_{21}(x_1) & +f_{22}(x_2) & +\cdots & +f_{2p}(x_p) \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ \min & f_k(x) & \triangleq & f_{k1}(x_1) & +f_{k2}(x_2) & +\cdots & +f_{kp}(x_p) \end{array} \tag{1a}$$

subject to

$$\begin{array}{llllll} g_1(x) & \triangleq g_{11}(x_1) & +g_{12}(x_2) & +\cdots & +g_{1p}(x_p) & \leq b_1 \\ g_2(x) & \triangleq g_{21}(x_1) & +g_{22}(x_2) & +\cdots & +g_{2p}(x_p) & \leq b_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ g_m(x) & \triangleq g_{m1}(x_1) & +g_{m2}(x_2) & +\cdots & +g_{mp}(x_p) & \leq b_m \\ & h_1(x_1) & & & & \leq 0 \\ & & h_2(x_2) & & & \leq 0 \\ & & & \ddots & & \vdots \\ & & & & h_p(x_p) & \leq 0 \end{array} \tag{1b}$$

where $f_i(x)$, $i = 1, \ldots, k$, are k distinct objective functions of the decision vector $x = (x_1, \ldots, x_p) \in E^n$ $(p \leq n)$, $x_\ell \in E^{n_\ell}$, $\ell = 1, \ldots, p$, $(\sum_{\ell=1}^{p} n_\ell = n)$ are n_ℓ dimensional subvectors of decision variables, $g_j(x) \leq b_j$, $j = 1, \ldots, m$, are m coupling constraints, $h_\ell(x_\ell)$, $\ell = 1, \ldots, p$, are m_ℓ dimensional nonlinear vector-valued functions with respect to x_ℓ, and $f_{i\ell}(x_\ell)$, $g_{j\ell}(x_\ell)$, $i = 1, \ldots, k$, $j = 1, \ldots, m$, $\ell = 1, \ldots, p$, are nonlinear functions with respect to x_ℓ.

Throughout this paper, we make the following assumptions.

Assumption 1. Each element of the vector-valued functions $h_\ell(x_\ell)$, $\ell = 1$, $\ldots$, p are convex and differentiable with respect to x_ℓ, and the subset

$$S_\ell \triangleq \{x_\ell \in E^{n_\ell} \mid h_\ell(x_\ell) \leq 0\} \neq \emptyset, \quad \ell = 1, \ldots, p \tag{2}$$

is convex and compact.

In the following, we further define the set S as follows.

$$S \triangleq \prod_{\ell=1,\ldots,p} S_\ell \subset E^n \tag{3}$$

where the symbol $\prod$ means the direct product. Then the set S also becomes compact and convex.

Assumption 2. $f_{i\ell}(x_\ell)$, $i = 1,\ldots,k$ and $g_{j\ell}(x_\ell)$, $j = 1,\ldots,m$, are convex and differentiable on the subset S_ℓ, $\ell = 1,\ldots,p$.

By considering the vague nature of human subjective judgements, it is quite natural to assume that the decision maker (DM) may have a fuzzy goal for each of the objective functions [1,5,7,8]. These fuzzy goals can be quantified by eliciting the corresponding membership functions $\mu_i(f_i(x)), i = 1,\ldots,k$ through the interaction with the DM. Concerning the membership functions $\mu_i(f_i(x))$, $i = 1,\ldots,k$, we make the following assumption.

Assumption 3. The membership function defined by

$$\mu_i(f_i(x)) = \begin{cases} 1 & ; \quad f_i(x) \le f_i^1 \\ \dfrac{f_i(x) - f_i^0}{f_i^1 - f_i^0} & ; \quad f_i^1 \le f_i(x) \le f_i^0 \\ 0 & ; \quad f_i(x) \ge f_i^0 \end{cases} \tag{4}$$

for each of the objective functions is strictly monotone decreasing and linear function on the subset S, where f_i^0 and f_i^1 mean the maximum value of the unacceptable level and the minimum value of the acceptable level of the DM for the objective function $f_i(x)$ respectively. These values are subjectively assessed by the DM, which have to satisfy the following inequalities for any $x \in S$.

$$f_i^1 \le f_i(x) \le f_i^0, \quad i = 1,\ldots,k, \quad \forall x \in S \tag{5}$$

After determining the membership functions, if we adopt the add-operator [7] for combining them, the compromise solution for the DM can be obtained by solving the following large-scale fuzzy decision making problem (LS-FDMP).

$$\max_{x \in S} \left\{ \sum_{i=1}^{k} \mu_i(f_i(x)) \right\} \tag{6a}$$

subject to

$$\begin{array}{lllllll} g_1(x) & \triangleq g_{11}(x_1) & +g_{12}(x_2) & +\cdots & +g_{1p}(x_p) & \le b_1 \\ g_2(x) & \triangleq g_{21}(x_1) & +g_{22}(x_2) & +\cdots & +g_{2p}(x_p) & \le b_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ g_m(x) & \triangleq g_{m1}(x_1) & +g_{m2}(x_2) & +\cdots & +g_{mp}(x_p) & \le b_m \end{array} \tag{6b}$$

Here, from the definition of the membership functions, the objective function $\sum_{i=1}^{k} \mu_i(f_i(x))$ can be expressed as:

$$\sum_{i=1}^{k} \mu_i(f_i(x)) = -\sum_{\ell=1}^{p}\left\{\sum_{i=1}^{k} A_i f_{i\ell}(x_{i\ell})\right\} + C \tag{7}$$

where

$$A_i = \frac{1}{f_i^0 - f_i^1} > 0, \qquad C = \sum_{i=1}^{k} \frac{f_i^0}{f_i^0 - f_i^1}. \tag{8}$$

Therefore, LS-FDMP can be equivalently expressed as:

$$\min_{x \in S} \sum_{\ell=1}^{p}\left\{\sum_{i=1}^{k} A_i f_{i\ell}(x_\ell)\right\} \tag{9a}$$

subject to

$$\begin{array}{lllll} g_1(x) & \triangleq g_{11}(x_1) & +g_{12}(x_2) & +\cdots & +g_{1p}(x_p) & \leq b_1 \\ g_2(x) & \triangleq g_{21}(x_1) & +g_{22}(x_2) & +\cdots & +g_{2p}(x_p) & \leq b_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ g_m(x) & \triangleq g_{m1}(x_1) & +g_{m2}(x_2) & +\cdots & +g_{mp}(x_p) & \leq b_m \end{array} \tag{9b}$$

It should be noted here that $\sum_{i=1}^{k} A_i f_{i\ell}(x_\ell)$ is convex and differentiable on the set S_ℓ from $A_i > 0$, $i = 1, \ldots, k$, and Assumption 2.

For this problem, if the number of the coupling constraints m is much smaller than the total number of constraints $h_\ell(x_\ell) \leq 0$, $\ell = 1, \ldots, p$, the decomposition method by right-hand-side allocation [3] is especially efficient from the computational viewpoint.

Now, denote the vectors of the coupling constraint functions and the corresponding allocation vectors as follows:

$$g_\ell(x_\ell) \triangleq (g_{1\ell}(x_\ell), \ldots, g_{m\ell}(x_\ell))^T, \quad \ell = 1, \ldots, p \tag{10}$$

$$y_\ell \triangleq (y_{1\ell}, \ldots, y_{m\ell})^T \in R^m, \quad \ell = 1, \ldots, p \tag{11}$$

Then the allocation vectors y_ℓ, $\ell = 1, \ldots, p$, must satisfy the feasibility condition:

$$\sum_{\ell=1}^{p} y_{j\ell} \leq b_j, \quad j = 1, \ldots, m. \tag{12}$$

Given the allocation vectors $y_\ell, \ell = 1, \ldots, p$ satisfying the above conditions, we can decompose the LS-FDMP into the following subproblems $\ell = 1, \ldots, p$:

$$\min_{x_\ell \in S_\ell} \sum_{i=1}^{k} A_i f_{i\ell}(x_\ell) \tag{13a}$$

subject to

$$g_\ell(x_\ell) \leq y_\ell. \tag{13b}$$

Since all of the above subproblems must be feasible, it is necessary to satisfy the following condition for the allocation vectors $y_\ell, \ell = 1, \ldots, p$:

$$y_\ell \in Y_\ell \triangleq \{y_\ell \in R^m \mid \text{there exists } x_\ell \in S_\ell \text{ such that } g_\ell(x_\ell) \leq y_\ell\}. \tag{14}$$

We also define the minimal subproblem objective values and the sum of them as follows.

$$w_\ell(y_\ell) \triangleq \left\{ \min_{x_\ell} \sum_{i=1}^{k} A_i f_{i\ell}(x_\ell) \;\middle|\; g_\ell(x_\ell) \leq y_\ell,\ x_\ell \in S_\ell \right\} \tag{15}$$

$$w(y) \triangleq \sum_{\ell=1}^{p} w_\ell(y_\ell) \tag{16}$$

Then the LS-FDMP can be equivalently transformed into the following form, where the allocation vectors y_ℓ, $\ell = 1, \ldots, p$, are decision variables.

$$\min_{y} \quad w(y) = \sum_{\ell=1}^{p} w_\ell(y_\ell) \tag{17a}$$

subject to

$$\sum_{\ell=1}^{p} y_{j\ell} \leq b_j, \quad j = 1, \ldots, m, \quad y_\ell \in Y_\ell, \quad \ell = 1, \ldots, p \tag{17b}$$

where y is the $(m \times p)$-matrix defined by

$$y = (y_1, \ldots, y_p) = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{21} & y_{22} & \cdots & y_{2p} \\ \cdots & \cdots & \cdots & \cdots \\ y_{m1} & y_{m2} & \cdots & y_{mp} \end{bmatrix}. \tag{18}$$

It should be noted here that the transformed problem (17) becomes the convex programming problem with respect to y. From the properties of convexity and differentiability (Assumptions 1, 2 and 3), as a locally best usable direction, we can adopt a feasible direction which minimizes the directional derivative [3] of the transformed problem (17). Such a direction matrix can be obtained by solving the following direction-finding problem.

Direction-Finding Problem

$$\min_{s=(s_1,\ldots,s_p)} \quad Dw(y; s) \triangleq \sum_{\ell=1}^{p} Dw_\ell(y_\ell; s_\ell) \tag{19a}$$

subject to

$$\sum_{\ell=1}^{p} s_{j\ell} \leq 0, \quad j \in B, \qquad \|s\| \leq 1 \tag{19b}$$

where $Dw(y;s)$ is a directional derivative of the objective function $w(\cdot)$ at y in the direction s,

$$s = (s_1, \ldots, s_p) = \begin{bmatrix} s_{11} & s_{12} & \ldots & s_{1p} \\ s_{21} & s_{22} & \ldots & s_{2p} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots & \ldots\ldots \\ s_{m1} & s_{m2} & \ldots & s_{mp} \end{bmatrix} \tag{20}$$

is an $(m \times p)$-search direction matrix at the allocation matrix y,

$$B = \left\{ j = (1, \ldots, m) \;\middle|\; b_j - \sum_{\ell=1}^{p} y_{j\ell} = 0 \right\} \tag{21}$$

is an index set of the binding constraints of (12) at y, and $\|s\|$ is a norm of s.

Concerning the allocation matrix y, we make the following assumptions.

Assumption 4. Any allocation vector y_ℓ belongs to the interior of Y_ℓ, i.e.,

$$y_\ell \in \text{int } Y_\ell, \quad \ell = 1, \ldots, p. \tag{22}$$

Assumption 5. The minimal subproblem objective values $w_\ell(y_\ell), \ell = 1, \ldots, p$ are differentiable with respect to y_ℓ.

Then the direction-finding problem (19) can be reduced to the following simple linear programming problem [3].

$$\min_{s=(s_1,\ldots,s_p)} \; -\sum_{\ell=1}^{p} [\lambda_{1\ell}, \ldots, \lambda_{m\ell}] \begin{bmatrix} s_{1\ell} \\ s_{2\ell} \\ \ldots \\ s_{m\ell} \end{bmatrix} \tag{23a}$$

subject to

$$\sum_{\ell=1}^{p} s_{j\ell} \leq 0, \; j \in B, \quad -1 \leq s_{j\ell} \leq 1, \; j = 1, \ldots, m, \; \ell = 1, \ldots, p \tag{23b}$$

where $[\lambda_{1\ell}, \ldots, \lambda_{m\ell}]$ is the vector of the Lagrange multipliers of the constraint $g_\ell(x_\ell) \leq y_\ell$ in the subproblem (13).

Given a usable direction matrix s, a new allocation matrix y is generated by choosing an appropriate step size $\alpha > 0$.

$$y^{t+1} = y^t + \alpha s \tag{24}$$

To obtain such an optimal step size $\alpha > 0$, we solve the following convex one-dimensional search problem.

One-Dimensional Search Problem

$$\min_{\alpha \geq 0} \left\{ w(y + \alpha s) \;\middle|\; \sum_{\ell=1}^{p} (y_\ell + \alpha s_\ell) \leq b, \;\; y_\ell + \alpha s_\ell \in Y_\ell, \;\; \ell = 1, \ldots, p \right\} \tag{25}$$

From the above discussion, we can now construct the fuzzy decomposition algorithm by right-hand-side allocation to the LS-MONLP for deriving the compromise solution for the DM.

Step 1: Elicit a linear membership function $\mu_i(f_i(x))$ satisfying the condition (5) from the DM for each of the objective functions $f_i(x)$, $i = 1, \ldots, k$, of the LS-MONLP in a subjective manner.

Step 2: Set the iterative notation $t = 0$, and set the initial values of the allocation matrix

$$y^t = (y_1^t, \ldots, y_p^t) = \begin{bmatrix} y_{11}^t & y_{12}^t & \cdots & y_{1p}^t \\ y_{21}^t & y_{22}^t & \cdots & y_{2p}^t \\ \cdots & \cdots & \cdots & \cdots \\ y_{m1}^t & y_{m2}^t & \cdots & y_{mp}^t \end{bmatrix} \tag{26}$$

satisfying the conditions (12) and (14) appropriately.

Step 3: Solve the p subproblems (13) and obtain an optimal solution $x_\ell^t, \ell = 1, \ldots, p$, and the vector of the Lagrange multipliers $\lambda_\ell^t = [\lambda_{1\ell}^t, \ldots, \lambda_{m\ell}^t]$, $\ell = 1, \ldots, p$ for the constraints $g_\ell(x_\ell) \leq y_\ell$, respectively.

Step 4: Given the vector of the Lagrange multipliers λ_ℓ^t, $\ell = 1, \ldots, p$ in Step 3, solve the corresponding linear programming problem (23) to obtain the search direction matrix, and set the search direction matrix as

$$s^t = (s_1^t, \ldots, s_p^t) = \begin{bmatrix} s_{11}^t & s_{12}^t & \cdots & s_{1p}^t \\ s_{21}^t & s_{22}^t & \cdots & s_{2p}^t \\ \cdots & \cdots & \cdots & \cdots \\ s_{m1}^t & s_{m2}^t & \cdots & s_{mp}^t \end{bmatrix} \tag{27}$$

Step 5: For the given search direction matrix s^t in Step 4, solve the following one-dimensional search problem to obtain the optimal step size $\alpha^t \geq 0$.

$$\min_{\alpha \geq 0} \left\{ w(y^t + \alpha s^t) \;\middle|\; \sum_{\ell=1}^{p} (y_\ell^t + \alpha s_\ell^t) \leq b, \;\; y_\ell^t + \alpha s_\ell^t \in Y_\ell, \ell = 1, \ldots, p \right\} \tag{28}$$

Step 6: If $\alpha^t \approx 0$, or $\|w(y^t) - w(y^{t-1})\| < \epsilon$, then stop. Otherwise, set $y^{t+1} = y^t + \alpha^t s^t$, $t = t + 1$, and return to Step 3.

3. Numerical Example

Based on the proposed algorithm, we have developed a computer program in FORTRAN to solve the LS-MONLP. In our computer program, having elicited a membership function for each of the objective functions, when the initial value of the allocation matrix is selected appropriately, the corresponding compromise solution for the DM can be obtained by solving the subproblems iteratively. To show the feasibility and efficiency of both the proposed algorithm and the developed computer program, consider the following three-objective nonlinear programming problem with the block angular structure.

$$
\begin{aligned}
\min_{x\geq 0} f_1(x) &\triangleq f_{11}(x_1,x_2) + f_{12}(x_3,x_4)\\
&= \left\{(x_1-8)^2+(x_2-12)^2\right\} + \left\{(x_3-30)^2+(x_4-10)^2\right\}\\
\min_{x\geq 0} f_2(x) &\triangleq f_{21}(x_1,x_2) + f_{22}(x_3,x_4)\\
&= \left\{(x_1-10)^2+(x_2-7)^2\right\} + \left\{(x_3-8)^2+(x_4-25)^2\right\}\\
\min_{x\geq 0} f_3(x) &\triangleq f_{31}(x_1,x_2) + f_{32}(x_3,x_4)\\
&= \left\{(x_1-35)^2+(x_2-10)^2\right\} + \left\{(x_3-12)^2+(x_4-7)^2\right\}
\end{aligned}
$$

subject to

$$
\begin{aligned}
g_1(x) &\triangleq g_{11}(x_1,x_2) + g_{12}(x_3,x_4)\\
&= \{x_1/3+x_2/10\} + \{x_3/7+x_4/8\} \leq 1\\
g_2(x) &\triangleq g_{21}(x_1,x_2) + g_{22}(x_3,x_4)\\
&= \{x_1/15+x_2/12\} + \{x_3/5+x_4/10\} \leq 1\\
g_3(x) &\triangleq g_{31}(x_1,x_2) + g_{32}(x_3,x_4)\\
&= \{x_1/10+x_2/12\} + \{x_3/8+x_4/4\} \leq 1
\end{aligned}
$$

For this numerical example, suppose that the hypothetical DM establishes the following linear membership functions for the three objectives.

$$
\begin{aligned}
\mu_1(f_1(x)) &= \frac{f_1^0 - f_1(x)}{f_1^0 - f_1^1} = \frac{1250 - f_1(x)}{1250 - 900}\\
\mu_2(f_2(x)) &= \frac{f_2^0 - f_2(x)}{f_2^0 - f_2^1} = \frac{850 - f_2(x)}{850 - 650}\\
\mu_3(f_3(x)) &= \frac{f_3^0 - f_3(x)}{f_3^0 - f_3^1} = \frac{1550 - f_3(x)}{1550 - 1300}
\end{aligned}
$$

Then we can formulate the corresponding LS-FDMP to obtain the compromise solution for the DM .

Since this is a simple nonlinear programming problem, we first solve it directly by applying the well-known nonlinear programming code, the revised version of the generalized reduced gradient program, called GRG2 [4]. As a

result we obtain an optimal decision values $x_1^* = 0.449168$, $x_2^* = 1.82974$, $x_3^* = 3.31024$, $x_4^* = 1.55530$, and the corresponding objective value $w(y^*) = -2.41332$, respectively.

With this result in mind, we now apply the proposed algorithm for solving it. For the selected initial values of the allocation matrix $y_{11}, y_{12}, y_{21}, y_{22}, y_{31}, y_{32}$ satisfying the conditions (12) and (14), the corresponding subproblems become

Subproblem 1 (y_{11}, y_{21}, y_{31})

$$\min_{(x_1,x_2)\geq 0} -\left\{ \frac{1250 - f_{11}(x_1,x_2)}{1250-900} + \frac{850 - f_{21}(x_1,x_2)}{850-650} + \frac{1550 - f_{31}(x_1,x_2)}{1550-1300} \right\}$$

subject to

$$g_{j1}(x_1,x_2) \leq y_{j1}, \quad j = 1,2,3$$

Subproblem 2 (y_{12}, y_{22}, y_{32})

$$\min_{(x_3,x_4)\geq 0} -\left\{ \frac{-f_{12}(x_3,x_4)}{1250-900} + \frac{-f_{22}(x_3,x_4)}{850-650} + \frac{-f_{32}(x_3,x_4)}{1550-1300} \right\}$$

subject to

$$g_{j2}(x_3,x_4) \leq y_{j2}, \quad j = 1,2,3$$

For these subproblems, applying the developed computer program yields the following result summarized in Table 1. From Table 1, we can see that the objective function value $w(y)$ at the 30th iteration approximately coincides with the calculation result through GRG2 [4].

4. Conclusion

In this paper, in order to deal with large-scale multiobjective nonlinear programming problems with the block angular structure in a fuzzy environment, a fuzzy decomposition approach by right-hand-side allocation has been proposed. In the proposed approach, after eliciting the linear membership functions for the objective functions from the DM, if the initial value of the allocation matrix y is selected appropriately, the compromise solution for the DM is obtained by solving the small-size subproblems iteratively. Based on the proposed algorithm, a computer program has been written in FORTRAN and an illustrative numerical example demonstrated the feasibility and efficiency of the proposed method. Applications of the proposed method as well as extensions to nonlinear membership functions will require further investigation.

Table 1. Iteration Processes

iteration	1	10	20	30
y_{11}	0.3	0.319922	0.314064	0.311050
y_{12}	0.6	0.680078	0.685936	0.688950
y_{21}	0.1	0.139066	0.150020	0.153120
y_{22}	0.8	0.829297	0.841038	0.843579
y_{31}	0.1	0.160547	0.167971	0.169868
y_{32}	0.8	0.839453	0.832029	0.830138
x_1	0.84375	0.604132	0.529106	0.502541
x_2	0.18750	1.185444	1.376956	1.435360
x_3	2.48889	3.239931	3.359016	3.408335
x_4	1.95556	1.737847	1.648609	1.616363
$w(y)$	−2.155384	−2.399757	−2.408099	−2.410648

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V

Multiobjective Integer Programming

An invited leading paper by Clímaco, Ferreira and Captivo presents an introduction to the subject of this chapter. The authors of this paper make a tentative classification of different algorithmic approaches for multicriteria integer programming problems. A characterization of the different approaches is also outlined.

Vassilev, Narula, Vladimirov and Djambov describe a decision support system (MOIP) designed to model and solve multiple objective integer linear programming problems interactively, efficiently, and without putting undue demands on the decision maker.

Ulungu and Teghem consider a classical knapsack problem with two objectives, in which concepts of supported and non supported efficient solutions are pointed out. A branch and bound procedure is described to solve this bicriteria knapsack problem.

The paper by Azizoglu, Kondakci and Koksalan considers the bicriteria problem of minimizing total flowtime and maximum earliness penalties for a given set of jobs on a single machine. The authors discuss some properties of the efficient solutions and develop a heuristic procedure to generate all efficient solutions when inserted idle time is not allowed.

Multicriteria Integer Programming: An Overview of the Different Algorithmic Approaches

João Clímaco
Faculdade de Economia, Universidade de Coimbra and INESC
3000 Coimbra, Portugal

Carlos Ferreira
Departamento de Matemática, Universidade de Aveiro, C. I. O. and INESC
3810 Aveiro, Portugal

M. Eugénia Captivo
Departamento de Estatística e Investigação Operacional and C. I. O.
Faculdade de Ciências da Universidade de Lisboa
1700 Lisboa, Portugal

Abstract: In this paper a tentative classification of different algorithmic approaches for Multicriteria Integer Programming Problems is presented. A characterization of the different approaches is also outlined.

1. INTRODUCTION

After the second world war linear programming (LP) models started being used in some real world problems. Since then the research on mathematical programming has followed different paths, from the theoretical point of view, to the development of new algorithms. Integer and non-linear mathematical programming problems are used nowadays in the real world in many situations where the LP approach was not satisfactory. Concerning the integer programming (excluding the particular cases where the constraints matrix is unimodular and the LP codes are useful to solve the problems) it is very difficult to develop algorithms to tackle problems of real world dimension. This is particularly true if the integer variables are not 0-1 or if the model is mixed-integer, i.e. where some variables are integer and others are real variables. Implicit enumeration methods, cutting plan methods and different types of heuristics have been developed in order to increase more and more the dimension of the problems running in acceptable time.

More recently, and following the steps made previously by multiobjective linear programming (MOLP), i.e. considering explicitly several conflituous objective functions, the theory and algorithms dedicated to multiobjective integer programming (MOIP), is being developed.

In general, for this multiobjective approach, an optimal solution does not exist. However, a special set of solutions usually designated by efficient solutions set can be defined. A solution is efficient if it is impossible to find another feasible solution improving one or more objective functions without worsening at least one of them. From an operational point of view these type of problems can be studied using the following approaches:

- Calculation of the whole efficient set of solutions (generating methods);
- A priori articulation of preferences of the decision maker (utility function methods);
- Progressive articulation of the decision maker preferences (interactive methods) searching for a "satisfactory" efficient solution.

It must be remarked that even for bicriteria problems (qualitatively simpler than the general multicriteria situations), both generating methods and interactive approaches are computationally much more complex than the method for single criteria optimization problems.

To illustrate this issue, the following example for a bicriteria case is outlined. Suppose it is considered as a minimizing bicriteria problem whose efficient solutions are represented in fig.1.

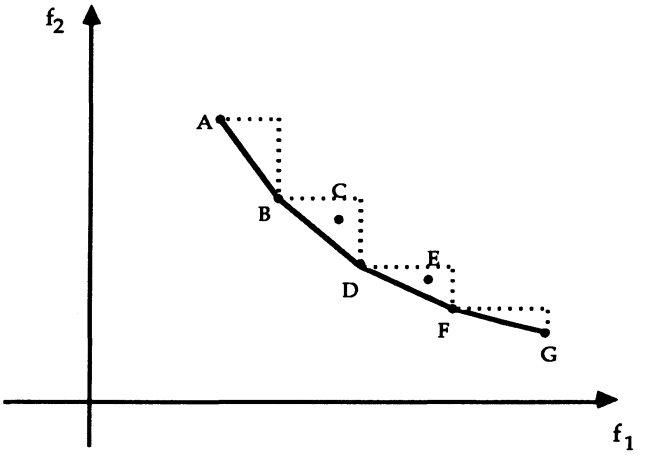

Figure 1

Solutions C and E are unsupported efficient solutions, since they are inside the duality gaps. So, it is impossible to obtain them using parametric approaches solving a single criteria problem. In these circumstances we are referring to a difficult area of research concerning theoretical results and algorithmic development. From the point of view of the applicability of the results the state of the art is still very poor for most of the situations and certainly in the general case. In our opinion this is clear by the analysis of the review section presented in the next section.

Due to space limitations the tentative classification and characterization of different approaches is restricted to the general integer programming case. Particular cases, taking into account the special structure of the problems, such as those occurring in location or scheduling problems are not considered. This limitation is mitigated by the inclusion in this chapter of contributed papers, selected from the works presented at the XIth International Conference on MCDM. In fact, two contributed papers are concerned to special integer programming problems and the third one shows how the development of a decision support system can improve the acceptability of an interactive procedure.

2. ALGORITHMS CLASSIFICATION AND CHARACTERIZATION

In the classification the bicriteria and the multicriteria cases will be dealt with separately, because its difficulty is qualitatively different. In each case, it considers the linearity or the non-linearity of the approaches; the type of aggregation for the decision maker preferences used; and the single criterion procedure used to calculate efficient solutions. Concerning the interactive methods it must be referred to an issue related to the human-computer protocol of communication. Finally it must be noticed that this review does not encompass goal programming articles.

Schematically one can say that:

- in some methods it is assumed that there exists an implicit utility function of the decision maker, and the interactive process intends to converge to the optimal solution for this "function";
- in other methods it is not assumed the existence of an implicit utility function of the decision maker. In these cases the process can be semi-structured or unstructured enabling a more or less free search for a satisfactory efficient solution.

This is a study for integer programming models. We will assume that the model has just integer variables, so, just more general or particular cases of integer programming (mixed and binary) will be explicitly identified.

2.1 BICRITERIA LINEAR PROBLEMS

2.1.1 NON-INTERACTIVE METHODS

(a) Pasternak and Passy (1973)

- algorithm for 0-1 programming
- the method for finding the efficient solutions is a combined approach of implicit enumeration and the use of constraints
- the method generates the complete set of efficient solutions
- the article does not present computational testing

(b) Chalmet et al (1986)

- the method for finding the efficient solutions is a combined approach of implicit enumeration and the use of constraints
- the method generates the complete set of efficient solutions

- the article presents computational results for multidimensional knapsack problems having up to 10 constraints and 15 variables

2.1.2 INTERACTIVE METHODS

(a) Ramesh et al (1990)
- a branch-and-bound for finding the efficient solutions it is used
- the decision maker is assumed to have an implicit utility function and his/her preference structure is explored and determined using pairwise comparison questions
- the article presents computational results for problems having up to 40 constraints and 80 variables

(b) Solanki, R. (1991)
- algorithm for mixed integer programming
- a combined approach of a augmented weighted Tchebycheff metric and the Noninferior Set Estimation (NISE) method (Cohon 1978) is used for finding the efficient solutions
- the method does not assume an implicit utility function and proposes the generation of an approximate representation of the non dominated set. This information can be interactively used by the decision maker to restrict the search for additional non dominated points
- the article does not present computational testing

(c) Ferreira et al. (1994)
- this paper addresses the Location-Covering Problem but the underlying methodology can be used on any bicriteria integer programming model
- the method for finding the efficient solutions is a combined approach of parametric programming and the use of constraints. These constraints are introduced accordingly with the indications of the decision maker
- the method does not assume an implicit utility function. It allows a free (non structured) progressive and selective search in order to find an efficient satisfactory solution
- the article does not present computational testing

2.2 BICRITERIA NON LINEAR PROBLEMS

2.2.1 NON-INTERACTIVE METHODS

Non-interactive methods for bicriteria non linear problems weren't found.

2.2.2 INTERACTIVE METHODS

(a) Walker, J. (1978)
- to determine the efficient solutions the method uses Generalized Lagrange Multipliers
- the decision maker is assumed to have an implicit utility function and he/she is required to provide yes or no answers to questions
- the article does not present computational testing

(b) Shin and Allen (1994)
- the method for finding the efficient solutions is a combined approach of parametric programming and the use of constraints
- the method does not determine unsupported nondominated points

- the decision maker is assumed to have an implicit utility function and his/her preferences structure is explored and determined using pairwise comparison questions
- the article does not present computational testing concerning mathematical programming problems

2.3 MULTICRITERIA LINEAR PROBLEMS

2.3.1 NON-INTERACTIVE METHODS

(a) Bitran, G. (1977), (1979)
- algorithm for 0-1 programming
- implicit enumeration is used for finding the efficient solutions
- the article dated 1979 is an attempt to improve the computational performance of the approach presented at 1977
- the method generates the complete set of efficient solutions
- the article dated 1977 presents computational results for problems having up to 4 constraints, 9 variables and 4 criteria
- the article dated 1979 presents computational results for problems having up to 4 constraints, 18 variables and 3 criteria

(b) Villarreal and Karwan (1981)
- a combined approach of dynamic programming and branch-and-bound rules is used for finding the efficient solutions
- the method generates the complete set of efficient solutions
- computational results are reported for bicriteria knapsack problems with 4 constraints and 10 variables and for problems having up to 5 constraints, 8 variables and 3 criteria

(c) Bitran and Rivera (1982)
- algorithm for 0-1 programming
- implicit enumeration is used for finding the efficient solutions
- the method generates the complete set of efficient solutions (the procedure is complemented with the use of the decision maker utility function to identify a subset of efficient points candidates for the final selection)
- computational results for a class of facility location problems having up to 9 constraints, 35 variables and 3 criteria are presented; computational testing concerning general 0-1 models is reported for problems having up to 4 constraints, 14 variables and 3 criteria

(d) Klein and Hannan (1982)
- implicit enumeration is used for finding the efficient solutions
- the method generates the complete set of efficient solutions
- computational results for problems having up to 4 constraints, 18 variables and 2 criteria and problems having up to 4 constraints, 10 variables and 4 criteria are presented

(e) Sherali (1982)
- the method uses parametric programming in order to determine the solution of a lexicographic model for finding the efficient solutions
- the method does not determine unsupported nondominated points
- the article does not present computational testing

(f) Deckro and Winkofsky (1983)
- algorithm for 0-1 programming
- implicit enumeration is used for finding the efficient solutions
- the method generates the complete set of efficient solutions
- computational results for problems having up to 90 constraints, 25 variables and 3 criteria are presented

(g) Kiziltan and Yucaoglu (1983)
- algorithm for 0-1 programming
- implicit enumeration is used for finding the efficient solutions
- the method generates the complete set of efficient solutions
- computational results for problems having up to 20 constraints, 20 variables and 4 criteria are presented

(h) Ignizio and Thomas (1984)
- the method uses parametric programming in order to determine the solution of a lexicographic model for finding the efficient solutions
- the method does not determine unsupported nondominated points
- the article does not present computational testing

2.3.2 INTERACTIVE METHODS

(a) Villarreal and Karwan (1981)
- dynamic programming is used for finding the efficient solutions
- the decision maker is assumed to have an implicit utility function
- computational results are reported for bicriteria knapsack problems with 4 constraints and 15 variables

(b) White, D. (1985)
- a combined approach of a branch-and-bound method and Lagrangean relaxation is used for finding the efficient solutions
- the decision maker is assumed to have an implicit utility function
- the article does not present computational testing

(c) Gabbani and Magazine (1985)
- parametric programming is used for finding the efficient solutions. Because a heuristic is used, there is no guarantee of an optimal solution for the auxiliary problem. Moreover the method does not determine unsupported nondominated points
- the decision maker is assumed to have an implicit utility function
- computational results are reported for problems having up to 30 constraints, 60 variables and 3 criteria and for problems having up to 2 constraints, 105 variables and 3 criteria

(d) Gonzalez et al. (1985)
- the method for finding the efficient solutions uses parametric programming
- the method does not assume an implicit utility function. The decision maker's preferences structure is explored using pairwise comparison questions
- the article does not present computational testing

(e) Karwan et al. (1985)
- implicit enumeration is used for finding the efficient solutions
- the decision maker is assumed to have an implicit utility function

- computational results are reported for problems having up to 8 constraints, 20 variables and 4 criteria

(f) Ramesh et al. (1986a) (1986b)
- these papers represent a further development of the earlier method proposed in Karwan et al. (1985)
- computational results are reported for 0-1 problems having up to 4 constraints, 30 variables and 3 criteria (1986a); the (1986b) article presents computational testing with randomly generated 0-1 problems having up to 12 constraints, 30 variables and 6 criteria

(g) Rueda, A. (1986)
- algorithm for 0-1 programming
- the method uses an extension of Balas additive algorithm to solve the lexicographic model for finding the efficient solutions
- the decision maker is not assumed to have an implicit utility function
- the article does not present computational testing

(h) Marcotte and Soland (1986)
- implicit enumeration is used for finding the efficient solutions
- the decision maker is not assumed to have an implicit utility function
- computational results are reported for a 0-1 problem with 10 constraints, 10 variables and 3 criteria

(i) Ramesh et al. (1989)
- a combined approach of parametric programming and convex cones is used for finding the efficient solutions
- the decision maker is assumed to have an implicit utility function
- computational results are reported for bicriteria 0-1 problems having up to 40 constraints, 80 variables

(j) Karaivanova et al. (1992) (1993)
- a augmented weighted Tchebycheff metric is used for finding the efficient solutions. To accelerate the procedure, the authors propose the use of a heuristic algorithm to solve the single objective integer linear programming problem (in this case there is no guarantee of an optimal solution for the auxiliary problem)
- the decision maker is not assumed to have an implicit utility function
- the (1992) article does not present computational results; the (1993) article presents computational testing with problems having up to 15 constraints, 15 variables and 6 criteria

(k) Vassilev and Narula (1993), Narula and Vassilev (1994)
- the method uses a reference direction approach and aspiration levels for finding the efficient solutions
- the decision maker is not assumed to have an implicit utility function
- computational testing is not reported

(l) Karaivanova et al (1995)
- the method uses a scalarizing function, a reference direction approach and aspiration levels for finding the efficient solutions
- the decision maker is not assumed to have an implicit utility function
- computational results are reported for problems having up to 30 constraints, 30 variables and 6 criteria

2.4. MULTICRITERIA NON LINEAR PROBLEMS

2.4.1 NON-INTERACTIVE METHODS

(a) Bowman, V. (1976)
- algorithm for 0-1 programming
- the method uses a combined approach of Tchebycheff metric and parametric programming for finding the efficient solutions
- the method generates the complete set of efficient solutions
- the article does not present computational testing

(b) White, D. (1984)
- algorithm for 0-1 programming
- a branch-and-bound method is used for finding the efficient solutions
- the method generates the complete set of efficient solutions
- the article does not present computational testing

2.4.2 INTERACTIVE METHODS

(a) Steuer and Choo (1983)
- several weighted Tchebycheff metrics are proposed for finding the efficient solutions
- the decision maker is assumed to have an implicit utility function. Operationally the method uses 'filtering' techniques
- the article does not present computational results

3. COMPUTATIONAL EFFICIENCY COMMENTS

Algorithms for Multiple Objective Mathematical Programming need to solve single objective programming problems several times. In the discrete case, as Integer Programming problems are NP-hard, the algorithms for MOIP are far from being efficient.

To overcome this difficulty MOIP approaches should minimize the number of single objective Integer Programming problems (IP) that need to be solved and also improve the algorithms for IP used.

The great majority of the available algorithms for MOIP uses tree-search procedures to solve the IP sub-problems (see, for instance [3,27]). A small number of procedures apply "pure" dynamic programming or dynamic programming combined with branch and bound (see, for instance [11,18]). Some authors report better results solving some of the integer programming sub-problems by parametric programming and only a small part as integer problems (see, for instance [30,31]).

More recently heuristic methods are included in implicit enumeration techniques increasing the efficiency of the resulting algorithms (see, for instance [28,29,32]).

The computational results presented in several papers show a clear increase on the dimension of the solved problems. While the first results reported for problems with 2 or 3 criteria, no more than 4 constraints and 9 variables, we have reached now problems with 6 criteria, 30 variables and 30 constraints. The improved efficiency of well structured tree search procedures using reference

points and the Tchebycheff metric to determine efficient solutions in interactive algorithms for MOIP, seems to be the most promising class of algorithms for this type of problems.

In conclusion the MOIP approaches are not yet able to deal with real world dimension problems in reasonable time. The computational experience referred to in the bibliography is still poor.

4. CONTRIBUTED PAPERS

The remainder of this chapter concerns the three contributed papers selected by the referees.

The first paper describes a Decision Support System (DSS) dedicated to an interactive approach reviewed in section 3 [30,31]. It must be emphasised the importance of the DSS to improve the acceptability of the interactive procedure.

The second paper is dedicated to a particular bicriteria problem, i.e. a knapsack problem. An adaptation of the Martello and Toth algorithm for single objective problems is used. This approach is computationally heavy since the efficient set calculation implies to visit the whole set of nodes of the combinatorial tree.

The third paper is dedicated to a bicriteria scheduling problem. Some theoretical results and an algorithm are presented.

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MOIP: A DSS FOR MULTIPLE OBJECTIVE INTEGER PROGRAMMING PROBLEMS

V.Vassilev

Institute of Information Technologies – BAS,
Acad. G. Bonchev Str., 1113, Sofia, Bulgaria

S. Narula

School of Business – Virginia Commonwealth University
Richmond, Virginia 23284–4000, USA

P. Vladimirov, V. Djambov

Institute of Information Technologies – BAS,
Acad. G. Bonchev Str., 1113, Sofia, Bulgaria

Abstract: Multiple objective integer linear programming models are often used to formulate real world problems. Our objective, in this paper, is to describe a decision support system, MOIP, designed to model and solve multiple objective integer linear programming problems interactively, efficiently, and without putting undue demands on the DM. The system is designed for IBM compatible personal computers working under DOS operating system.

1. INTRODUCTION

Many practical problems, e.g., man-power planning, media selection, staff scheduling, project selection, and location- allocation problems, etc., can be formulated as multiple objective integer linear programming problems. For a survey of the multiple objective programming formulation and methodology for some of these problems, the interested reader may refer to Blazewicz, Dror and Weglarz (1991), Current, Min and Schelling (1990), Fry, Armstrong, and Lewis (1989) and Sayibo (1985). Because multiple objective programming problems do not admit an "optimal" solution, it is imperative to develop decision support systems (DSS) to help a decision maker choose an acceptable compromise solution.

A major part of the multiple objective decision support systems are the software systems used to model and to solve multiple objective programming problems interactively. At present, such software systems, Dror, Shoral, and Yellin (1991), Korhonen and Wallenius (1988, 1990) and Vassilev et al. (1991), have been developed to interactively solve multiple objective linear programming problems, the widest application having gained systems, including interactive algorithms for solving multiple objective linear programming problems. It is desirable that this class of algorithms satisfies the following requirements:

- minimizes and formalizes the demands placed on the DM;
- uses substitute single objective linear programming problems;
- finds arbitrary (weak) efficient ((weak) non-dominated) solutions.

Besides these considerations, an interactive algorithm for multiple objective integer programming problems should mimimize the number of single objective (mixed) integer linear programming problems that need to be solved and reduce the time needed to solve these problems.

In this paper we discuss a decision support system MOIP, designed to model and solve multiple objective integer linear programming problems. It is based on an hybrid algorithm, which is built on two interactive algorithms described in Vassilev and Narula (1993) and Narula and Vassilev (1994). The hybrid algorithm and the basic algoritms

satisfy the preceding requirement. The first algorithm, Vassilev and Narula (1993), uses a specially constructed single-objective integer parametric problem which is solved at each iteration to obtain an integer (weak) efficient solution; its corresponding (weak) nondominated solution is shown to the DM. The second algorithm, Narula and Vassilev (1994), is based on the use of two scalarizing problems. The first scalarizing problem has the same form as the scalarizing problem of the first algorithm, except that it solves a continuous parametric problem. This problem is solved at each iteration; the (weak) efficient solutions and their respective (weak) nondominated solutions are continuous. The second scslarizing problem is an integer problem which is obtained from the first problem by reduction and replacement operations. This problem is solved only when the DM wants to find an integer (weak) efficient solution and corresponding (weak) nondominated solution.

The rest of the paper is organized as follows. The multiple objective integer linear programming problem is stated in section 2. The main characteristics of the two basic algorithms and the hybrid algorithm are discussed in section 3. The structure, the functions and the user-interface of MOIP decision support system are described in section 4.

2. PROBLEM STATEMENT

The MOIP decision support system is designed to solve multiple objective integer linear programming problems. The mathematical statement of these problems is:

$$\text{"max"}\ z_k = f_k(x) = \sum_{j \in J} c_{kj} x_j,\ k \in K, \tag{1}$$

subject to

$$\sum_{j \in J} a_{ij} x_j \le b_i,\ i \in M, \tag{2}$$

$$0 \le x_j \le d_j,\ j \in N, \tag{3}$$

$$x_j - \text{integers},\ j \in N, \tag{4}$$

where $K = \{1, 2, ..., k\}$, $M = \{1, 2, ..., m\}$, $N = \{1, 2, ..., n\}$, and "max" means that all the objectives have to be maximized simultaneously.

Let X_1 and X_2 denote the sets of solutions which satisfy constraints (2–4) and constraints (2–3) respectively. Let I and C denote problem (1–4) and problem (1–3) respectively.

3. INTERACTIVE ALGORITHMS

The MOIP decision support system includes an hybrid algorithm, which is based on two interactive algorithms, MOIA and MOHIA, as described in Vassilev and Narula (1993) and Narula and Vassilev (1994). The first, pure integer algorithm, MOIA, solves the following single objective mixed integer linear programming problem at each iteration:

$$\max\ (z_1 - z_2) \tag{5}$$

subject to

$$f_k(x) - (\overline{f}_k - f_k)(z_1 - z_2) \ge f_k,\ k \in K_1, \tag{6}$$

$$f_k(x) - \overline{f}_k \ge \alpha(\overline{f}_k - f_k),\ k \in K_2, \tag{7}$$

$$f_k(x) = f_k,\ k \in K_3, \tag{8}$$

$$x \in X_1, \tag{9}$$

$$z_1, z_2 \ge 0. \tag{10}$$

where

$$K_1 = \{ k \in K \mid \overline{f}_k > f_k \},$$

$$K_2 = \{ k \in K \mid \overline{f}_k < f_k \},$$

$$K_3 = \{ k \in K \mid \overline{f}_k = f_k \},$$

$$K = K_1 \cup K_2 \cup K_3,$$

α is a positive parameter, $\overline{f}_k$ is the aspiration value of the criterion k, given by the decision maker (DM); and f_k is the value of the k^{th} criterion obtained at the previous iteration.

In general, the solution of problem (5–10) is a (weak) efficient ((weak) nondominated) solution of problem *I*. To get efficient (nondominated) solutions of problem I, we solve the following modification of problem (5–10):

$$\max(z_1 - z_2 + \beta \sum_{k \in K_1 \cup K_2} y_k), \tag{11}$$

subject to

$$f_k(x) - f_k = y_k, \; k \in K_1, \tag{12}$$

$$f_k(x) - f_k = -y_k, \; k \in K_2, \tag{13}$$

$$y_k \geq 0, \; k \in K_1 \cup K_2, \tag{14}$$

and constraints (6–10).

The MOIA algorithm consists of three steps:

Step 1: Determine an initial (weak) efficient solution for (I).

Step 2: Show the solution to the DM. If the DM is satisfied with the solution, Stop; otherwise, ask the DM to specify a new reference point $\overline{f}_k$ and go to Step3.

Step 3: Based on the values of $\overline{f}_k$ and f_k, solve (5–10) or (11–14) and find a new intermediate (weak) efficient solution $f_k(x)$: go to Step 2.

It should be noted, that when $\overline{f}_k > f_k$ for every $k \in K$, problems (5–10) and (11–14) have to be slightly modified as described in Vassilev and Narula (1993).

The MOIA algorithm has the following advantages:

a) The number of the single objective integer programming problems solved is equal to the number of reference points considered (the definition of an ideal point or the solution of additional auxiliary integer problems is not necessary).

b) The solution of problem (5–10) or of problem (11–14) found at the previous iteration is a feasible solution of the problem at the next iteration. This is important, because the use of an initial feasible solution in an exact or a heuristic integer algorithm can decrease the solution time considerably.

c) The feasible solutions of problems (5–10) or (11–14) lie near or on the (weak) efficient surface of the multiple objective integer linear problem I. This is important because the DM can interrupt the operation of an exact algorithm (when the solution time is long) or he can use an approximate algorithm to solve the single objective integer problem. The approximate solution of problem (5–10) or (11–14) is a near efficient solution and it can be used to formulate a new problem (5–10) or (11–14) at the next iteration.

The (weak) nondominated solutions computed at each iteration of the MOIA algorithm may contain continuous components (depending on objective function coefficients), but the components of the corresponding (weak) efficient solutions are integer.

To find weak efficient (weak nondominated) solutions of problem I using the second algorithm, MOHIA, two problems are solved. One problem is a parametric linear programming problem:

$$\max (z_1 - z_2) \tag{15}$$

subject to

$$f_k(x) - (\overline{f}_k - f_k)(z_1 - z_2) \geq f_k, \; k \in K_1, \tag{16}$$

$$f_k(x) \geq \overline{f}_k + \alpha(\overline{f}_k - f_k), \; k \in K_2, \tag{17}$$

$$f_k(x) = f_k, \; k \in K_3, \tag{18}$$

$$x \in X_2, \tag{19}$$

$$z_1, z_2 \geq 0, \tag{20}$$

where α is a positive parameter. Each solution of problem (15–20) is a weak efficient (weak nondominated) solution of problem C.

The other problem is the following mixed integer linear programming problem:

$$\max (z_1 - z_2), \tag{21}$$

subject to

$$f_k(x) - 0.1(z_1 - z_2) \geq f^*_k, \tag{22}$$

$$x \in X_1, \tag{23}$$

$$z_1, z_2 \geq 0, \tag{24}$$

Problem (21–24) is obtained from problem (15–20) by making the following substitutions:

$$f_k = f^*_k,$$

$$\overline{f}_k = f^*_k + 0.1,$$

$$x \in X_2 \Rightarrow x \in X_1,$$

where f^*_k is the last solution of problem C. The solution of problem (21–24) is a weak efficient (weak nondominated) solution of problem I.

In order to find an efficient (nondominated) solution of the multiple objective problems I and C, the modifications of problems (15–20) and (21–24) are solved. The modifications are similar to the ones for problems (5–10) and (11–14).

For most of the time the DM works in the criteria space. Therefore, we solve easier (from a computational viewpoint) linear parametric programming problems. Only when the DM approves a (weak) nondominated solution to obtain a (weak) efficient solution with integer values, we solve a mixed integer programming problem (21–20).

The MOHIA algorithm consists of four steps:

Step 1: Determine an initial (weak) efficient solution for (C).

Step 2: Show the solution to the DM. If the DM finds this solution satisfactory, go to Step 4; otherwise go to Step 3.

Step 3: Ask the DM to specify a new reference point $\overline{f}_k$. Find a new (weak) efficient solution for (C) by solving problem (15-20) or its modification. Go to Step 2.

Step 4: Find a (weak) efficient integer solution for (I), closest to the (weak) efficient solution for (C), by solving problem (21-24) or its modification. Show the solution to the DM. If the DM accepts this solution, Stop; otherwise go to Step 3.

The MOHIA algorithm has the following advantages:

a) The number of the single objective problems solved is equal to the number of the reference points considered by the DM, and only a few single objective mixed integer problems are solved.

b) The DM can evaluate more than one (weak) nondominated continuous solution at each iteration, because linear parametric programming problem is solved at each iteration.

The MOHIA algorithm is computationally more efficient than the MOIA algorithm, but it is possible that some integer (weak) efficient solutions may be overlooked; and the search for continuous (weak) efficient solutions, when an integer multiple objective problem is being solved, may annoy some DM's, i. e. they may feel uncomfortable during the solution process.

Clearly, the two algorithms have their advantages and disadvantages. Their combined use, which preserves the advantages of the basic algorithms, may be more appropriate, i.e., maintain the quality of the (weak) nondominated solutions, the needs of the DM, and reduce the total computational time for the multiobjective integer programming problem I. The combined use of the basic algorithms can be represented as an hybrid algorithm. The hybrid algorithm has the following two steps:

Step 1: When an integer (weak) efficient solution is found at an iteration of the MOHIA algorithm and it satisfies the DM, Stop. If the DM wants to find a (weak) efficient integer solution at the next iteration, go to Step 2. Otherwise, use MOHIA algorithm.

Step 2: Find an integer (weak) efficient solution using MOIA; if it satisfies the DM, Stop. If the DM wishes to find a continuous (weak) efficient solution at the next iteration, use MOHIA, go to Step 1. Otherwise, use MOIA algorithm.

The decision support system MOIP, proposed in the paper, is developed on the basis of the hybrid algorithm. The charactersistics of this algorithm, the software environment and the user-friendly interface make MOIP an appropriate system for multiple objective decision making with integer variables.

4. STRUCTURE, FUNCTIONS AND USER - INTERFACE

MOIP decision support system consists of three modules:

- a control module,
- an editing module, and
- an optimization module.

The control module provides the software environment, to support the following basic functions of the system:

- to create, update and store the files, to represent a description of the multiple objective integer linear programming problems;
- to solve the multiple objective integer linear programming problems entered;
- to analyse and print the various solutions of the problems solved;
- to teach the operations of MOIP and to execute system commands.

The control module is menu-driven. The interface with the DM consists of three menus. The software programs, which execute the functions listed above, are chosen in the main menu. The two more menus select the editing and optimization procedures. The operation with MOIP is started and stopped in the main menu.

We use the following multiple objective integer linear programming problem to demonstrate the operation with MOIP system:

$$\max f_1(x) = 1.4x_1 + 2.1x_2$$
$$\max f_2(x) = -2x_1 + 5.3x_2$$

subject to:

$$A: \quad x_1 + 2x_2 \le 12.6,$$
$$B: \quad 8x_1 + 3x_2 \le 54,$$
$$C: \quad -2x_1 + 6x_2 \le 30,$$
$$D: \quad 2x_1 - 3x_2 \le 6.8,$$
$$0 \le x_1 \le 6,$$
$$0 \le x_2 \le 100,$$
$$x_1, x_2 - \text{integers}.$$

The data of the multiple objective problem to be solved are entered, altered and stored by using MOIP-E editing module. The DM executes these operations with the help of several screens. The main screen for the example is shown in Fig. 1. The screen can be divided into two parts. The upper part of the screen is the spread sheet of the problem. The first column contains the names of the criteria, the constraints, the names of the rows, labels for the upper and lower bounds on the variables and their type. In the second column, the type of the optimization criteria and the constraints are entered. The third column contains

Name	Type	RHS	X_1	X_2		
LB			0	0		
UB			6	100		
Type			I	I		
f	max		1.4	2.1		
f	max		-2	5.3		
A:	<	12.6	1	2		
B:	<	54	8	3		
C:	<	30	-2	6		
D:	<	6.8	2	-3		

M=4 K=2 N=2

MOIP - E F1 - Help

Fig. 1. Main screen of the editing program for the example

the right hand side values of the constraints and probably the upper (lower) limits of the criteria. In the remaining columns the variable names, the coefficients of the constraints matrix, the values of the upper and lower limits and the variable type are given.

The lower part of the screen is a command row. The data are entered and corrected with the help of commands, in this row, using different keys.

The editing module works with operating memory only. The size of the problems, we may enter and edite with MOIP-E depends on the operating memory available.

To solve a given multiple objective integer linear problem, MOIP system provides control of the solution process to the DM via different screens. The following message appears on the screen at the beginning:

Initial solution
(N)o, (Y)es
(N)ew, (O)ld, (V)erify, (C)ontinue, (E)scape

By pressing N key, the DM lets the system define an initial (weak) nondominated solution. If he presses Y key, the DM also has to select either (N)ew or (O)ld. In (N)ew mode, the DM sets the aspiration levels of the objective functions; it is assumed that the DM does not have a solution of the problem. When the DM wants to continue solving an old problem from the point where the solution process was interrupted, he selects (O)ld mode. In this mode, the values of the objective functions obtained at a previous step are entered and new aspiration levels defined. In each case, the DM has to specify whether he desires a continuous or an integer solution.

After a (weak) nondominated solution has been found by the MOIP system, it is displayed on the screen. The values of the objective functions are displayed in numerical format and as bar graphs in the upper left part of the screen. The deviations of the objective functions from their previous values are given. The optimization type for each objective function is also displayed.

The left lower part of the screen shows the type of the solution obtained (continuous, parametric or integer) and a set of commands, questions, replies, as well as prompts for continuation or cancellation of the process. The DM may also enter aspiration levels or the desired alterations of the criteria values in this window.

The right part of the screen is used to display the information, entered by the DM and to display the values of the decision variables in the (weak) efficient solutions.

Fig. 2 shows the screen after computing the second continuous (weak) nondominated solution for the example; it also shows the new aspiration levels. The second continuous solution, approved by the DM is $(F_1, F_2) = (14.84, 12)$ and the deviation from the last (weak) nondominated solution is $(\Delta F_1, \Delta F_2) = (0.198, -2.641)$.

After command (A)sp. Lev. or (D)elta is executed, the DM is offered the possibility to enter the preferred aspiration levels or the alterations in the values of the objective functions. This is achieved by the appearance of two windows in which the DM may enter the criterion number and its aspiration level (altered value) sequentially. The (V)erify command enables the DM to check the entered values. After (C)ontinue command is executed, the DM specifies the type of the new (weak) efficient, (integer or continuous) solution that is desired and the computing process starts. When the number of the objective functions is greater than eight, the DM can enter aspiration levels for all the objective functions by using the (M)ore command. (E)scape command returns the operation to the control menu.

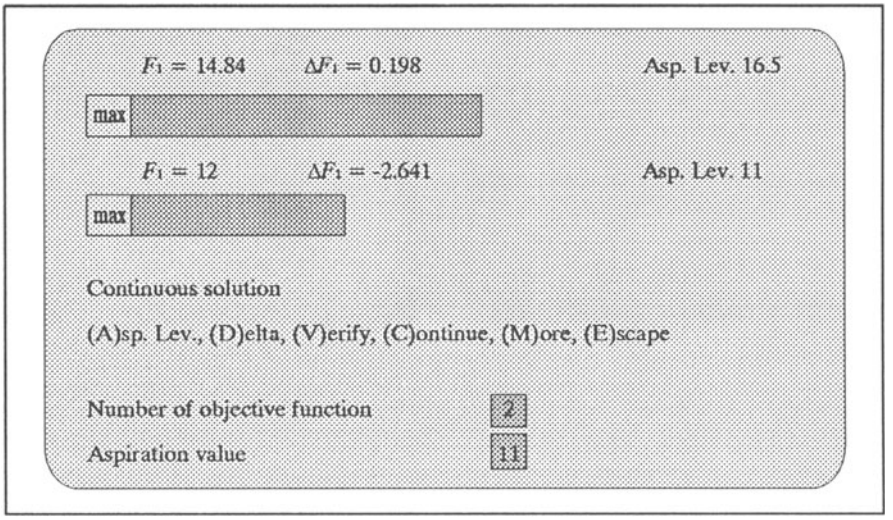
F1 = 14.84 ΔF1 = 0.198 Asp. Lev. 16.5
max
F1 = 12 ΔF1 = -2.641 Asp. Lev. 11
max
Continuous solution
(A)sp. Lev., (D)elta, (V)erify, (C)ontinue, (M)ore, (E)scape
Number of objective function 2
Aspiration value 11

Fig. 2. Entry and check of new aspiration levels

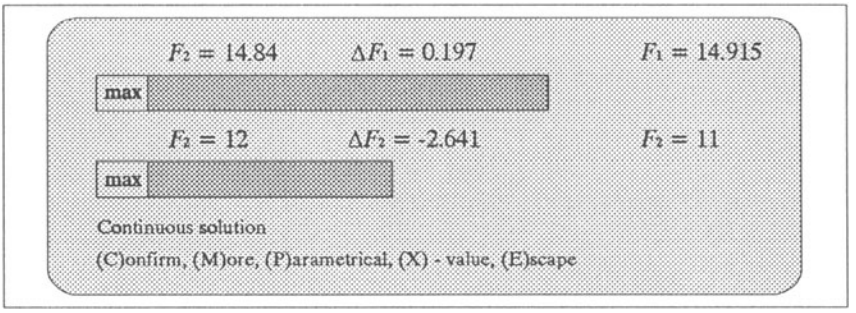

Fig. 3. Consideration and approval of continuous (weak) nondominated solution

Fig. 3 shows a screen that appears after the DM has tried to find a new continuous (weak) nondominated solution, corresponding to the aspiration point, shown in Fig. 2. One of the continuous (weak) nondominated solutions found is displayed in the upper right part of the screen; the DM evaluates it. The other continuous solutions (parametric), appear at the same place on the screen after pressing the P key that starts the (P)arametric command. With the help of (X)-value command, the corresponding (weak) efficient solutions can be displayed there. When the number of the objective funcions is more than eight, the (M)ore command enables the display of all functions and variables. The DM may approve any of the (weak) nondominated solutions found by using the (C)onfirm command; it results in updating the values of the objective function, the deviations of these functions and the alteration of the bar graphs on the left upper part of the screen. Then the new aspiration levels can be entered or an integer (weak) efficient solution close to the last continuous (weak) efficient solution found can be computed.

Fig. 4 shows a screen, where the DM has found, but has not yet approved, an integer (weak) efficient solution (x_1,x_2) = (4,4). This integer (weak) efficient solution is close to

the last continuous (weak) efficient solution found – (x_1, x_2) = (4.815, 3.892), and its corresponding (weak) nondominated solution (F_1, F_2) = (14.915, 11).

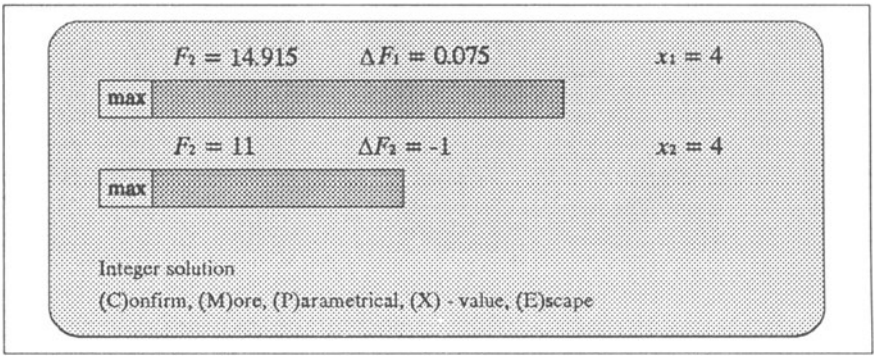

Fig. 4. Consideration and approval of integer (weak) efficient solution

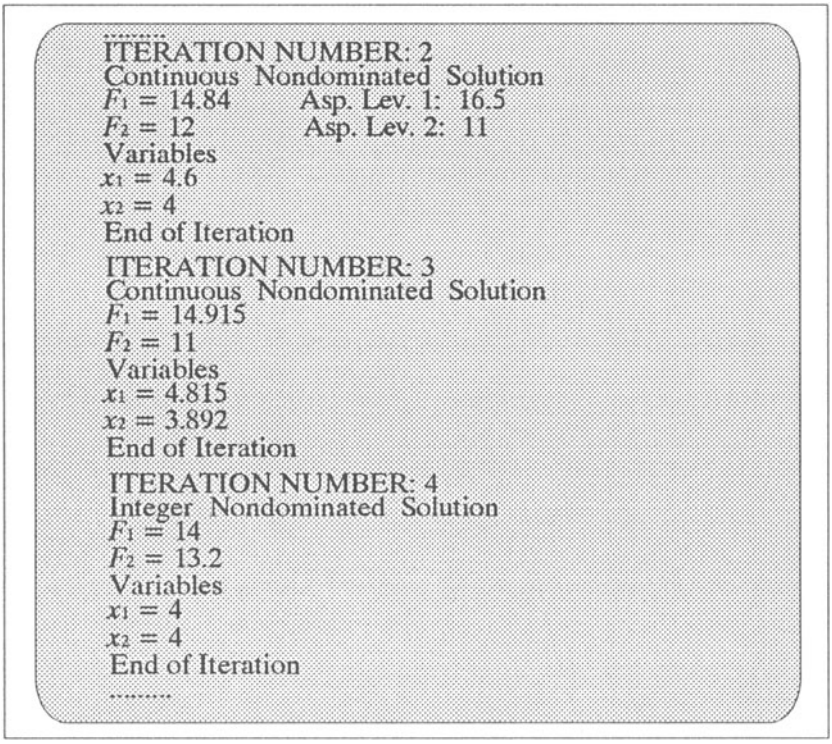

Fig. 5. A part of the protocol file for the example

All the (weak) nondominated solutions and their corresponding (weak) efficient solutions are stored in a separate protocol file. The DM has access to this file through the main menu. He can analyze and print all the solutions obtained. This file gives DM the possibility to start solving the problem again, using any of the stored intermediate solutions

as an initial solution. Fig. 5 shows a part of the protocol file for the example. This part of the protocol file contains three iterations, which were discussed in describing the operation of the MOIP system.

5. REMARKS

The MOIP decision support system is written in C and FORTRAN 77 programming languages. It is designed to model and solve multiple objective integer linear programming problems. The combined use of the two algorithms MOIA and MOHIA, helps reduce the total solution time for the multiobjective integer problems while preserving the quality of the (weak) nondominated solutions, and offering flexibility to the DM. The user-friendly interface provides ease in teaching, modelling and solving (including interruptions) of real multiple criteria integer problems. It may be observed that MOIP can also be used to solve multiple objective linear programming problems.

The MOIP decision support system is intended for IBM compatible personal computers, working under DOS operating system. The approach used in its software development makes it comparatively easy to develop versions for other operating systems.

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Solving Multi-Objective Knapsack Problem by a Branch-and-Bound Procedure

Ulungu-Ekunda Lukata and Jacques Teghem

Department of Mathematics & Operational Research, Faculté Polytechnique de Mons, 9, rue de Houdain 7000 Mons, Belgium

Abstract. A classical knapsack problem with two objectives is considered in which concepts of supported and non supported efficient solutions are pointed out.
A branch-and-bound procedure is described to solve this bicriteria knapsack problem. The proposed method can be interpreted as an adaptation of the Martello & Toth's classical method to the multi-objective context. Its main features are:

1. no any unicriterion optimization is performed during the procedure.
2. the specific structure of the multi-objective knapsack problem is preserved.

A didactic example is presented to illustrate the method and an extension of the procedure to more than two criteria is discussed.

1 Introduction

The knapsack problem is a well known operational research problem (Martello and Toth 1990). Up to now, this classical combinatorial optimization problem is not spread over multicriteria domain. Only two papers devoted to particular version of the problem exist in the literature (Dyer, Foreman, and Mustafa 1988; Rosenblatt and Sinuany-Stern 1989).
The aim of the present paper is to provide a tool treating the classical version of that problem (Ulungu E.L. 1993).
The general structure of MultiObjective Knapsack Problem (MOKP) is

$$\left[\begin{array}{l} \text{“max”} z_k(X) = \sum_{j=1}^{n} c_j^{(k)} x_j \quad k = 1, \ldots, K \\ \sum_{j=1}^{n} w_j x_j \leq W \\ x_j = (0,1) \end{array}\right] \qquad (P)$$

where

- Coefficients c_j^k, w_j and W are non negative integers.
- $w_j \leq W, \forall j = 1, \ldots, n.$
- $\sum_{j=1}^{n} w_j \geq W.$

These hypothesis are necessary to avoid trivial solutions. Several contexts are possible to apply this model (e.g. problems of media selection (Dyer, Foreman, and Mustafa 1988) and capital budgeting (Rosenblatt and Sinuany-Stern 1989)).
Efficient solutions are referred to as solutions of problem (P). Along this paper, we intend to present a branch-and-bound procedure for the explicit generation of the entire efficient set, $\mathcal{E}(\mathcal{P})$, of bicriteria case.
A solution $X^\star$ is "*efficient*" if there does not exist any other feasible solution X such that:
$z_k(X) \leq z_k(X^\star), \forall k$; with at least, one strict inequality.
In multi-objective integer linear programming, we distinguish two kinds of efficient solutions (Ulungu E.L. and Teghem 1994):

- the "*supported efficient solutions*", $\mathcal{SE}(\mathcal{P})$, which are optimal solutions of the single parametrized objective problem:

$$\left.\begin{array}{l} \max \sum_{k=1}^{K} \alpha_k z_k(X) \\ \sum_{j=1}^{n} w_j x_j \leq W \\ 0 \leq x_j \leq 1 \\ \text{with } \sum_{k=1}^{K} \alpha_k = 1 \text{ and } \alpha_k > 0 \end{array}\right\} (LP_\alpha)$$

- and the "*non supported efficient solutions*", $\mathcal{NSE}(\mathcal{P}) = \mathcal{E}(\mathcal{P})$ - $\mathcal{SE}(\mathcal{P})$, which cannot be found by simply optimizing problem (LP_α).

- In the following section, we describe a "branch-and-bound" procedure, whose spirit looks like Kiziltan & Yucaoğlu method (Kiziltan and Yucaoğlu 1983), in the sens that, an unicriterion optimization is never treated when optimizing problem MOKP. The procedures does not lose particularity of MOKP by destroying its unique constraint structure.
This method can be interpreted as an adaptation of the Martello & Toth method's to the multicriteria framework.
The method is illustrated by an example. Finally, we discuss an extension of the procedure to more than two criteria.

2 The adaptation of Martello & Toth method

2.1 Ranking variables

When modelling a classical knapsack problem, the discrete alternatives are evaluated against them (see e.g. (Martello and Toth 1990)). Let us recall that the items are ordered according to their "*value to weight*" ratio before the resolution procedure is applied. So, it is necessary to sort the variables. The efficiency of the "branch-and-bound" procedure and evaluation of the boundaries depend upon this order. In a multicriteria problem, this order is not unique. Here, each object is separately ranked according to different criteria. To decide which object to put into the knapsack, an overall ranking of objects is needed based upon the individual ranking of each criterion. Consequently, the key difficulty of MOKP is to implement technique for obtaining the overall preferences for alternatives. A wide variety of techniques have been proposed to carry out this task (Roubens and Vincke 1985; Pastijn and Leysen 1989).

The following typical definitions will be used to outline the procedure. For $k = 1, 2$:

- define O_k : variables order according to decreasing values of $\frac{c_j^k}{w_j}$.
- Let $r_j^{(k)}$ be the rank (mean in ex aequo case) for criteria k.
- define $\mathcal{O}$ the overall preferences for variables according to increasing values of $\frac{r_j^{(1)}+r_j^{(2)}}{2}$, the arithmetic mean of ranks.

We assume that variables are indexed according to ordinal preference $\mathcal{O}$.
Remark: If the values scales of the criteria are different, then the order $\mathcal{O}$ is obtained as follows:

1. reduce the ratios of $\frac{c_j^k}{w_j}$ in the interval $[0, 1]$ for $k = 1, 2$

$$\text{with} \begin{cases} 0 = \min\limits_j \dfrac{c_j^k}{w_j} & k = 1, 2 \\ 1 = \max\limits_j \dfrac{c_j^k}{w_j} & k = 1, 2 \end{cases}$$

2. define $\mathcal{O}$ according to decreasing values of $\frac{c_j^{(1)}+c_j^{(2)}}{w_j}$

2.2 Analysis of a node of the combinatorial tree

2.2.1 Definitions

The following typical definitions will be used to outline the procedure.

- A *node* of the enumeration tree is a *partial* solution, which is an assignment of 0/1 values to a subset of n variables of the problem.
- Variables which are not assigned values are called *free* variables.
- A solution formed by assigning 0/1 values to all free variables is called *completion* of the partial solution.
- A partial solution is said to be *fathomed*, if it is established that none of its completions require further examination.

2.2.2 Notations

Notations and other concepts defined here are connected to the node considered but with a view to simplify formula script we don't mark node index.
At any node, some variables are set to 0 or 1 and the remaining are free.
Let B_0 and B_1 denote the index sets of variables assigned the values 0 and 1 respectively, and let F be the index set of free variables.
In $\mathcal{O}$, the first variables ranked are those belonging to B_1 and then those of B_0; the free variables are setted at the end.
Let $i-1$ be the last index of fixed variables so that we have:

$$\mathcal{O} \equiv \{B_1 \cup B_0\} \cup F$$

$$\text{with} \begin{cases} B_1 \cup B_0 & \equiv & \{1, \ldots, i-1\} \\ F & \equiv & \{i, \ldots, n\} \end{cases}$$

Note by $i_k (k = 1, 2)$, the first variable index in F according to partial preferences O_k.
Let

- $\overline{W} = W - \sum_{j \in B_1} w_j \geq 0$ be the leftover capacity of the knapsack.
- $\underline{Z} = \begin{pmatrix} \underline{z}_1 = \sum_{j \in B_1} c_j^{(1)} \\ \underline{z}_2 = \sum_{j \in B_1} c_j^{(2)} \end{pmatrix}$ be the criteria values vector obtained with already fixed variables.
- $\overline{Z} = \begin{pmatrix} \overline{z}_1 \\ \overline{z}_2 \end{pmatrix}$ be the vector whose components are upper bounds of feasible values respectively for each criterion at considered node. These bounds are evaluated separately.
- s_k be the index of variable define by $\max\{l_k \in F | \sum_{j_k=i_k}^{s_k} w_{j_k} < \overline{W}\}$ where notation j_k means that variables in F are considered taking into account the partial preference O_k.

The evaluation function $\overline{z}_k$ of criterion z_k at this node is computed with

Martello & Toth (Martello and Toth 1990) bound's as below:

$$\overline{z}_k = \underline{z}_k + \sum_{j_k=i_k}^{s_k} c_{j_k}^{(k)} + \max\left(\left[\overline{W}_k \frac{c_{s_k+2}^{(k)}}{w_{s_k+2}} \right]; [c_{s_k+1}^{(k)} - (w_{s_k+1} - \overline{W}_k) \frac{c_{s_k}^{(k)}}{w_{s_k}} \right] \right)$$

with $\overline{W}_k = \overline{W} - \sum_{j_k=i_k}^{s_k} c_{j_k}^{(k)}$; the indices $s_k + 1$ and $s_k + 2$ are taken from O_k.

Thus the problem considered is:

$$\left[\begin{array}{ll} \text{“max”} \sum_{j \in F} c_j^{(k)} x_j + \sum_{j \in B_1} c_j^{(k)} & k = 1, 2 \\ \sum_{j \in F} w_j x_j \leq \overline{W} & \\ x_j = (0, 1) & \end{array} \right]$$

2.2.3 The incumbent efficient values list L

The “*incumbent value*” of unicriterion methods –i.e the best feasible value already found for criterion z until considered node– will be replaced by list L of incumbent non dominated feasible values $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ i.e. feasible values which are candidates to be non dominated. More precisely, at each iteration, L contains the set of generated feasible values which till now are non dominated by the values generated during the previous iterations.

At each new step, list L is updated by comparison with value $\underline{Z}$ found at this new node: $\underline{Z}$ is introduced in L if it is non dominated by any vector of L and any vector of L dominated by the new one $\underline{Z}$ is removed from L. Finally, when all nodes of the combinatorial tree have been fathomed, $E(P) \equiv L$.

2.3 Algorithm

2.3.1 Initialisation (Root)

$$B_1 = \emptyset; \quad B_0 = \emptyset; \quad F = \{1, \ldots, n.\}$$
$$\underline{z} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}; \quad \overline{z} = \begin{pmatrix} \infty \\ \infty \end{pmatrix}; \quad L = \emptyset$$

2.3.2 General iteration

Each node is characterized by B_1, B_0 and F (according to the order $\mathcal{O}$). New values of $\overline{W}$ and $\underline{z}$ are determined. A node is fathomed in the two following situations:

$$\left\{ \begin{array}{ll} (a) & \text{Obviously, if } \{j \in F | w_j < \overline{W}\} = \emptyset, \\ & \text{no supplementary object can be selected.} \\ (b) & \text{Otherwise, if } \overline{z} \text{ is dominated by a vector of L,} \\ & \text{further investigations are useless.} \end{array} \right.$$

The backtraking process (cf. §2.3.3) is performed when a node is fathomed. In case (a), the update of $\underline{z}$ is done before that processus.

• To continue, let us suppose that the node is not fathomed.

So, we define s the index variable such that $\max\{l \in F | \sum_{j=i}^{l} w_j < \overline{W}\}$ where variables of F are considered according to $\mathcal{O}$.
If $w_i > \overline{W}$, set $s = i - 1$.
A new node of the combinatorial tree is build up for next iteration by calculating new indices sets B_0, B_1 and F in the following way:

$\underline{1^{st}\text{case} : \ s > i}$
$B_1 \longleftarrow B_1 \cup \{i, \ldots, s\}$
$B_0 \longleftarrow B_0$
$F \longleftarrow F \backslash \{i, \ldots, s\}$
$\underline{2^{nd}\text{case} : \ s = i - 1}$
Let $r = \min\{j \in F | w_j < \overline{W}\}$ (according to the order $\mathcal{O}$)
$B_1 \longleftarrow B_1 \cup \{r\}$
$B_0 \longleftarrow B_0 \cup \{i, \ldots, r-1\}$
$F \longleftarrow F \backslash \{i, \ldots, r\}$

2.3.3 Backtracking

When a node is fathomed, a new node is built up by setting to zero the variable corresponding to the last index in B_1. If t denotes that index, then:

$$\begin{aligned} B_1 &\longleftarrow B_1 \backslash \{t\} \\ B_0 &\longleftarrow (B_0 \cap \{1, \ldots, t-1\}) \cup \{t\} \\ F &\longleftarrow \{t+1, \ldots, n\} \end{aligned}$$

Remark: If $t = n$, let
u be the smaller index such that $\{u, u+1, \ldots, t-1, t\} \subset B_1$
v be the last index of $B_1 \backslash \{u, \ldots, t\}$, then

$$\begin{aligned} B_1 &\longleftarrow B_1 \backslash \{v, u, u+1, \ldots, t-1, t\} \\ B_0 &\longleftarrow (B_0 \cap \{1, \ldots, v-1\}) \cup \{v\} \\ F &\longleftarrow \{v+1, \ldots, n\} \end{aligned}$$

The procedure stops when the initial node is fathomed. At that moment, the list L contains both supported and non supported efficient solutions of the bicriteria knapsack problem.

2.4 Didactic example

$$\text{“max”} \begin{cases} z_1(X) = 11x_1 + 5x_2 + 7x_3 + 13x_4 + 3x_5 \\ z_2(X) = 9x_1 + 2x_2 + 16x_3 + 5x_4 + 4x_5 \end{cases}$$
$$4x_1 + 2x_2 + 8x_3 + 7x_4 + 5x_5 \leq 16$$
$$x_j = (0, 1) \quad j = 1, \ldots, 5.$$

The partial preferences are $\begin{cases} O_1: & x_1, & x_2, & x_4, & x_3, & x_5 \\ O_2: & x_1, & x_3, & x_2, & x_5, & x_4 \end{cases}$

Thus O : $x_1,\ x_2,\ x_3,\ x_4,\ x_5$

(P.S. : the indices are choosen so that reordering is not needed).

(0) $B_1 = \emptyset;\ B_0 = \emptyset;\ F = \{1,2,3,4,5\}$

(1) $\underline{B_1 = \{1,2,3\}};\ \underline{F = \{4,5\}};\ \overline{W} = 2;\ \underline{z} = \begin{pmatrix} 23 \\ 27 \end{pmatrix} \equiv L$

$\{j \in F | w_j < \overline{W}\} = \emptyset \Longrightarrow$ the node is fathomed.

(2) $\underline{B_1 = \{1,2\}};\ \underline{B_0 = \{3\}};\ \underline{F = \{4,5\}}; \overline{W} = 10;$
computation of $\overline{z}$ wouldn't fathom this node $(*)^1$

(3) $\underline{B_1 = \{1,2,4\}};\ \underline{B_0 = \{3\}};\ \underline{F = \{5\}}$
$\overline{W} = 3;\ \underline{z} = \begin{pmatrix} 29 \\ 16 \end{pmatrix} \Longrightarrow \quad L = \left\{ \begin{pmatrix} 23 \\ 27 \end{pmatrix}, \begin{pmatrix} 29 \\ 16 \end{pmatrix} \right\}$

(4) $\underline{B_1 = \{1,2\}};\ \underline{B_0 = \{3,4\}};\ \underline{F = \{5\}}$
$\overline{W} = 10;\quad \overline{z} = \begin{pmatrix} 16+3 \\ 11+4 \end{pmatrix} = \begin{pmatrix} 19 \\ 15 \end{pmatrix}$
$\overline{z}$ is dominated by at least one solution of L $\Rightarrow$ the node is fathomed.

(5) $\underline{B_1 = \{1\}};\quad \underline{B_0 = \{2\}};\quad \underline{F = \{3,4,5\}}$
$\overline{W} = 12;$ computation of $\overline{z}$ wouldn't fathom this node$(*)$

(6) $\underline{B_1 = \{1,3\}};\ \underline{B_0 = \{2\}};\quad \underline{F = \{4,5\}}$
$\overline{W} = 3;\quad \underline{z} = \begin{pmatrix} 18 \\ 25 \end{pmatrix}$
$\{j \in F | w_j < \overline{W}\} = \emptyset \Longrightarrow$ the node is fathomed.

(7) $\underline{B_1 = \{1\}};\quad \underline{B_0 = \{2,3\}};\ \underline{F = \{4,5\}}$
$\overline{W} = 12;$ computation of $\overline{z}$ wouldn't fathom this node$(*)$

(8) $\underline{B_1 = \{1,4,5\}};\ \underline{B_0 = \{2,3\}};\ \underline{F = \emptyset}$
$\overline{W} = 0;\quad \underline{z} = \begin{pmatrix} 27 \\ 18 \end{pmatrix} \Longrightarrow L = \{ \begin{pmatrix} 23 \\ 27 \end{pmatrix}, \begin{pmatrix} 27 \\ 18 \end{pmatrix}, \begin{pmatrix} 29 \\ 16 \end{pmatrix} \}$

(9) $\underline{B_1 = \emptyset};\quad \underline{B_0 = \{1\}};\quad \underline{F = \{2,3,4,5\}}$
$\overline{W} = 16;\ \overline{z} = \begin{pmatrix} 0+5+13+\max([\frac{5}{8}.3],[7-\frac{1}{7}.13]) \\ 0+16+2+4+\max([0],[5-\frac{6}{5}.4]) \end{pmatrix} = \begin{pmatrix} 23 \\ 22 \end{pmatrix}$
given $s_1 = 4$ and $s_2 = 5$

$\overline{z}$ is dominated by the solutions in list L $\Longrightarrow$ the node is fathomed.

Procedure is finished and

$$E(P) = \left\{ \begin{pmatrix} 23 \\ 27 \end{pmatrix}, \begin{pmatrix} 27 \\ 18 \end{pmatrix}, \begin{pmatrix} 29 \\ 16 \end{pmatrix} \right\}$$

corresponds respectively to the following three solutions in decision space.

$$\begin{aligned} \tilde{X}_2 &\equiv (1,1,1,0,0) \\ X_3 &\equiv (1,0,0,1,1) \\ \tilde{X}_1 &\equiv (1,1,0,1,0) \end{aligned}$$

Figure 1 shows that solution X_3 is non supported

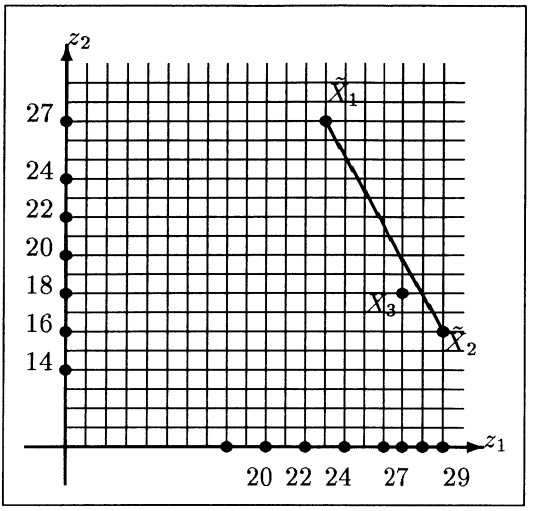

Figure 1: $\mathcal{E}(\mathcal{P})$ for the didactic example

3 Extension

With $K \geq 3$ the problem (P) can be stated as

$$\left[\begin{array}{l} \text{“}\max_{X \in D}\text{”} CX \\ \\ \sum_{j=1}^{n} w_j x_j \leq W \\ \\ x_j = (0,1) \end{array}\right]$$

where C is a $(K \times n)$ matrix.

All the concepts defined before are just extend to K compounds instead of two:

- $O_k \quad (k = 1, \ldots, K)$;
- $r_j^{(k)} \quad (k = 1, \ldots, K)$ and $(j = 1, \ldots, n)$;
- $\mathcal{O}$ is defined according to the increasing values of $\dfrac{\sum_{k=1}^{K} r_j^{(k)}}{n}$;

- $\underline{Z} \equiv (\underline{z}_1, \underline{z}_2, \ldots, \underline{z}_K)$;
- $\overline{Z} \equiv \overline{z}_1, \overline{z}_2, \ldots, \overline{z}_K$

Then the problem considered is:

$$\left[\begin{array}{l} \text{"max"} \sum_{j \in F} C^j x_j + \sum_{j \in B_1} C^j \\ \sum_{j \in F} w_j x_j \leq \overline{W} \\ x_j = (0,1) \end{array}\right]$$

where C^j represent the j^{th} column of matrix C.

The branch-and-bound procedure developped in section §2.3 still always valuable. We can apply it to obtain $\mathcal{SE}(\mathcal{P})$ as well as $\mathcal{NSE}(\mathcal{P})$.

4 Some concluding remarks

The survey done by the authors in the multi-objective combinatorial optimization (Ulungu E.L. and Teghem 1994) has oriented futur research in this field in three directions. It was claimed that some important progress can still be made in the so-called "methodological direction". The aim of this work is to investigate that direction which consists to adapt existing methods of combinatorial optimization to the multicriteria case especially for the knapsack problem.

In this paper, it is observed that through a vector interpretation, it is possible to use most of the concepts defined for the implicit enumeration to solve the knapsack problem in a multicriteria context.

As the cardinality of the feasible set is finite, the procedure described before is a finite one, because each step eliminates at least one feasible solution. It is easy to extend the method to more than two criteria so that high dimensional problems can be theoretically solved. It possibly remains to increase the algorithm capacity to curtail enumerations.

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Bicriteria Scheduling: Minimizing Flowtime and Maximum Earliness on a Single Machine

Meral Azizoglu, Suna Köksalan Kondakci and Murat Köksalan

Industrial Engineering Department Middle East Technical University, Ankara 06531, TÜRKIYE

Abstract: We consider the bicriteria problem of minimizing total flowtime and maximum earliness penalties for a given set of jobs on a single machine. We discuss some properties of the efficient solutions and develop a heuristic procedure to generate all efficient solutions when inserted idle time is not allowed.

1. INTRODUCTION

The research done in scheduling mostly involves regular measures as performance criteria, i.e., measures that are nondecreasing in completion times. Recently, especially with the growing interest in just-in-time (JIT) production strategies, manufacturers began to realize the costs associated with finishing a job early i.e., the importance of measures that are not regular. Flowtime is one of the most widely used regular criterion. Through the minimization of flowtime, manufacturer's concern of keeping inventory at low levels is tried to be achieved. However, minimization of flowtime only leads to the minimization of work-in-process (WIP) inventory. The processed items are assumed to leave the system, i.e., no

finished goods inventory. On the other hand, receipt of orders earlier than their due dates causes an inventory holding cost for the customer.

In this paper, we consider the problem of minimizing both total flowtime and maximum earliness. Total flowtime, by minimizing WIP inventory, acts as a measure representing manufacturer's concerns. Minimizing maximum earliness, on the other hand, represents either customer's concerns of inventory holding cost or manufacturer's concerns of holding finished goods inventory in case early shipments are not allowed.

Flowtime and maximum earliness measures are conflicting. Minimizing flowtime leads to completing jobs as soon as possible. Minimizing maximum earliness, on the other hand, leads to completing jobs close to their due dates.

Several multicriteria scheduling studies have considered flowtime together with maximum tardiness (Heck and Roberts 1972, Van Wassenhove and Gelders 1980, Sen and Gupta 1983, Nelson et al. 1986, Kondakci et al. 1994), number tardy (Emmons 1975, Nelson et al. 1986, Kiran and Ünal 1991) or total tardiness (Lin 1983).

There is a limited amount of research dealing with the maximum earliness measure. Maximum earliness problem has been addressed by Monma (1980) as a single criterion problem under the presence of precedence constraints. Sidney (1977) and Lakshminarayan et al. (1978) studied maximum earliness penalties in conjuction with maximum tardiness penalties.

A number of articles addressed a combination of flowtime and earliness criteria. Fry, Leong and Rakes (1987) study a model that uses total

flowtime, total earliness and total tardiness to form a single objective function. They formulate a mixed integer programming model and propose a branch and bound algorithm to solve the problem. Fry and Leong (1987) also propose a mixed integer programming formulation and its solution considering total flowtime and total earliness penalties.

We define the problem in Section 2. We discuss the properties of efficient solutions in Section 3. We present heuristic procedure to generate all efficient schedules in Section 4.

2. PROBLEM DEFINITION

Let

F_j: flowtime of job j, j=1,...,n. It is the same as the completion time of job j (C_j) since all jobs are available at time zero.

F: total flowtime; $F = SF_j$.

p_j: processing time of job j, j=1,...,n.

d_j: due date of job j, j=1,...,n.

E_j: earliness of job j, j=1,...,n; $E_j = \max(0, d_j - C_j)$.

E_{max} = maximum earliness; $E_{max} = \max_j \{E_j\}$

We consider a single machine, static job scheduling system where total flowtime and maximum earliness are minimized, i.e., the problem is

$1 // F, E_{max}$

It is always possible to create sequences having E_{max} values of zero by inserting sufficient amount of idle times so that no job is early. It is only when inserting idle time is not allowed can we have positive E_{max} values. In this study we assume that inserting idle times between jobs is not allowed.

Total flowtime and maximum earliness problems are solved optimally in polynomial time. The Shortest Processing Time (SPT) rule (i.e., ordering the jobs in non-decreasing order of their processing times) minimizes total flowtime. Maximum earliness is minimized if jobs are sequenced according to nondecreasing order of their slack times i.e.,

$$d_{[1]} - p_{[1]} \leq d_{[2]} - p_{[2]} \leq .... \leq d_{[n]} - p_{[n]}$$

where $d_{[i]} - p_{[i]}$ indicates the slack time of the job scheduled at the i th position (see Hogeveen and Van de Velde 1995)

The ordering of jobs mentioned above is known as the minimum slack time rule (MST).

A sequence S is efficient with respect to total flowtime and maximum earliness if there does not exist another sequence S' with

$$F(S') \leq F(S) \text{ and } E_{max}(S') \leq E_{max}(S)$$

with at least one of the above holding as a strict inequality. If there exists such a sequence, S', then we say that S' dominates S and S is an inefficient (or dominated) sequence.

3. CHARACTERIZING EFFICIENT SOLUTIONS

The SPT sequence may not be efficient. If each job's processing time is distinct then the SPT rule results in a unique sequence having minimum flowtime. Hence the sequence is efficient. If there are jobs with equal processing times, then the SPT sequence is not unique. Let the ties be broken by the MST rule for one of these sequences. Then this sequence is an efficient sequence as it dominates the other SPT sequences since the E_{max} value of this sequence is smaller than the E_{max} value of other sequences. Let the E_{max} value of this sequence be $E_{max}(SPT)$.

The sequence produced by the problem

(P) Min F

s.t. $E_{max} = E_{max}(MST)$

is efficient but it has been shown that (P) is NP-hard (see Hogeveen, 1992). An approximate solution of (P) is obtained employing a rule that is explained in the next section. We will refer to the total flowtime value of the resulting sequence as F(P). Assuming that the processing times and due dates are integers it can be seen that the maximum number of efficient sequences is

$$\text{Min } \{F(P) - F(SPT) + 1, E_{max}(SPT) - E_{max}(MST) + 1\}.$$

As we emphasized before, we assumed that inserted idle time is not allowed. If idle time could be inserted efficient sequences having E_{max} values smaller than $E_{max}(MST)$ could be found.

Since flowtime is minimized by the SPT rule and maximum earliness is minimized by the MST rule the following theorem directly follows.

Theorem 1. If $p_j \leq p_k$ and $d_j - p_j \leq d_k - p_k$ then there exists an efficient sequence where job j precedes job k.

It is trivial to prove Theorem 1 by showing neither flowtime nor maximum earliness decreases by interchanging the positions of two adjacent jobs that satisfy the conditions of the theorem.

If we replace $p_j \leq p_k$ with $p_j < p_k$ in Theorem 1 we can say that job j precedes job k in every efficient sequence. When Theorem 1 is implemented then there may exist efficient sequences where job k precedes job j. However, for each such sequence, S, there will be at least one sequence, S', where job j precedes job k and $F(S')=F(S)$ and $E_{max}(S')=E_{max}(S)$.

If the SPT and the MST sequences are the same then this sequence is the unique efficient sequence.

The following precedence theorem can be repeatedly applied to reduce the problem size.

Theorem 2. If $d_k - p_k \leq E_{max}(MST)$ where $p_k \leq p_i$ for all i then there exists an efficient sequence where job k is the first job.

Proof: Assume that job k does not occupy the first position in the schedule. Taking job k from its current position and putting it ahead of all jobs will not increase total flowtime since $p_k \leq p_i$ for all i. When job k is assigned to the first position its earliness is $\text{Max}\{0, d_k - p_k\}$ which is less than or equal to

E_{max}(MST). The earliness of other jobs will not increase since their completion times do not decrease. Hence E_{max} value of the resulting sequence will not increase.

Q.E.D.

4. GENERATING APPROXIMATELY EFFICIENT SEQUENCES

The following forward sequencing rule approximately solves the problem

Min F

s.t. $E_{max} \leq E$.

The rule : Starting from the first position, to the next available position assign the job that has the smallest processing time among the jobs that satisfy the earliness constraint when scheduled to this position. Use the MST rule as a tie breaker.

The rule does not necessarily yield an efficient solution. By selecting the smallest processing time job among the eligible jobs, a smaller number of jobs will be eligible to be assigned to the next position. We expect the rule to produce sequences having close to the minimum F value if not the minimum F value.

Using this heuristic procedure we can generate the approximately efficient solutions corresponding to the integer values of E within the range [E_{max}(MST), E_{max}(SPT)].

The Procedure

Step 1: Calculate E_{max}(MST), E_{max}(SPT) and let k=1.

If $E_{max}(MST)=E_{max}(SPT)$, stop.
Otherwise, let $E= E_{max}(SPT) - 1$.

Step 2: Approximately solve Min F

s.t. $E_{max} \leq E$

Let k=k+1 and E^*_{max} be the maximum earliness value of the resulting sequence.

If $E^*_{max}=E_{max}(MST)$, stop. All distinct approximately efficient schedules are generated and there are k such schedules .

Step 3: Let $E=E^*_{max}-1$ and go to Step 2.

Note that the precedence relations created using Theorem 1 can be utilized when finding the sequence in Step 2.

An example

To see how the procedure works, the following example with four jobs is presented. The processing times and the due dates of the jobs are as follows:

Job i	1	2	3	4
p_i	2	4	5	8
d_i	6	7	10	10

Step 1. Let t denote the time. The SPT sequence, {1-2-3-4}, yields $\Sigma F_i=2+6+11+19=38$ and $E_{max}(SPT)=max\{4,1,0,0\}=4$. The MST sequence, {4-2-1-3}, yields $\Sigma F_i=8+12+14+19=53$ and $E_{max}(MST)=max\{2,0,0,0\}=2$. Then the range for E is [2,4]. $E= E_{max}(SPT)-1=3$

Step 2:

. At t=0 jobs 2 and 4 are eligible, assign job 2 to the first position since $p_2< p_4$.

. At t=4 (job 2 is completed at 4) all remaining jobs are eligible for the second and later positions. Order the remaining jobs in the nondecreasing order of their processing times to positions 2, 3 and 4. That is, assign jobs 1,3 and 4 to positions 2, 3 and 4, respectively.

Resulting sequence is {2-1-3-4} with ΣF_i=4+6+11+19=40 and E_{max}=3.

Step 3: E=3-1=2

Step 2:

. At t=0 only job 4 is eligible. Assign job 4 to the first position.

. At t=8 all remaining jobs are eligible for positions 2, 3 and 4. Assign jobs 1, 2 and 3 to positions 2, 3 and 4, respectively (i.e., in the nondecreasing order of their processing times).

Resulting sequence is {4-1-2-3} with ΣF_i=8+10+14+19=52 and E_{max}=2.

A careful examination of the problem data reveals that the procedure has found all three efficient solutions.

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VI

Multicriteria Interactive Methods

An invited leading paper by Gardiner and Vanderpooten presents an introduction to the subject of multicriteria interactive methods. The authors of this paper present some reflections on the evolution of interactive multiple criteria procedures. They trace their development from early roots in mathematical programming (where the theoretical and technical challenges of computing candidate solutions dominated the research) to major challenges that more recently arose from consideration of the behavioral aspects of decision processes. Current challenges and some directions for future developments are also outlined.

Kirilov and Vassilev describe an interactive method for solving multiple objective linear programming problems. The method develops an idea for successive reduction of the set of normalized weighting coefficients, and the dialog is made in terms of aspiration levels in the objective space.

Miettinen and Makela present an interactive method, called NIMBUS, for nondifferentiable multiobjective optimization problems. The method is capable of handling several nonconvex locally Lipschitzian objective functions subject to nonlinear constraints. The idea of NIMBUS is that the decision maker can easily indicate what kind of improvements are desired and what kind of impairments are tolerable at the point considered. At each iteration the decision maker is asked to classify the objective functions into five different classes, and a new problem is formed according to this classification. A multiobjective proximal bundle method is employed to solve the new optimization problem.

Jaszkiewicz and Slowinski present an interactive procedure, called LBS-Discrete (Light Beam Search) for multiple-criteria analysis of decision problems with an explicitly given set of alternatives. The procedure is an extension of the Light Beam Search Method for linear and non-linear multiple-objective programming to the discrete alternative case.

Interactive Multiple Criteria Procedures: Some Reflections

Lorraine R. Gardiner
Department of Management
Lowder College of Business
Auburn University
Auburn, AL 36849 USA
e-mail: gardilr@mail.auburn.edu

Daniel Vanderpooten
Université Paris Dauphine
LAMSADE
Place du Maréchal De Lattre de Tassigny
75775 Paris Cedex 16, France
e-mail: vdp@lamsade.dauphine.fr

Abstract

This paper offers personal reflections on the evolution of interactive multiple criteria procedures. We trace their development from early roots in mathematical programming. During this early period, the theoretical and technical challenges of computing candidate solutions dominated the research. Later, major challenges arose from consideration of the behavioral aspects of decision processes. We further discuss what we consider to be current challenges and suggest some directions for future developments.

1 Introduction

Interactivity is used in many areas of operations research either to provide assistance in the modelling process or to guide the solution process (Pollack, 1976; Fischer, 1985). In the field of multiple criteria decision analysis, interactivity has been particularly emphasized in the solution process, giving rise to a specific and well-defined class of methods referred to as 'interactive multiple criteria procedures'. These are computer-aided procedures supporting a user[1] in his/her exploration of a set of decision alternatives in order to select one or a limited number of them. The general structure of an interactive procedure is that of an iterative algorithm which alternates:

- *dialog phases* where a proposal (usually consisting of one or several candidate alternatives) is presented to the user in order to obtain his/her reaction,
- *computational phases* where the reaction is analysed and used as a basis for generating a new proposal,

and includes a more or less formal *stopping condition* which indicates that the exploration can be stopped either successfully because a satisfactory proposal

[1] For the sake of simplicity, we will refer to a single user or decision maker. We appreciate that this may be a 'myth' (Zionts, 1994).

is reached or unsuccessfully for many possible reasons, including the weariness of the user.

Although the mathematical roots of multiple criteria procedures date back to the 1700s (Stadler, 1979), the preponderance of research has occurred since the late 1960s. Interactive procedures began to appear in the early 1970s (Benayoun *et al.*, 1970, 1971; Geoffrion *et al.*, 1972). Since that time, researchers have been quite active, producing hundreds of interactive procedures. Surveys of interactive multiple criteria procedures include Vanderpooten and Vincke (1989); Aksoy (1990); Lieberman (1991); Shin and Ravindran (1991).

Two perspectives have dominated the design of interactive procedures: *search-oriented* and *learning-oriented* (Vanderpooten, 1992). The perspective taken by a procedure's designer(s) strongly influences its assumptions and structural details. A *search-oriented* perspective assumes that the decision maker has an existing, unchanging preference structure which can be expressed with no contradictions (contradictions being considered in this perspective as 'inconsistencies'). An optimal solution based on this preference structure is 'waiting' to be found. The search orientation is closely related to the description of a parlor game by Roy (1987) in which a known story is 'discovered' by questioning a respondent who has chosen the story in advance. With a search-oriented perspective, the dialog and computational phases are controlled by the procedure in its efforts to converge to this optimal solution. The stopping condition is also controlled by the procedure with a stopping rule similar to those used in single objective optimization.

A *learning-oriented* perspective, on the other hand, does not assume the existence of a well-formed preference structure. Instead of searching for a solution obeying a strong form of rationality, this perspective emphasizes preference learning. It does, however, assume that the decision maker is motivated to progress toward a final, satisfactory decision. The decision process is assumed to be more evolutionary in nature, with changes of mind certainly a possibility. The analogy made by Roy (1987) is that of a story being 'constructed' within the framework of interaction between the game players (questioner and respondent). The dialog, computations and stopping condition are all controlled by the decision maker.

Vanderpooten (1992) proposed a *mixed perspective* that includes both learning and search stages and suggested that it may be more closely aligned with decision processes. This is similar to the dual modes of control suggested by Daellenbach and Buchanan (1989). A mixed perspective allows for the free exploration of a learning-oriented design within a framework that can be called upon to help guide the search.

In considering the development of interactive multiple criteria procedures over the past 25 years, it appears to us that the field has evolved from early roots in optimization. The search, learning and mixed design perspectives, themselves, reflect different stages of evolution in the field. In this paper, we trace the evolution of interactive multiple criteria procedures from the early years dominated by the theoretical and technical challenges of generating proposals (section 2), through later years when major challenges arose from the

consideration of decision processes (section 3), up to today when challenges to our field emerge from a desire to support real decision making (section 4). We share reflections on interactive multiple criteria procedures that we have individually accumulated over time. Our intention is to provoke discussion and thought about the purpose, design and evaluation of these procedures.

2 Early years: theoretical and technical challenges

Interactive multiple criteria procedures have their roots in mathematical programming, either in vector optimization or goal programming. Early theoretical work in vector optimization such as Koopmans (1951) and Kuhn and Tucker (1951) established efficient solutions as the set of interest. Later, researchers developed algorithms for multiple objective linear programming to generate all efficient extreme points and, in some cases, all maximally efficient facets. These algorithms include the procedures of Evans and Steuer (1973); Yu and Zeleny (1975); Gal (1977); Isermann (1977); Ecker and Kouada (1978). From computational experimentation, it became obvious that even the set of efficient extreme points is often extremely large. Consequently, attempts to describe the set of nondominated criterion vectors meaningfully to a decision maker would encounter great difficulty due to its size. In addition, the adequacy of efforts to elicit a decision maker's preference structure *a priori* and determine the corresponding most preferred solution (e.g., by constructing and optimizing an overall value function) seemed doubtful in many cases. Interactive multiple criteria procedures arose primarily in response to these difficulties.

In the early years of interactive multiple objective programming procedures, the major challenges were associated primarily with the computational phase. Research proceeded initially with multiple objective linear programs and, later, with other types of models. The dominant questions were: What mathematical programming formulations will guarantee the generation of nondominated (or at least weakly nondominated) alternatives? What mechanisms can be used to direct the search to a stopping point? In the following, we outline some of the major early developments. Due to space limitations, the outline is by no means comprehensive.

Prominent early mathematical programming formulations for generating nondominated alternatives include

- *weighted-sums*: Geoffrion *et al.* (1972); Belenson and Kapur (1973); Steuer (1977); Steuer and Schuler (1981); Zionts and Wallenius (1976, 1983),
- *ideal point*: Benayoun *et al.* (1971); Zeleny (1974); Roy (1976); Vincke (1976); Choo and Atkins (1980); Steuer and Choo (1983),

- *reference point projection*: Wierzbicki (1977, 1980); Nakayama and Sawaragi (1984),
- *ε-constraint*: Chankong and Haimes (1978); Sakawa (1982).

We listed the ideal point formulation separately above due to its distinct early emphasis. It is, however, clearly a member of the broader class of reference point projection formulations. More general theoretical descriptions of formulations for generating nondominated solutions can be found in Wierzbicki (1977, 1986); Soland (1979); Jahn (1985).

Early strategies for directing the search based on input from the user include the use of

- *criterion value constraints*: Benayoun *et al.* (1971); Chankong and Haimes (1978); Sakawa (1982),
- *reference point specification*: Wierzbicki (1977, 1980); Nakayama and Sawaragi (1984),
- *criterion cone reduction*: Steuer (1977),
- *weighting vector space reduction*: Belenson and Kapur (1973); Zionts and Wallenius (1976, 1983); Steuer and Schuler (1981); Steuer and Choo (1983),
- *local marginal indifference tradeoffs*: Geoffrion *et al.* (1972); Chankong and Haimes (1978); Sakawa (1982).

Interactive goal programming procedures evolved in response to difficulties in the *a priori* determination of target levels and their associated weights or priority levels. Early interactive goal programming procedures include Dyer (1972); Monarchi *et al.* (1973); Franz *et al.* (1981); Spronk (1981); Nakayama and Sawaragi (1984).

The early theoretical and technical developments resulted in sound foundations for the computational phase of interactive multiple criteria procedures. In particular, these developments guaranteed methods for generating solutions which exhibited certain properties that were considered important. Chief among these properties was nondominance : any solution generated should be nondominated and all nondominated solutions should be obtainable using the method (Wierzbicki, 1986).

Interactive procedures' birthplace in optimization strongly impacted their design philosophies. In the early years, the search-oriented design perspective dominated. Given an optimization orientation, emphasis on mathematical convergence is understandable. Almost certainly, the philosophies of operations research journals also encouraged the proof of mathematical convergence for interactive multiple criteria procedures. Open questioning of its importance in real problems came later. The dialog phase was also generally assumed to be troublefree. The cognitive ability of decision makers to absorb a method's output or provide the necessary input was evidently not

an early concern. In procedures where the search-oriented perspective dominated strongly, it can be argued that what was called interaction was, in large part, not really interaction but rather a one-sided interrogation for obtaining the information needed for the procedure's mathematical convergence.

3 Later years: challenges from consideration of decision processes

With the increasing maturation of the theoretical and technical aspects of the field, attention turned toward consideration of decision processes in the design of interactive multiple criteria procedures. Questions emerged (White, 1983; French, 1984) regarding the role of interactivity and the underlying assumptions of most interactive procedures. Comparative studies of interactive procedures did not tend to support search-oriented approaches (Wallenius, 1975; Buchanan and Daellenbach, 1987; Michalowski, 1987; Buchanan, 1994). In particular, comparative studies pointed to users' desire for control over the process. Search-oriented approaches allow very little user control. Researchers such as Larichev and Nikiforov (1987) investigated the cognitive difficulty of the types of tasks required of users by interactive multiple criteria procedures. Behavioral studies of human judgment and decision processes (Tversky, 1969; Kahneman and Tversky, 1979; Einhorn and Hogarth, 1981; Kahneman *et al.*, 1982; Kahneman and Tversky, 1984; Tversky and Simonson, 1993) began to permeate the interactive multiple criteria literature.

The cumulative result was a diminished willingness to allow the assumption of the existence of a value function to drive interactive procedures. Mathematical convergence began to receive a much decreased emphasis and, with it, search-oriented procedures. Mathematical convergence becomes irrelevant if one no longer views the decision maker as possessing a value function. There is no longer an optimal solution toward which the procedure can converge.

Learning-oriented procedures proliferated. In this regard, reference point methodology and philosophies (Wierzbicki, 1983) made a significant impact on the nature of interactive multiple criteria procedures. New procedures such as Nakayama and Sawaragi (1984); Korhonen and Laakso (1986); Korhonen and Wallenius (1988) emphasized flexibility and user control. Researchers performed studies of decision behavior in the interactive multiple criteria context (Korhonen *et al.*, 1990). Developers of interactive procedures began to consider behavioral issues explicitly in their design (Michalowski and Szapiro, 1992). In line with a mixed perspective, comparative studies (Brockhoff, 1985; Buchanan and Daellenbach, 1987) also indicated that a mixture of approaches might be desirable to users. Among others, Buchanan (1991); Antunes *et al.* (1992); Gardiner and Steuer (1994) proposed hybrid approaches.

Emphasis on decision processes also served to decrease the identification of interactive multiple criteria procedures with interactive multiple objective programming procedures. The distinction between mathematical program-

ming and discrete alternative models, while important from a computational point of view, is not important in the overall consideration of the use of interactivity in multiple criteria methodology. Interactivity becomes important for problems where the set of alternatives is so large that it cannot be easily grasped by the decision maker.

While some interactive procedures are especially designed for the mathematical programming case (Geoffrion *et al.*, 1972; Vincke, 1976) and others for discrete alternative models (Lévine and Pomerol, 1986; Korhonen, 1988; Lotfi *et al.*, 1992), many of the proposed procedures can be easily adapted to both cases. Even if technical details strongly influence the design of a procedure, the element to be emphasized is *interactivity.*

4 Current challenges: an application-oriented perspective

Research on interactive multiple criteria procedures has resulted in well-developed theoretical and technical foundations as well as an increased consideration of the psychological aspects of decision processes. While further progress can be made in both directions, we believe that the most important challenges are now to be found elsewhere.

Thanks to previous developments, it is now relatively easy to build an interactive multiple criteria procedure with satisfactory computational and dialog phases. A number of such procedures have been and are still being proposed in the literature. For all our wealth of interactive procedures, we are forced to acknowledge, however, that these tools are not part of the common practice of decision makers and managers (White, 1990). We must consider why this is the case. There are, of course, a number of hypotheses. Perhaps decision makers are not familiar with the tools and their potential benefits. On the other hand, perhaps the tools are not as beneficial as we would like to think. The challenge now is for us to investigate more thoroughly what happens when decision makers attempt to use the tools which are the result of 25 years of research. We are convinced that this endeavor will provide rich opportunities for future developments. In the remainder of this section, we indicate several directions which should be helpful in integrating interactive procedures into actual decision processes.

As a first direction, let us investigate how interactive multiple criteria procedures can be integrated into the current practice of decision makers as opposed to the imposition of new approaches. An excellent opportunity is offered by spreadsheets, since they are used widely and already provide a certain form of interactivity. Integrating an interactive multiple criteria procedure into a spreadsheet is quite natural and particularly easy (with the possibilities provided by most spreadsheets of using built-in or add-on mathematical programming solvers and a variety of graphical displays). Moreover, the possibility given to the user of mixing classical what-if analyses with interactive exploration of solutions adds a new dimension to the tool.

Another direction towards a better integration of interactive procedures comes from a real acknowledgement of the fact that the underlying multiple criteria model is nothing but a model. As such, it may (can) not capture all the aspects of the decision. Let us indicate two points here.

As discussed earlier in the paper, the ability to generate nondominated solutions is one of the theoretical backbones of interactive multiple criteria procedures. This is certainly a sound basis. As indicated by Haimes (1985); Zionts (1994), it is clear, however, that owing to the imprecision, uncertainty and even arbitrariness involved in the model (e.g., regarding the data, the exact meaning of criteria, measurement issues), dominated solutions may be good candidates. Similar ideas are considered explicitly in other multiple criteria approaches using thresholds (e.g., outranking methods). We would like to insist, however, that such arguments should not be used to support procedures unable to explore properly the set of nondominated solutions. We are interested here in giving the additional possibility of generating (slightly) dominated solutions, in particular, when a specific subregion seems to be of interest to the user.

The second point, which is partly related to the previous one, refers to the need often expressed by users to consider other features of a solution in addition to its criterion values. In a mathematical programming context, these features correspond to the value of some decision variables or of some combinations of decision variables. This illustrates the fact that all relevant aspects are usually not captured by the decision criteria. For example, consider a manufacturing setting where a solution may not be considered interesting even if satisfactory values are reached for all criteria because it involves a zero production level for one or several products. The ability to interact in decision space is made all the more important because there is not a one-to-one mapping between solutions in criterion space and their inverse images. The same solution in criterion space may correspond to very different solutions in decision space. Given these considerations, interaction should not be directed exclusively towards criterion values but also towards the structure of solutions. More generally, supporting interaction in decision space gives the opportunity to play with and change the model (e.g., by modifying, adding and/or removing constraints), which provides a dimension of 'interactive modelling' to the procedure.

Other issues of importance for the integration of interactive procedures in managerial practices are those of conviction and communication. The issue of conviction refers to the necessity of reaching a state where the user is convinced that the selected solution is the best one or at least among the best ones. We commented in the previous section on the illusive nature of the concept of mathematical convergence in this respect. Notice also that even if mathematical convergence were meaningful, it would not be really convincing to practitioners. Arguments of a practical nature must be provided to support the conviction that the final selection is satisfactory. These can be based on a sufficient level of evidence that there is no other solution that is significantly better. This evidence can be obtained when the user has the impression

that he/she has performed a sufficiently exhaustive exploration. This leads to the apparently paradoxical fact (with respect to a classical perspective of convergence of iterative procedures) that the procedure must induce the user to perform new explorations. In studies comparing interactive procedures (Wallenius, 1975; Michalowski, 1987; Buchanan, 1994), the median number of iterations performed by users is between three and eight. These values are surprisingly low and are perhaps due either to rapid convergence artificially imposed by the respective procedures or because the users did not know how to continue further. Rather than incorporating *converging* devices which narrow the exploration, it seems more useful for the purpose of conviction to include *diverging* devices inviting the user to explore other solutions.

The issue of communication is certainly a major challenge for which innovative research is needed. Even if the user is convinced that the final solution is satisfactory, it is important in some decision contexts to provide support for sharing this conviction with others. In this respect, it might be interesting to design *a posteriori* analyses aiming at answering questions like:

- Can we derive information from the exploration performed by the user?
- Can we establish specific features or formal properties characterizing the solution finally retained?

5 Concluding remarks

As any newcomer to the field can attest, the number of interactive multiple criteria procedures is overwhelmingly large. There are numerous possibilities in developing interactive procedures which combine different dialog and computational components. This is sometimes mistakenly confused with technical challenge. At this point, offering new methods is easy from a combinatorial point of view. We need to resist the urge to create new methods for the sake of it and focus instead on the needs of real people with real decision problems. It is time to pay more attention to applying the methodology and reporting both successes and failures. In this paper, we have tried to show that new exciting challenges, both practical and theoretical, will naturally derive from an application- oriented perspective. This is in many ways a call to return to the early roots of operations research itself, when real problems drove the research.

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A Method for Solving Multiple Objective Linear Programming Problems

Leonid M. Kirilov and Vassil S. Vassilev *

Institute of Information Technologies – Bulgarian Academy of Sciences
Acad. G. Bonchev St., bl. 29A
1113 Sofia, BULGARIA

Abstract: An interactive method is presented for solving multiple objective linear programming problems. The method develops an idea for successive reduction of the set of normalized weighting coefficients. A set of Pareto efficient solutions is generated at each iteration. The dialog is in terms of aspiration levels in the objective space. A theoretical comparison with other related methods is done.

1. INTRODUCTION

In multiple objective linear programming (MOLP) problem, the Decision Maker (DM) attempts to maximize (or to minimize, or both) more than one objective function. Without loss of generality, MOLP can be formulated as:

$$\text{"max"}_{X} \quad \{ (c^i, x) = \sum_{j=1}^{n} c_j^i \, x_j, \;\; i=1, 2, \ldots, k \}, \qquad (1.1)$$

where the feasible set X is a convex polygon, defined by
$X = \{x \in R^n | \, Ax \le b, \, x \ge \mathbf{0} \}$, A is a matrix of dimension (m, n), $b \in R^m$, $\mathbf{0} \in R^n$.

Usually at least two objectives are conflicting. For this reason a unique solution of the problem (1.1) rarely exists. Therefore, the DM chooses a solution among the set of efficient solutions.

We will use the traditional conception of Pareto efficiency:
Definition. A point x^e is (Pareto) efficient iff there does not exist another $x' \in X$ such that $(c^i, x') \ge (c^i, x^e)$ for $i = 1,2,\ldots,k$ and $(c^i, x') > (c^i, x^e)$ for at least one $i \in \{1,2,\ldots,k\}$.

Further the set of normalized weighting vectors will be used. It can be defined as:

$$\Lambda = \{\alpha \in R^k | \; \sum_{i=1}^{k} \alpha_i = 1, \, \alpha_i \ge 0, \, i = 1, 2, \ldots, k\} \qquad (1.2)$$

The rest of the paper is organized as follows. We give the general idea of the proposed method in section 2. An algorithmic scheme is presented in section 3. A discussion and theoretical comparison with other related methods are done in section 4. We give some final remarks in the conclusion.

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2. GENERAL DESCRIPTION OF THE METHOD

A number of interactive methods exist for solving MOLP problems. They differ in the approaches used. For example, the generating of the solution(s) at an iteration can be done by reduction of the decision space, or the objective space, or the set of normalized weighting vectors. This can also be realized by searching along the direction of DM's preferences increase and so on. Without having the possibility for full representation (and this is not the purpose of this paper) we can mention some typical methods: interactive surrogate worth trade-off method of Chankong and Haimes (1978), method of Geoffrion-Dyer-Feinberg (1972), the reference point approach of Wierzbicki (1980), the visual interactive approach of Korhonen and Laakso (1986), the interactive weighted Tchebycheff procedure of Steuer and Choo (1983) and Steuer (1986), method of Zionts and Wallenius (1976, 1983). An overview of existing approaches and methods can be found for example in Evans (1984), Zionts (1988), Shin and Ravindran (1991).

The method presented here is developed on the base of results, described in Kirilov (1992). It follows the idea of successive reduction of the set of normalized weighting vectors Λ. In this connection some of its main characteristics can be outlined:

1. The DM expresses his preferences in terms of reference points in the objective space - Wierzbicki (1983), Nakayama (1985). The coordinates of the reference point have the meaning of aspiration levels for the separate objectives, i.e. every solution dominating the reference point (in the objective space) is satisfactory.
2. If we denote the current set of weights as Λ^h at iteration h, then the new set Λ^{h+1} is formed taking into account the reference point set by the DM.
3. In order to generate the efficient point(s) at each iteration an appropriate augmented Tchebycheff problem is used.

In summary, the method consists of the following steps:

1) Set $h = 0$ and $\Lambda^h = \Lambda$. Generate an initial set of efficient points;
2) The DM evaluates the efficient set presented. If at least one point is satisfactory – stop. Otherwise, the DM gives his aspirations in terms of a reference point in the objective space;
3) Set $h = h+1$. Perform a suitable reduction and translation of Λ^{h-1} according to the DM's preferences to form a new set Λ^h;
4) Generate a new set of efficient points using the new set Λ^h. Go to step 2.

3. THE ALGORITHMIC SCHEME

The method has the following form in details:

Step 1 The DM gives the maximal number of iterations IT for which he/she wishes to investigate problem (1.1). If it is difficult to set we recommend $k \leq \text{IT} \leq 2k$, where k is the number of objectives.

Step 2 The following numbers are computed:

a) $h = 0$ – an iteration counter;

b) $u_i^0 = 1$, $l_i^0 = 0$, i=1, 2, ... , k – upper and lower bounds of element coordinate exchanges of the set Λ.

c) $d_0 = u_1^0 - l_1^0$ – exchange interval width.

d) EPS = machine precision.

e) $R = 1 - \text{EPS}^{1/\text{IT}}$ – a reduction factor.

f) rescale (normalize) the objective functions if necessary.

Step 3 A number of k single criterion (SC) problems are solved. They have the form:

$$\max (c^i, x)$$

s.t.

$$Ax \le b \qquad (2.1)$$
$$x \ge 0$$
$$x \in R^n,$$

for i=1, 2, ... , k.

Let x^{i0}, i=1, 2,..., k are their optimal solutions. The ideal vector f^* is computed. It has the components:

$$f_i^* = (c^i, x^{i0}),\ i=1, 2, \dots, k \qquad (2.2)$$

and the point f^{**} with components:

$$f_i^{**} = f_i^* + \mu_i,\ i=1, 2, \dots, k \qquad (2.3)$$

where $\mu_i \in (0, 1)$.

Set parameter $l = k$ (2.4)

Step 4 A set of vectors is computed in the objective space by the generated set of points $\{x^{ih}\}$, i=1, 2, ... , k from the argument space R^n:

$$F^h = \{f^{ih},\ i=1, 2,..., l\} = \{(c^j, x^{ih}); j=1, 2,..., k,\ i=1, 2,..., l\} \qquad (2.5)$$

Step 5 (dialog) The DM evaluates every vector $f^{ih} = ((c^1, x^{ih}), \dots, (c^k, x^{ih}))$, i = 1, 2, ..., l. If any of them is/are acceptable for him, go to step 11. Otherwise, the DM sets (according to his/her preferences) a reference point f^r in the objective space with coordinates

$$f_i^r \le f_i^*,\ i=1, 2,..., k.$$

Compute the weighting vector:

$$\alpha^r = (1/(f_1^{**} - f_1^r)T^{-1}, \dots, 1/(f_k^{**} - f_k^r)T^{-1}) \qquad (2.6)$$

where $T = \left(\sum_{i=1}^{k} (1/(f_i^{**} - f_i^r))\right)$.

If d_h > EPS, Go to step 6. Otherwise, Go to step 7.

Step 6 (reduction) Set the iteration counter $h = h+1$. The following numbers are computed:

$$l_i^h = l_i^{h-1} + (1/k)\, d_{h-1}\, R \qquad (2.7)$$
$$u_i^h = u_i^{h-1} - ((k-1)/k)\, d_{h-1}\, R$$
$$\text{for } i = 1,\ 2, \dots, k.$$

u_i^h and l_i^h are the upper and lower bounds of the element coordinate exchanges of the new set Λ^h respectively.

$$d_h = u_1^h - l_1^h \text{ - exchange interval width.} \tag{2.8}$$

Compute gravity center of Λ^h:

$$\alpha^{ch} = ((1/k)(u_1^h + (k-1)l_1^h), \ldots, (1/k)(u_k^h + (k-1)l_k^h)). \tag{2.9}$$

Step 7 A parameter t of the maximal feasible moving is defined following the scheme:

1. Compute
   $t = 1$; $i = 1$;
   $D^r = \alpha^r - \alpha^{ch} \in R^k$.
2. If $D_i^h > 0$, Go to 5.
3. If $D_i^r < 0$, Go to 6.
4. If $D_i^r = 0$, Go to 8.
5. If $u_i^h + tD_i^r > 1$, then $t_1 = (1 - u_i^h)/D_i^r$; Go to 7.
   else Goto 8.
6. If $l_i^h + tD_i^r < 0$, then $t_1 = -l_i^h/D_i^r$; Go to 7.
   else Go to 8.
7. $t := \min(t, t_1)$
8. $i = i + 1$
   If $i \le k$ Go to 2.
   else Go to 9.
9. stop – t is a parameter of maximal feasible moving.

Step 8 (translation of Λ^h in direction $D^r = \alpha^r - \alpha^{ch}$)

$$\begin{aligned} L_i^h &= l_i^h + tD_i^r \\ l_i^h &= L_i^h \\ U_i^h &= u_i^h + tD_i^r \\ u_i^h &= U_i^h \end{aligned} \tag{2.10}$$

for $i = 1, 2, \ldots, k$. The parameter t from step 7 assures that Λ^h is still in Λ after performing step 8.

Step 9 A number of $k+1$ weighting coefficients are computed following the formula:

$$\alpha^{ih} = (l_1^h, l_2^h, \ldots, u_i^h, \ldots, l_k^h) \text{ for } i = 1, 2, \ldots, k \tag{2.11}$$

(these are the extreme points of Λ^h).

$\alpha^{k+1\,h} = \alpha^r$

Step 10 A number of $k+1$ SC problems are solved for $i = 1, 2, \ldots, k+1$. They have the form:

$$\min \{ y + \tau \sum_{j=1}^{k} (f_j^{**} - (c^j, x)) \} \tag{2.12}$$

s.t.

$$\alpha_j^{ih} (f_j^{**} - (c^j, x)) \le y, \; j = 1, 2, \ldots, k$$
$$Ax \le b$$
$$x \ge 0$$

where $x \in R^n$, $y \in R^1$, τ is a small positive number. Usually, $\tau \in [0.0001, 0.01]$

Let x^{ih}, i =1, 2, ... , k+1 are their optimal solutions. Set parameter $l = k+1$ and go to step 4.

Step 11 End of the algorithm. The compromise solution is chosen by the DM at step 5.

The algorithm is presented for MOLP problems in which all the objectives have to be simultaneously maximized. It can be also applied without any difficulties for problems in which some of the objectives have to be minimized.

4. DISCUSSION

As it can be seen from step 9 (see Section 2), the method operates mainly with the extreme points of the old set Λ^{h-1} to generate a new set Λ^h. Therefore it is called an Extreme Points Method (EPM). Of course this does not mean that the method produces only extreme efficient points (see the augmented Tchebycheff problems in step 10).

In step 1 of the algorithm the DM sets the maximal number of iterations IT for which (s)he wants to investigate problem (1.1). In some cases this can be unrealistic for the DM. Therefore the algorithm does not force the conclusion of the interactive process in some prespecified number of iterations. The DM decides when to stop the problem solution.

A problem (2.12) is used in step 10 since it is shown in Kirilov (1992) that such augmented Tchebycheff problem with the weighting vectors computed as in (2.6) and (2.11) satisfies the general requirements for scalarizing functions described in Sawaragi, Nakayama, Tanino (1985)). Namely:

1) Every efficient point of problem (1.1) can be optimal solution to the problem (2.12) for suitable chosen parameters, i.e. the problem (2.12) can cover all Pareto solutions;

2) The optimal solutions of (2.12) are efficient ones;

3) If the reference point is feasible, then at least one solution generated in step 10 is satisfactory.

In order to see the place of the EPM method among the other related methods, we will use the classification of Shin and Ravindran (1991) according to the following criteria: *interaction style*, *solution approach*, *applicability* and *mathematical programming required.*

Interaction style.

The interaction style is in terms of vector comparison and setting reference points in the objective space. According to many authors this kind of a dialog is preferred by most DM's – Wierzbicki (1985), Nakayama (1985), Korhonen, Moskowitz, Wallenius (1990).

Solution approach.

We will consider this criterion in more details. The solution approaches (in particular for solving MOLP problems) can be classified into two distinct groups: the point assessment methods and the interval reduction methods (see Shin and Ravindran (1991)).

We can name the methods of Steuer and Choo (1983) (Interactive Weighted Tchebycheff Procedure (IWTP)) and Steuer and Wood (1986) (Interactive Weighted-Sums/Filtering Approach (IWS)) as examples of methods close to the EPM, that use interval reduction approach as well. The two methods reduce the set Λ in a finite number of steps. Generating randomly the weights, a representative subset of the current set Λ^h is generated at each iteration. After filtering the weights, one Tchebycheff scalarizing problem (or weighted sums in case of IWS) is solved for each remaining weighting vector. After filtering the computed efficient points are reduced to the number defined by the DM. The cluster received is then evaluated and the best preferred solution is chosen. After that a reduction of the set Λ with a center – the weighting vector corresponding to the chosen solution is performed and so on. These methods have error-correcting capability. But areas of set Λ (resp. areas of efficient frontier) exist that cannot be generated in case of essential change of DM's aspirations. In these cases, a restart is advised. The methods require also considerable CPU time. In addition, IWS method cannot generate nonextreme efficient points.

The method Z–W of Zionts and Wallenius (1983) also reduces the set Λ. The difference is that the set Λ^h in *Z–W* method is described by a system of linear equations (the so called EPS-constraints) and its dimension grows when increasing the iteration number. The method cannot generate nonextreme efficient points.

In comparison with ITP and IWS methods, it is not necessary to generate a representative set of efficient points in EPM because of its flexibility. The new efficient sample (resp. reduced set Λ^h) can be located anywhere in the efficient frontier (resp. set Λ) according to the DM's preferences. In this way the EPM method does not require considerable CPU time.

Another related method is the Combined Tchebycheff/Aspiration Criterion Vector Interactive Multiobjective Programming Procedure of Steuer, Silverman and Whisman (1993). In general, this method is an efficient combining of two methods: IWTP – Steuer (1986) and Reference Point Approach – Wierzbicki (1983) as they are originally formulated, i.e. without changing their internal logics (philosophy). In the EPM (if we consider it as a continuation of the IWTP method) the reduction of the set Λ is performed according to the current reference point, converted suitably into the weigting vector.

On the other hand, in relation to the point assessment methods, the EPM method can also be considered as a variant of the reference point approach of Wierzbicki (1983). Other examples of the point assessment methods are the VIG – Korhonen and Laakso (1986) and Pareto Race – Korhonen and Wallenius (1988) methods. At each iteration in the EPM, a number of efficient points is presented to the DM according to his preferences: one "central" point f^m and k additional points

f^i, i=1, 2, ..., k around it. Thus the DM can evaluate how the objectives change in the neighbourhood of the point f^m and in this way he is supported in selecting the next reference point.

Applicability.

The EPM method is designed to solve multiple objective linear programming problems.

Mathematical programming required.

Any linear programming solution method can be used for solving SC problems (2.12).

5. CONCLUSION

An interactive method for solving multiple objective linear programing problems is suggested in the paper. The method has the following main characteristics: 1) easy dialog in terms of aspiration levels; 2) successive reduction and translation of the set of normalized weighting vectors according to the DM's preferences; 3) no special linear programming solution method is required. Having in mind properties 1) and 2) we can say that the method proposed combines the features of the point assessment approach and the interval reduction approach.

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Interactive Method NIMBUS for Nondifferentiable Multiobjective Optimization Problems

Kaisa Miettinen and Marko M. Mäkelä

Department of Mathematics, University of Jyväskylä, P.O. Box 35,
FIN-40351 Jyväskylä, Finland

Abstract. An interactive method, NIMBUS, for nondifferentiable multiobjective optimization problems is introduced. The method is capable of handling several nonconvex locally Lipschitzian objective functions subject to nonlinear (possibly nondifferentiable) constraints. The idea of NIMBUS is that the decision maker can easily indicate what kind of improvements are desired and what kind of impairments are tolerable at the point considered.

The decision maker is asked to classify the objective functions into five different classes: those to be improved, those to be improved down to some aspiration level, those to be accepted as they are, those to be impaired till some upper bound, and those allowed to change freely. A new problem is formed according to this classification.

An MPB (Multiobjective Proximal Bundle) method is employed to solve the new (multiobjective) optimization problem. The MPB method is a generalization of a proximal bundle approach for nondifferentiable single objective optimization into the multiobjective case. The objective functions are treated with the help of an improvement function.

Finally, numerical experiments with an academic test problem are reported. The aim is to briefly demonstrate how the method works.

1 Introduction

During the last decades, problems with several objective functions have been widely studied. However, considerably little attention has been paid to such multiobjective optimization problems where some of the functions involved are nondifferentiable. Thus, there is still room for interactive methods for nondifferentiable multiobjective optimization.

Earlier, we have described a method based on the well-known GDF method, called a subgradient GDF method, in [7]. In this paper, we present a method called NIMBUS (see also [6] and [8]). It has been aimed to be easy to use. No complicated inputs, such as marginal rates of substitution, are required from the decision maker.

The decision maker is asked to classify the objective functions into different classes according to her/his desires. After the classification, a new multiobjective optimization problem is formed. This new problem is solved by an MPB routine, which produces weakly Pareto optimal solutions. The values of all the current objective functions are decreased as much as possible. In Section 2, we state some concepts and notations needed. In Section 3, we sketch the basic ideas of the NIMBUS method, and the detailed NIMBUS algorithm is presented in Section 4. The MPB routine is introduced in Section 5 in general outline, and Section 6 illustrates the functioning of the NIMBUS method by a simple numerical example. Lastly, some conclusions are drawn in Section 7.

2 Concepts and Notations

We study a multiobjective optimization problem of the form

$$\begin{array}{ll} \text{minimize} & \{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_k(\mathbf{x})\} \\ \text{subject to} & \mathbf{x} \in S, \end{array} \tag{2.1}$$

where we have k *objective functions* $f_i\colon \mathbf{R}^n \to \mathbf{R}$. The *decision vector* $\mathbf{x}$ belongs to the (nonempty) feasible set $S = \{\mathbf{x} \in \mathbf{R}^n \mid (g_1(\mathbf{x}), g_2(\mathbf{x}), \ldots, g_m(\mathbf{x}))^T \leq \mathbf{0}\}$. The word "minimize" means that we want to minimize all the objective functions simultaneously. To avoid trivial cases, we suppose that there does not exist a single solution which is optimal with respect to every objective function.

In the following, we denote the image of the feasible set by $Z \subset \mathbf{R}^k$. The elements of Z are called *criterion vectors* and denoted by $\mathbf{z} = (z_1, z_2, \ldots, z_k)^T$, where $z_i = f_i(\mathbf{x})$.

As the concepts of optimality we employ Pareto optimality and weak Pareto optimality. We denote an open ball around a point $\mathbf{x}^*$ with radius $\delta > 0$ by $B(\mathbf{x}^*, \delta)$.

Definition 2.1. *A decision vector $\mathbf{x}^* \in S$ is locally Pareto optimal if there does not exist another decision vector $\mathbf{x} \in S \cap B(\mathbf{x}^*, \delta)$ (for certain $\delta > 0$) such that $f_i(\mathbf{x}) \leq f_i(\mathbf{x}^*)$ for all $i = 1, \ldots, k$ and $f_j(\mathbf{x}) < f_j(\mathbf{x}^*)$ for at least one objective function f_j.*

A decision vector $\mathbf{x}^* \in S$ is locally weakly Pareto optimal if in the definition above the inequalities are all strict.

A criterion vector $\mathbf{z}^* \in Z$ is (weakly) locally Pareto optimal if the feasible decision vector corresponding to it is (weakly) locally Pareto optimal.

Because of the possible nonconvexity of the problem (2.1), (weak) Pareto optimality is to be understood in the local sense in what follows. We assume that we have a *single decision maker* or a unanimous group of decision

makers. By solving a multiobjective optimization problem we here mean finding a feasible decision vector such that it is Pareto optimal and satisfies the needs and requirements of the decision maker. Such a solution is called a *final solution*.

Next, we state some concepts of nondifferentiable analysis.

Definition 2.2. *A function* $f\colon \mathbf{R}^n \to \mathbf{R}$ *is locally Lipschitzian at a point* $\mathbf{x}^* \in \mathbf{R}^n$ *if there exist scalars* $K > 0$ *and* $\delta > 0$ *such that*

$$|f(\mathbf{x}^1) - f(\mathbf{x}^2)| \leq K\|\mathbf{x}^1 - \mathbf{x}^2\| \quad \text{for all } \mathbf{x}^1, \mathbf{x}^2 \in B(\mathbf{x}^*, \delta).$$

In the sequel, we employ the concept of a subdifferential as defined by Clarke in [1]. It corresponds to the gradient in a differentiable case.

Definition 2.3. *Let the function* $f\colon \mathbf{R}^n \to \mathbf{R}$ *be locally Lipschitzian at the point* $\mathbf{x}^* \in \mathbf{R}^n$. *The set*

$$\partial f(\mathbf{x}^*) = conv\{\xi \in \mathbf{R}^n \mid \xi = \lim_{l\to\infty} \nabla f(\mathbf{x}^l);\ \mathbf{x}^l \to \mathbf{x}^*,\ \nabla f(\mathbf{x}^l)\ exists\},$$

where conv denotes a convex hull, is called a subdifferential of the function f *evaluated at the point* $\mathbf{x}^*$. *In addition, the vectors* $\xi \in \partial f(\mathbf{x}^*)$ *are called subgradients.*

3 NIMBUS Method

The NIMBUS method is an interactive multiobjective optimization method designed to be able to handle nondifferentiable functions efficiently. The interaction phase is aimed at being comparatively simple and easy to understand for the decision maker.

The assumptions in the NIMBUS method are

- All the objective and the constraint functions are locally Lipschitzian.
- The feasible region S is convex.
- Less is preferred to more to the decision maker.

Notice that continuously differentiable functions are always locally Lipschitzian.

The idea is that the decision maker examines the values of the objective functions calculated at a point $\mathbf{x}^h$ at the iteration h and divides the objective functions into up to five classes. Those classes are functions f_i whose values should be decreased ($i \in I^<$), functions whose values should be decreased down till some aspiration level ($i \in I^\leq$), functions whose values are satisfactory at the moment ($i \in I^=$), functions whose values are allowed to increase up till some upper bound ($i \in I^>$), and functions whose values are allowed to change freely ($i \in I^\diamond$).

The decision maker is asked to specify the aspiration levels $\bar{z}_i$ for $i \in I^{\leq}$ and the upper bounds ε_i for $i \in I^{>}$. The difference between the classes $I^{<}$ and $I^{\leq}$ is that the functions in $I^{<}$ are to be minimized as far as possible but the functions in $I^{\leq}$ only till the aspiration level.

The classification is the core of NIMBUS. However, the decision maker can tune the order of importance inside the classes $I^{<}$ and $I^{\leq}$ with optional positive weighting coefficients w_i. Notice that the weighting coefficients do not change the primary orientation specified in the classification phase.

As a method NIMBUS is quite general. The classification part of the objective functions can be performed as if the ε-constraint method, the weighting method, lexicographic ordering or goal programming were used to produce weakly Pareto optimal solutions.

According to the classification and the connected information, we form a new problem

$$
\begin{aligned}
&\text{minimize} && \left\{ w_i f_i(\mathbf{x}) \ (i \in I^{<}),\ \max_{j \in I^{\leq}} \left[w_j \max \left[f_j(\mathbf{x}) - \bar{z}_j,\ 0 \right] \right] \right\} \\
&\text{subject to} && f_i(\mathbf{x}) \leq f_i(\mathbf{x}^h), \quad i \in I^{=} \\
&\text{(3.1)} && f_i(\mathbf{x}) \leq \varepsilon_i, \qquad i \in I^{>} \\
& && \mathbf{x} \in S, \\
&\text{where} && \sum_{i \in I^{<} \cup I^{\leq}} w_i = 1 \quad \text{and} \quad w_i > 0,\ i \in I^{<} \cup I^{\leq}.
\end{aligned}
$$

The aspiration levels $\bar{z}_i$ must satisfy the condition $\bar{z}_i < f_i(\mathbf{x}^h)$ for $i \in I^{\leq}$, and the upper bounds ε_i must satisfy the condition $\varepsilon_i > f_i(\mathbf{x}^h)$ for $i \in I^{>}$.

Notice that the aspiration functions are handled such as in goal programming. The distance is, however, measured with the L_∞-metric instead of the more common L_1-metric because nondifferentiable functions cause no additional problems here. The L_1-metric could be employed as well.

When the problem (3.1) is solved (employing the MPB routine), a solution, denoted by $\hat{\mathbf{x}}^h$, is obtained. If the decision maker dislike it for some reason, no dead end is at hand. (S)he can explore solutions between $\mathbf{x}^h$ and $\hat{\mathbf{x}}^h$. This means that we form a search direction $\mathbf{d}^h = \hat{\mathbf{x}}^h - \mathbf{x}^h$.

The step-size is determined by the decision maker by comparing different alternatives. Criterion vectors $(f_1(\mathbf{x}^h + t\mathbf{d}^h), \ldots, f_k(\mathbf{x}^h + t\mathbf{d}^h))^T$ are calculated with different values of $0 \leq t \leq 1$. Then weakly Pareto optimal solutions are produced from them (through MPB) and presented to the decision maker for selection.

4 NIMBUS Algorithm

Here we describe the NIMBUS algorithm in detail. Because NIMBUS contains so many different possibilities, the detailed algorithm becomes quite long and somewhat obscure. However, using NIMBUS is much simpler for the decision maker than one could assume when looking at the algorithm.

(1) Choose a starting point $\mathbf{x}^0 \in S$ and calculate its weakly Pareto optimal counterpart $\mathbf{x}^1 \in S$ and the corresponding criterion vector $\mathbf{z}^1 \in Z$ employing MPB with $I^< = \{1, \ldots, k\}$. Set the iteration counter $h = 1$.

(2) Ask the decision maker to divide the objective functions into the classes $I^<$, $I^\leq$, $I^=$, $I^>$, and $I^\diamond$ at the point $\mathbf{z}^h$ such that $I^> \cup I^\diamond \neq \emptyset$ and $I^< \cup I^\leq \neq \emptyset$. If either of the unions is empty, go to step (9). Ask the aspiration levels $\bar{z}_i^h$ for $i \in I^\leq$ and the upper bounds ε_i^h for $i \in I^>$ of the decision maker. Ask also the possible positive weighting coefficients w_i^h for $i \in I^< \cup I^\leq$ summing up to one.

(3) Calculate $\hat{\mathbf{x}}^h$ by solving the problem (3.1) by the MPB routine. If $\hat{\mathbf{x}}^h = \mathbf{x}^h$, ask the decision maker whether (s)he wants to try another classification. If yes, set $\mathbf{x}^{h+1} = \mathbf{x}^h$, $h = h + 1$ and go to step (2). If no, go to step (9).

(4) Let $\hat{\mathbf{z}}^h$ be the criterion vector corresponding to $\hat{\mathbf{x}}^h$. Present $\mathbf{z}^h$ and $\hat{\mathbf{z}}^h$ to the decision maker. If the decision maker wants to see different alternatives between $\mathbf{z}^h$ and $\hat{\mathbf{z}}^h$, set $\mathbf{d}^h = \hat{\mathbf{x}}^h - \mathbf{x}^h$ and go to step (6). If the decision maker prefers $\mathbf{z}^h$, set $\mathbf{x}^{h+1} = \mathbf{x}^h$, $h = h + 1$ and to go step (2).

(5) Now the decision maker wants to continue from $\hat{\mathbf{z}}^h$. If $I^< \neq \emptyset$ set $\mathbf{x}^{h+1} = \hat{\mathbf{x}}^h$, $h = h + 1$ and go to step (2). Otherwise ($I^< = \emptyset$), the weak Pareto optimality must be checked, setting $I^< = \{1, \ldots, k\}$ and employing MPB. Let the solution be $\check{\mathbf{x}}^h$. Set $\mathbf{x}^{h+1} = \check{\mathbf{x}}^h$, $h = h+1$ and go to step (2).

(6) Calculate $P-2$ new criterion vectors $(f_1(\mathbf{x}^h + t_j \mathbf{d}^h), \ldots, f_k(\mathbf{x}^h + t_j \mathbf{d}^h))^T$, where $t_j = \left(\frac{j-1}{P-1}\right)$, $j = 2 \ldots, P-1$.

(7) Produce weakly Pareto optimal solutions from the criterion vectors, employing MPB (with $I^< = \{1, \ldots, k\}$).

(8) Present P alternative criterion vectors to the decision maker and let her or him choose the most preferred one among them. Denote the corresponding decision vector by $\mathbf{x}^{h+1}$ and set $h = h+1$. If the decision maker wants to continue, go to step (2).

(9) Check the Pareto optimality of $\mathbf{x}^h$ by solving the auxiliary problem (4.1). Let the solution be $(\tilde{\mathbf{x}}, \tilde{\boldsymbol{\mu}})$. Stop. The final solution is $\tilde{\mathbf{x}}$ with the corresponding $\tilde{\mathbf{z}}$.

Because the solutions produced by the MPB routine may be weakly Pareto optimal, it is good to make sure that the final solution is Pareto optimal. In the last step, the Pareto optimality is guaranteed by solving an additional

problem like

$$
\begin{aligned}
& \text{maximize} && \sum_{i=1}^{k} \mu_i \\
& \text{subject to} && f_i(\mathbf{x}) + \mu_i \leq f_i(\mathbf{x}^h) \quad \text{for all} \quad i = 1, \ldots, k, \\
& && \mu_i \geq 0 \quad \text{for all} \quad i = 1, \ldots, k, \\
& && \mathbf{x} \in S,
\end{aligned} \tag{4.1}
$$

with respect to $\mathbf{x} \in \mathbf{R}^n$ and $\mu_i \in \mathbf{R}$ for $i = 1, \ldots k$. If $\mathbf{x}^h$ is not Pareto optimal, then the solution $\tilde{\mathbf{x}}$ of the additional problem is. For clarity of notations, it has not been mentioned in the algorithm that the decision maker may check the Pareto optimality of any current solution.

Notice that the decision maker must be ready to give up something in order to attain improvement for some other objective functions. The search procedure stops if the decision maker does not want to improve any criterion value.

We must remember that the best we can guarantee is a local optimum. Because of its structure, the solution of the MPB method is quite sensitive with respect to the starting point. Then, if the solution obtained is not completely satisfactory, one can always solve the problem again from a different starting point. It is also advised if the decision maker has to stop the search process after the step (3).

5 MPB Routine

Here we briefly sketch the MPB (Multiobjective Proximal Bundle) method. For details, see [4]. The most advanced version of the bundle family for nondifferentiable single objective optimization, namely, the proximal bundle method, is presented in [3]. It has been generalized to handle nonconvex and constrained problems in [5]. The strategy of handling several objective functions is based on the ideas presented in [10].

Although the MPB routine is employed in the NIMBUS algorithm to solve the problem (3.1), we describe it for a more general problem (2.1) for notational simplicity. Theoretically, we produce solution candidates with the help of an *improvement function* $H: \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$ defined by

$$H(\mathbf{x}; \mathbf{y}) = \max \{f_i(\mathbf{x}) - f_i(\mathbf{y}),\ g_l(\mathbf{x}) \mid i = 1, \ldots, k,\ l = 1, \ldots, m\}.$$

It can be proved (see [8]) that a necessary condition for a point $\mathbf{x}^*$ to be a weakly Pareto optimal solution of the problem (2.1) is that $\mathbf{x}^*$ minimizes the function $H(\cdot\,; \mathbf{x}^*)$. If the problem is convex, the condition is also sufficient.

Because optimality cannot be guaranteed easily in nonconvex cases, we do not want to waste computational capacity in assuring the Pareto optimality

of each alternative. Instead, the Pareto optimality is checked only at the end of the algorithm (and whenever the decision maker so desires). Notice once again that optimality is local.

In the following, the iteration counter s refers to the inner iterations of the MPB routine. Let $\mathbf{x}^s$ be the current approximation to the solution of (2.1) at the iteration s. Then we seek for the search direction $\mathbf{d}^s$ as a solution of the unconstrained optimization problem

$$\begin{aligned} &\text{minimize} && H(\mathbf{x}^s + \mathbf{d}; \mathbf{x}^s) \\ &\text{subject to} && \mathbf{d} \in \mathbf{R}^n. \end{aligned} \tag{5.1}$$

Since (5.1) is a nondifferentiable problem, we must approximate it somehow. Let us assume for a moment that the problem is convex. We suppose that at the iteration s besides the iteration point $\mathbf{x}^s$, we have some auxiliary points $\mathbf{y}^j \in \mathbf{R}^n$ from the past iterations and subgradients $\xi_{f_i}^j \in \partial f_i(\mathbf{y}^j)$ for $j \in J^s$, $i = 1, \ldots, k$ and $\xi_{g_l}^j \in \partial g_l(\mathbf{y}^j)$ for $j \in J^s$, $l = 1, \ldots, m$, where J^s is a nonempty subset of $\{1, \ldots, s\}$. We linearize the objective and the constraint functions at the point $\mathbf{y}^j$ by

$$\bar{f}_{i,j}(\mathbf{x}) := f_i(\mathbf{y}^j) + (\xi_{f_i}^j)^T(\mathbf{x} - \mathbf{y}^j) \quad \text{for all} \quad i = 1, \ldots, k, \ j \in J^s \quad \text{and}$$

$$\bar{g}_{l,j}(\mathbf{x}) := g_l(\mathbf{y}^j) + (\xi_{g_l}^j)^T(\mathbf{x} - \mathbf{y}^j) \quad \text{for all} \quad l = 1, \ldots, m, \ j \in J^s.$$

Now we can define a convex piecewise linear approximation to the improvement function by

$$\hat{H}^s(\mathbf{x}) := \max\{\bar{f}_{i,j}(\mathbf{x}) - f_i(\mathbf{x}^s), \ \bar{g}_{l,j}(\mathbf{x}) \mid i = 1, \ldots, k, \ l = 1, \ldots, m, \ j \in J^s\}$$

and we get an approximation to (5.1) by

$$\begin{aligned} &\text{minimize} && \hat{H}^s(\mathbf{x}^s + \mathbf{d}) + \tfrac{1}{2}u^s\|\mathbf{d}\|^2 \\ &\text{subject to} && \mathbf{d} \in \mathbf{R}^n, \end{aligned} \tag{5.2}$$

where $u^s > 0$ is some weighting parameter. The penalty term $\frac{1}{2}u^s\|\mathbf{d}\|^2$ is added to guarantee the existence of the solution of (5.2) and to keep the approximation local enough. Notice that (5.2) is still a nondifferentiable problem, but due to its minimax-nature it is equivalent to a certain (differentiable) quadratic problem (see [8]). We utilize so-called subgradient locality measures to handle the nonconvexity of the objective and the constraint functions (see [2]).

Let $\mathbf{d}^s$ be the solution of (5.2). We employ a line search algorithm described in [5], which detects discontinuities in the gradients of the objective functions. Roughly speaking, we try to find a step-size $t^s \in (0, 1]$ such that

$$t^s \approx \underset{t \in (0,1]}{\operatorname{argmin}}\{H(\mathbf{x}^s + t\mathbf{d}^s)\} \qquad \text{and} \qquad \mathbf{x}^s + t^s\mathbf{d}^s \in S.$$

The subgradient aggregation strategy due to [2] is used to bound the storage requirements, and a modification of the weight updating algorithm described in [3] is used to update the weight u^s.

6 Numerical Example

The numerical example to be solved here has been introduced in [9]. The problem is simple because the aim here is just to give some impression about how the method works. The problem describes a (hypothetical) pollution problem of a river. A fishery polluting the river is situated near the source of the river. A city of medium size is situated downstream from the fishery. There is both industrial and municipal waste in the river and there exist treatment plants in the fishery and in the city.

The waste in the river is described in pounds of biochemical oxygen demanding material (BOD). Two decision variables represent the proportional amount of BOD removed from the water in the fishery and in the city, respectively.

There are four objective functions in the original problem. The quality of the river water is measured by the concentration of dissolved oxygen (DO). It is surveyed in the city (f_1) and at the state line downstream from the city (f_2). Cleaning the water in the fishery decreases the return on the investments and cleaning the water in the city increases the taxation rate. Thus, the other objective functions are per cent return on investment at the fishery (f_3) and the addition to the tax rate in the city (f_4). The first three of the objective functions are to be maximized and the fourth to be minimized. For simplicity, each of them is transformed to be minimized.

We have added a nondifferentiable function (f_5) into the problem. The treatment plants work best if the proportional amount of BOD removed is as close as possible to the ideal value 0.65. Thus, our nondifferentiable multiobjective optimization problem consists of five objective functions

$$\begin{aligned}
\text{minimize} \quad & \Big\{ f_1(\mathbf{x}) = -4.07 - 2.27x_1, \\
& f_2(\mathbf{x}) = -2.60 - 0.03x_1 - 0.02x_2 - \frac{0.01}{1.39 - x_1^2} - \frac{0.30}{1.39 - x_2^2}, \\
& f_3(\mathbf{x}) = -8.21 + \frac{0.71}{1.09 - x_1^2}, \quad f_4(\mathbf{x}) = -0.96 + \frac{0.96}{1.09 - x_2^2}, \\
& f_5(\mathbf{x}) = \max\left[|x_1 - 0.65|, |x_2 - 0.65|\right] \Big\} \\
\text{subject to} \quad & 0.3 \leq x_1,\ x_2 \leq 1.0.
\end{aligned}$$

When solving this problem, the starting point has been set to be in the middle of the feasible set, $\mathbf{x}^0 = (0.65, 0.65)^T$. The corresponding criterion vector is $\mathbf{z}^0 = (-5.5455, -2.9529, -7.1463, 0.4782, 0.0)^T$. When all the objective functions are minimized simultaneously ($I^< = \{1, 2, 3, 4, 5\}$), it is noticed that the starting point is already weakly Pareto optimal, so $\mathbf{z}^1 = \mathbf{z}^0$.

To start with, the quality of the water is desired to improve and the first and the second objective function are minimized, $I^< = \{1, 2\}$, with the weighting

coefficients $w_1^1 = 0.7$ and $w_2^1 = 0.3$. The rest of the objective functions have to be allowed to increase. They become constraints with upper bounds, $I^> = \{3,4,5\}$, $\varepsilon_3^1 = -6.0$, $\varepsilon_4^1 = 0.5$ and $\varepsilon_5^1 = 0.3$. The result obtained is $\hat{\mathbf{z}}^1 = (-6.0603, -2.968, -6.0, 0.4942, 0.2268)^T$ and it is accepted for continuation ($\mathbf{z}^2 = \hat{\mathbf{z}}^1$).

Next, we still want the water quality to improve. The division into classes is the following, $I^< = \{2\}$, $I^\leq = \{1\}$, $\bar{z}_1^2 = -6.2$, $I^> = \{4\}$, $\varepsilon_4^2 = 1.0$ and $I^\diamond = \{3,5\}$. The resulting criterion vector is $\hat{\mathbf{z}}^2 = (-6.3265, -3.0491, -1.2379, 0.9923, 0.3441)^T$. The values of the third and the fifth objective function were allowed to change freely to see how the other objective functions behave. Five alternatives between $\mathbf{z}^2$ and $\hat{\mathbf{z}}^2$ are listed in Table 1.

Table 1.1. Five alternatives

	f_1	f_2	f_3	f_4	f_5
1	-6.0603	-2.9680	-6.0000	0.4942	0.2268
2	-6.1268	-2.9843	-5.5705	0.5868	0.2561
3	-6.1934	-3.0029	-4.9075	0.6969	0.2854
4	-6.2600	-3.0243	-3.7522	0.8296	0.3147
5	-6.3265	-3.0491	-1.2379	0.9923	0.3441

From this set the fourth alternative, denoted by $\mathbf{z}^3 = (-6.26, -3.0243, -3.7522, 0.8296, 0.3147)^T$, is the best for continuation. At this point, the water quality at the state line is wished to improve and $I^< = \{2\}$. Also the return of the investment at the fishery is to be improved and the treatment amounts should be closer to the ideal values. So the third and the fifth objective functions are desired to decrease till aspiration levels, $I^\leq = \{3,5\}$ with $\bar{z}_3^3 = -4.2$ and $\bar{z}_5^3 = 0.25$. The value of the first objective function is accepted as such $I^= = \{1\}$ and the fourth objective is allowed to increase, $I^> = \{4\}$ with $\varepsilon_4^3 = 1.6$. With this classification, no improved solutions can be obtained. Thus we have $\mathbf{z}^4 = \mathbf{z}^3$.

This implies that we must relax the first objective function to allow the other objective functions to improve. We revise the setting to be $I^< = \{2\}$ and $I^\leq = \{3,5\}$ with $\bar{z}_3^4 = -4.2$ and $\bar{z}_5^4 = 0.25$ and $I^> = \{1,4\}$ with $\varepsilon_1^4 = -6.2$ and $\varepsilon_4^4 = 1.6$.

This setting produces the solution $\hat{\mathbf{z}}^4 = (-6.2569, -3.1119, -3.8249, 1.6, 0.3134)^T$, and it is accepted as the final solution. The solution $\mathbf{z}^5 = \hat{\mathbf{z}}^4$ is also Pareto optimal. The corresponding decision vector is $\mathbf{x}^5 = (0.9634, 0.8456)^T$.

7 Conclusions

We have described NIMBUS – an interactive multiobjective optimization method which is able to handle nondifferentiable functions. Unlike many other interactive methods, the questions posed to the decision maker are not too demanding in NIMBUS. The decision maker is only asked to designate the desired changes in the objective function values. Any complicated concepts, such as marginal rates of substitution or trade-offs, are not dealt with. The calculations are not too massive, either, because the underlying MPB method is computationally efficient (see [4]).

NIMBUS is flexible to use. The decision maker is free to move around the weakly Pareto optimal set and also to change her or his mind if necessary. In addition to the simple test example of Section 6, NIMBUS has been applied in solving several complicated problems. Numerical tests give evidence that the method works reliably and efficiently. A World-Wide Web version of this method is available in the URL http://nimbus.math.jyu.fi/NIMBUS/.

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The LBS-Discrete Interactive Procedure for Multiple-Criteria Analysis of Decision Problems

Andrzej Jaszkiewicz and Roman Słowiński

Institute of Computing Science
Poznań University of Technology
60-965 Poznań, Poland

Abstract: An interactive procedure, called LBS-Discrete (Light Beam Search), for multiple-criteria analysis of decision problems with an explicitly given set of alternatives is presented. An alternative is defined as a non-dominated point in the space of criteria. The set of alternatives is assumed to be finite and relatively large. The procedure is an extension of the Light Beam Search method for linear and non-linear multiple-objective programming (Jaszkiewicz and Słowiński, 1992b) to the discrete case. While existing interactive procedures may involve the decision maker (DM) in too difficult comparisons of the candidates for the best compromise alternative, the LBS-Discrete procedure tries to overcome this inconvenience. It supports both learning about the problem and successive improvement of the current point. At the decision phase of the procedure, a sample of non-dominated points, composed of the current point and a number of alternative proposals, is presented to the DM. In order to ensure a relatively easy evaluation of the sample by the DM, it is generated in a way taking into account a preference information of intra- and inter-criteria type given by the DM with respect to the current point. The local preference model has the form of an outranking relation and defines a subregion of the non-dominated set. The sample presented to the DM comes from this subregion. The procedure can be compared to projecting a focused beam of light from a spotlight at the reference point onto the non-dominated set; the highlighted subregion changes when either the reference point or the point of interest in the non-dominated set are changed. This explains the name LBS-Discrete. Finally, a microcomputer implementation of the LBS-Discrete with an important graphical interface is characterized and its application to an agricultural problem is presented.

1. INTRODUCTION

It is commonly acknowledged that interactive procedures are very effective in searching over the non-dominated set of decision problems with a relatively large set of alternatives for the best compromise (Steuer, 1986). Procedures of this type are characterized by phases of decision alternating with phases of computation. At each computation phase, a point, or a sample of points, is generated for examination at the decision phase. As a result of the examination, the DM inputs some preferential information which intends to improve the point(s) generated in the next computation phase. So, at each decision phase the DM solves a subproblem of the original decision problem. It consists in a multiple-criteria

comparison of at least two points; the result of this comparison should guide the generation of a new sample of points. After solving a sequence of such subproblems, the DM either arrives to choose the best compromise point of the original decision problem or concludes that no compromise alternative exists for it.

A number of interactive procedures is characterized by iterative generation of samples of non-dominated points. At decision phases of such methods, the DM is expected to make a multiple-criteria comparison of the presented points and to supply some preferential information which often consists in specifying the best point in the presented sample. Let us notice that the problem of multiple-criteria comparison also concerns methods which propose a single new point to the DM. In these methods, the DM compares the new point with a reference (ideal) point and/or with previously generated points. So, in fact, the DM is expected to solve, at each decision phase, an MCDM subproblem with a small explicitly given set of alternatives. The interactive procedures mentioned above are based on an assumption, usually not declared, that pairwise comparison of a small explicitly given set of alternatives is relatively easy for the DM.

However, it follows from practical experience that evaluation of an even small explicitly given set of alternatives in the presence of multiple criteria can be difficult to the DM. Numerous psychological studies indicate the highly limited capability of the DMs in processing multi-criteria information (Kahneman et al., 1982). Taking into account results of the tests, Larichev et al. (1987) evaluated psychological difficulties of some elementary operations performed by the DMs at decision phases of interactive procedures. The selection of the best point from a finite sample was evaluated as one of the most complex operations. The complexity of this operation grows with the number of alternatives, the number of criteria and with the differences between alternatives. So, it should not be assumed that in all decision problems with a relatively large set of alternatives the comparison of points from a sample will be easy for the DM. Without satisfying some conditions on comparability and diversification of points creating the sample, the subproblems faced by the DM during the interactive process can be too difficult indeed. The authors claim that some elements of the methodology proposed for MCDM problems with a relatively small explicitly given set of alternatives can be used to help the DM in comparison of the points presented at the decision phase.

A number of methods and techniques has been proposed to help DMs in solving MCDM problems with a relatively small explicitly given set of alternatives. The techniques include Multiple Attribute Utility Theory (Keeney and Raiffa, 1976), Analytic Hierarchy Process (Saaty, 1980) and Outranking methods (Roy, 1990; Roy and Bouyssou, 1993). The Outranking methods use an *outranking relation* as a preference model (Roy, 1985).

The utility function can only be used if the available preference information is sufficient to compare alternatives using *indifference* and *strict preference* relations only. In real decision problems, however, one of the four following situations can appear while comparing two alternatives a and b (Vincke, 1990):

$a\ P\ b$ i.e. a is preferred to b,

$b\ P\ a$ i.e. b is preferred to a,

$a\ I\ b$ i.e. a and b are indifferent,

$a\ R\ b$ i.e. a and b are incomparable.

The *preference* P, *indifference* I and *incomparability* R relations are the sets of ordered pairs (a, b) such that $a\ P\ b$, $a\ I\ b$, $a\ R\ b$, respectively. The relations are not assumed to be transitive.

A significant number of practical applications of AHP (see e.g. Emshoff and Saaty, 1982; Zahedi, 1986) and methods from ELECTRE family (see e.g. Roy et al., 1986; Roy et al., 1992) shows that DMs are often unable or unwilling to make explicit distinctions between $a\ P\ b$, $a\ I\ b$ and $a\ R\ b$. Instead, they often prefer to specify some preference information of inter- and intra-criteria type. In ELECTRE methods, the inter-criteria information is usually expressed in the form of *indifference* and *veto thresholds* while the intra-criteria information in the form of *weights* and *veto thresholds* (Roy, 1990).

The preference information including the indifference, preference and veto thresholds can be used to define a preference model in the form of an outranking relation S. By definition, $a\ S\ b$ means that *a is at least as good as b*. Using the outranking relation one can express all the preference situations with respect to two alternatives a and b mentioned above:

$$a\ S\ b \text{ and } b\ \not S\ a \Rightarrow a\ P\ b,$$

$$a\ S\ b \text{ and } b\ S\ a \Rightarrow a\ I\ b,$$

$$a\ \not S\ b \text{ and } b\ \not S\ a \Rightarrow a\ R\ b.$$

In order to ensure a relatively easy comparison of points presented to the DM at the decision phase of an interactive procedure, the sample of points presented to the DM for an examination should meet some requirements. Specifically, the points in the sample should not be incomparable for the DM and also they should not be all indifferent for him/her. A "forced" comparison of points which do not meet these requirements can yield serious incoherence in the preferential information supplied by the DM. Furthermore, in such a case, the DM can stop the interactive procedure being unable to find a better proposal among the presented points even if the current point is far from the best compromise. If the DM is unable or unwilling to make explicit distinctions between $a\ P\ b$, $a\ I\ b$ and $a\ R\ b$, some additional preferential information of inter- and intra-criteria type should be used in order to find a sample of comparable points. Existing interactive procedures do not use, however, this additional information. The samples of points presented to the DM are usually constructed in a way that ensures mathematical or intuitive (Steuer, 1986) convergence to a final point. This understanding of convergence assumes, however, a pre-existence of a utility function in the mind of the DM. In consequence, all comparisons made by the DM are assumed to be coherent with the utility function. So, he/she should always be able to compare any points. We know, however, that it is not always true. Application of this assumption in practice can just create the situation of a "forced" comparison mentioned above with undesired consequences on the progress of the interactive process.

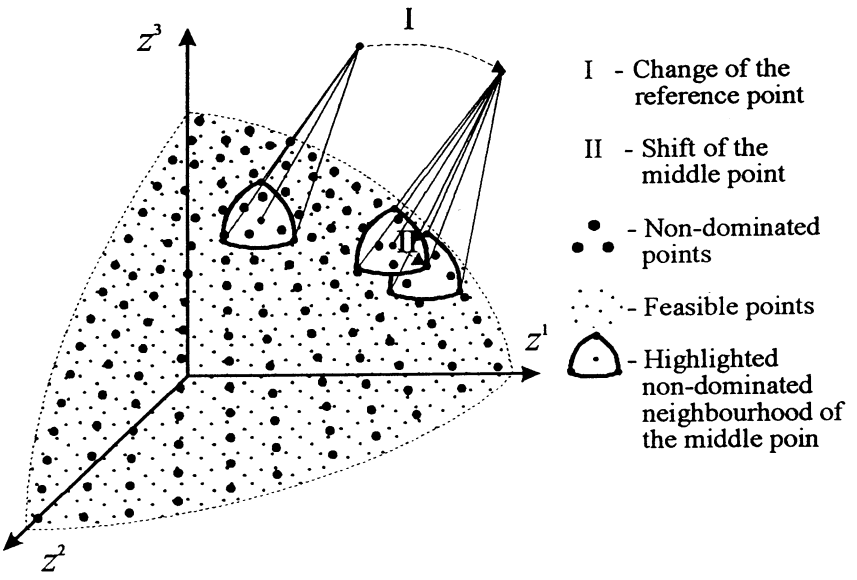

Figure 1. The Light Beam Search over a non-dominated set

The procedure presented in this paper tries to overcome the drawbacks of the above mentioned interactive procedures. Specifically,

- the search over the non-dominated set is organized through a sampling of a neighbourhood of the current point which moves in result of either a change of the DM's reference point or a shift of the current point within its neighbourhood.
- an outranking relation is used as a local preference model in a neighbourhood of the current point; its construction is based on a relatively weak preferential information of an inter- and intra-criteria type,
- the neighbourhood of the current point is composed of non-dominated points that outrank this point, so the neighbourhood includes points that are sufficiently different but comparable with the current point; the points from outside the neighbourhood are either incomparable with or outranked by the current point,

The last point submits some analogy with projection of a focused beam of light from a spotlight at the reference point onto the non-dominated set. For this reason the procedure is called the Light Beam Search or, shortly, LBS-Discrete (see figure 1).

The paper is organized in the following way. In the second section a formal statement of the problem and basic definitions will be presented. The main idea of the procedure will be presented in the third section. The general scheme of the procedure will be outlined in the fourth section. Some characteristic points of the procedure will be described more precisely in the fifth section. A microcomputer

implementation of the procedure and a real life application to an agricultural problem will be outlined in the sixth section, together with the final conclusions.

2. PROBLEM STATEMENT AND BASIC DEFINITIONS

The general multiple-criteria decision problem is formulated as:

$$\max\{\mathbf{z}\} \qquad \text{(P1)}$$

$$\text{s.t.} \quad \mathbf{z} \in Z$$

where $\mathbf{z} = [z_1,\ldots,z_J]$ is a vector of criteria and Z is the set of alternatives.

Point $\mathbf{z}' \in Z$ is *non-dominated* if there is no $\mathbf{z} \in Z$ such that $z_j \geq z_j' \ \forall j$, and $z_i > z_i'$ for at least one i. The set of all non-dominated points is the *non-dominated set*. For other definitions concerning non-dominance and efficiency, see e.g. Wierzbicki (1986) and Steuer (1986).

The point $\mathbf{z}^*$ composed of the best attainable criteria values is called the *ideal point*:

$$z_j^* = \max\{z_j \mid \mathbf{z} \in Z\} \qquad j = 1,\ldots,J.$$

Another useful definition is that of the *achievement scalarizing function* in the space of criteria:

$$s(\mathbf{z}^\circ, \Lambda, \rho, \mathbf{z}) = \max_j\left\{\lambda_j\left(z_j^\circ - z_j\right)\right\} + \rho\sum_{j=1}^{J}\left(z_j^\circ - z_j\right) \qquad (1)$$

where $\mathbf{z}^\circ=[z_1^\circ,\ldots,z_J^\circ]$ is a reference point, $\Lambda = [\lambda_1,\ldots,\lambda_J]$ is a weighting vector, $\lambda_j \geq 0$, $\sum_{j=1}^{J}\lambda_j = 1$ and ρ is a sufficiently small positive number.

3. MAIN IDEA OF THE LIGHT BEAM SEARCH PROCEDURE

At each computation phase of the LBS-Discrete procedure a sample of non-dominated points is selected. The sample is composed of a current point, called the *middle point*, obtained at a previous iteration, and a number of non-dominated points from its *neighbourhood*. In order to define the neighbourhood, an *outranking relation* S is used as a local preference model. Precisely, for a current middle point, the neighbourhood is defined as a set of non-dominated points that are not worse than the middle point, i.e. outrank the middle point. The neighbourhood is called the *outranking neighbourhood* of the middle point. In general, in the outranking neighbourhood there are also points outranked by the middle point. According to the local preference model, these points are *indifferent* to the middle point while the remaining points of the neighbourhood are *preferred* to the middle point.

In the LBS-Discrete, the DM can see either all the points from the outranking neighbourhood or J *characteristic neighbours* of the middle point showing to what extent the values of particular criteria can be improved in the outranking neighbourhood. Each point is presented together with an information if it is preferred or indifferent to the middle point. Notice that, in an extreme case, the outranking neighbourhood can be empty.

The formal expression of the conditions that must be satisfied to validate the assertion $a\ S\ b$ can be influenced by many factors. In the presented procedure, following the approaches proposed in various versions of ELECTRE methods (Roy, 1990), the following factors are taken into account:

- the discrimination power of the DM's preferences with respect to particular criteria that will be modelled with indifference (q_j) and preference thresholds (p_j) (i.e. the intra-criteria information),
- the inter-criteria information specified in the form of the veto thresholds.

Similarly to ELECTRE IV, the inter-criteria information does not include the weights of criteria which might be too difficult to define; it is assumed, however, that one criterion is not more important that all the others together. It means that in the case of two criteria their importance must be equivalent. It is worth noticing that the ratio of veto and preference thresholds of a criterion is related with its importance; the lower is the ratio the greater is the importance (Roy, 1990).

In general, the indifference and preference thresholds are functions of z_j; moreover:

$$p_j(z_j) \geq q_j(z_j) \geq 0$$

The indifference and preference thresholds allow to distinguish between the three following preference relations with respect to z_j, for any ordered pair (a, b) of alternatives:

$a\ I_j\ b$ i.e. a and b are *equivalent* $\Leftrightarrow$ $-q_j(z_j^a) \leq z_j^a - z_j^b \leq q_j(z_j^b)$,

$a\ Q_j\ b$ i.e. a is *weakly preferred* to b $\Leftrightarrow$ $q_j(z_j^b) < z_j^a - z_j^b < p_j(z_j^b)$,

$a\ P_j\ b$ i.e. a is *strictly preferred* to b $\Leftrightarrow$ $p_j(z_j^b) \leq z_j^a - z_j^b$.

Observe that the indifference and preference thresholds are defined for particular criteria independently with respect to the worst score of the two points. The DM is not required to take into account the other criteria while defining the indifference and preference thresholds.

The veto threshold v_j allows to take into account the possible difficulties of comparing the relative value of two alternatives when one is significantly better than the other on a subset of criteria, but much worse on at least one other criterion. By definition assumption that $a\ S\ b$ has to be denied if

$$z_j^b \geq z_j^a + v(z_j^a)$$

for at least one $j \in \{j,...,J\}$. In general, the veto threshold is also a function of z_j.

In the Light Beam Search procedure, the outranking relation is used as a local preference model in the neighbourhood of a middle point, so a simple value of each threshold is sufficient for a given middle point. These values are assumed to be valid in the whole outranking neighbourhood. Of course, the DM can update the values of the thresholds when middle point changes.

The outranking relation has already been used as a preference model in the Cone Contraction Method with Visual Interaction for Multiple-Objective Non-Linear Programmes (Jaszkiewicz and Słowiński, 1992a). In that method, however, it serves as a global preference model. The outranking relation has also been used as a local preference model in the method proposed by Lotfi et al. (1992). The authors do not use, however, any preferential information for the definition of the neighbourhood.

4. GENERAL SCHEME OF THE INTERACTIVE PROCEDURE

The following is a general scheme of the procedure presented in a Pascal-like form:

Fix the points of the best and the worst values of criteria; make the former one the first reference point;
if *the DM wants to specify the first reference point* **then**
    Ask the DM to specify the aspiration levels on particular criteria;
Find a starting middle point;
Ask the DM to specify the preferential information of inter- and intra-criteria type;
repeat
    Present the middle point to the DM;
    Find the sample of points from the outranking neighbourhood of the middle point and present them to the DM;
    Allow the DM to scan the inner of the current neighbourhood;
    if *the DM wants to store the middle point* **then**
        Add it to the set of stored points;
    case
        The DM wants to define a new reference point:
            Ask the DM to specify the aspiration levels on particular criteria;
            Project the reference point onto the non-dominated set;
        The DM wants a point from the neighbourhood to be the new middle point:
            Ask the DM to select the new middle point;
        The DM wants to return to one of the stored points:
            Use the selected stored point as a new middle point;

The DM wants to update the preferential information:
Ask the DM to specify the new preferential information;

end

until *the DM feels satisfied with a point found during the interactive process or concludes that no compromise point exists for the decision problem;*

5 CONSTRUCTION OF AN OUTRANKING NEIGHBOURHOOD

After finding a middle point $\mathbf{z}^c$, the DM is asked to give the preferential information for each criterion, i.e. the indifference and, optionally, the preference and veto thresholds. At this stage the DM should decide if he/she wants to specify the preference and/or veto thresholds, however, he/she is able to change these settings at every step of the procedure.

Then the non-dominated points belonging to the outranking neighbourhood are selected. In order to test if point $\mathbf{z}^a$ outranks point $\mathbf{z}^b$ the following numbers are used:

$m_q(\mathbf{z}^a, \mathbf{z}^b)$ - the number of the criteria for which point $\mathbf{z}^a$ is weakly preferred to $\mathbf{z}^b$,

$m_p(\mathbf{z}^a, \mathbf{z}^b)$ - the number of the criteria for which point $\mathbf{z}^a$ is strictly preferred to $\mathbf{z}^b$,

$m_v(\mathbf{z}^a, \mathbf{z}^b)$ - the number of the criteria being in a strong opposition to the assertion $\mathbf{z}^a \, S \, \mathbf{z}^b$, i.e. card $\{j: z_j^a - v_j \geq z_j^b, \, j = 1,..,J\}$.

The construction of the outranking relation depends on the type of preferential information supplied by the DM. If the DM has specified all of the three thresholds, the following definition of the outranking relation is proposed:

$$\mathbf{z} \, S^I \, \mathbf{z}^c \Leftrightarrow \begin{cases} m_v(\mathbf{z}^c, \mathbf{z}) = 0 \text{ and} \\ m_p(\mathbf{z}^c, \mathbf{z}) \leq 1 \text{ and} \\ m_q(\mathbf{z}^c, \mathbf{z}) + m_p(\mathbf{z}^c, \mathbf{z}) \leq m_v(\mathbf{z}, \mathbf{z}^c) + m_p(\mathbf{z}, \mathbf{z}^c) + m_q(\mathbf{z}, \mathbf{z}^c) \end{cases}$$

If the DM has decided not to specify the veto thresholds, one should not assume that for every criterion he/she is ready to accept any worsening of the criterion even if a subset of other criteria is significantly improved, i.e. one should not assume that the veto threshold does not exist. Such a situation indicates that at the particular stage of the interactive process, the DM is unable or unwilling to specify the value of this threshold explicitly. In this case, the following definition of the outranking relation is proposed:

$$\mathbf{z} \, S^{II} \, \mathbf{z}^c \Leftrightarrow \begin{cases} m_p(\mathbf{z}^c, \mathbf{z}) = 0 \text{ and} \\ m_q(\mathbf{z}^c, \mathbf{z}) \leq m_v(\mathbf{z}, \mathbf{z}^c) + m_p(\mathbf{z}, \mathbf{z}^c) + m_q(\mathbf{z}, \mathbf{z}^c) \end{cases}$$

Observe that $S^{II} \subseteq S^{I}$ and that $S^{II} = S^{I}$ if $v_j = p_j \; \forall j$. If, for a criterion z_j, the DM is ready to accept the worsening of its value greater than p_j, some points that outrank the middle point can be left outside the outranking neighbourhood. However, the neighbourhood will be still composed of points that are comparable with the middle point.

In a similar way one can analyze the situation when the DM has decided not to specify the preference threshold. In this case one should not assume that the DM feels no difference between the weak and strict preferences. Such a situation indicates that the DM is unable or unwilling to specify explicitly the value allowing to distinguish between the two relations. In this case, the following definition of the outranking relation is proposed:

$$\mathbf{z}\, S^{III}\, \mathbf{z}^c \Leftrightarrow \begin{cases} m_v(\mathbf{z}^c, \mathbf{z}) = 0 \text{ and} \\ m_p(\mathbf{z}^c, \mathbf{z}) \leq 1 \end{cases}$$

Observe that $S^{III} \subseteq S^{I}$ and that $S^{III} = S^{I}$ if $p_j = q_j \; \forall j$.

Finally, if the DM has decided to specify the preference thresholds only, the following definition of the outranking relation is proposed:

$$\mathbf{z}\, S^{IV}\, \mathbf{z}^c \Leftrightarrow m_p(\mathbf{z}^c, \mathbf{z}) = 0$$

Observe that $S^{IV} \subseteq S^{II} \subseteq S^{I}$ and $S^{IV} \subseteq S^{III} \subseteq S^{I}$, moreover, $S^{IV} = S^{I}$ if $v_j = p_j = q_j \; \forall j$.

6. APPLICATION ISSUES AND CONCLUSIONS

The LBS-Discrete procedure has been implemented under MS-Windows environment. This environment has been selected because it facilitates interaction with the user as well as presentation of large amounts of information in textual, numerical and graphical forms.

The LBS-Discrete procedure has been applied to an agricultural problem. The problem consists in multiple-criteria, resource-constrained scheduling of an agricultural project (see e.g. Słowiński, 1989). 40 activities composing the project compete for two types of renewable resources: manpower (R_1) and tractors (R_2).

Three kinds of criteria are used to evaluate alternative schedules: project completion time C_{max}, the maximal usage of manpower and the maximal usage of tractors.

The type of preferential information required by the LBS-Discrete procedure appeared relatively easy to specify for an agricultural expert. The DM was asked to answer questions of the following type:

- What is the minimal difference on the completion time that makes any difference for you?

- What is the minimal decrease of the maximal usage of manpower that makes the new alternative preferred with respect to this criterion?
- What is the maximal increase of the maximal usage of tractors that makes the new alternative unacceptable, regardless of the values of other criteria?

Answers to the above questions define the indifference, preference and veto thresholds, respectively.

In order to obtain the non-dominated points, the FPS decision support system (Hapke & Słowiński, 1993) was used. For each set of possible resource constraints a beam of twelve parallel priority heuristics, minimizing completion time, was used. This procedure yielded 568 non-dominated points.

It has been observed that the DM started with specifying a reference point which was then updated in several successive iterations. This led him to concentrate on the interesting region of the non-dominated set. Then, in further iterations, he was improving the middle point by shifting the neighbourhood on the non-dominated set. The same general pattern was observed in other applications of the LBS.

In conclusion, let us point out that the LBS-Discrete method gives the DM two degrees of freedom in searching over the non-dominated set: changing the reference point and shifting the middle point with its neighbourhood on the non-dominated set.

Moreover, the way of constructing the outranking relation in the neighbourhood of a middle point avoids specification of weights and allows to cover by the outranking neighbourhood different attitudes as to the importance of criteria.

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VII

Group Decision and Negotiation

An invited leading paper by Kersten presents an introduction to the subject of group decision and negotiation. Kersten discusses methods for the analysis and support of group decisions and negotiations from distinct perspectives. The continuation and outreach strategies are introduced; then methods and models are positioned within the modified process of negotiation proposed by Gulliver; subsequently, methods and systems developed within five areas of study are outlined.

Sakawa and Nishizaki consider solution concepts for an n-person cooperative game (represented by the set of all players and a characteristic function) with multiple scenarios. A characteristic function in a game with multiple scenarios associates a subset of players with its real vector value. They consider the nucleolus, which is one of the solution concepts based on the lexicographical framework in an n-person cooperative game, in the game with multiple scenarios. To define extended nucleoli, three aggregation methods using weighting coefficients, a minimum component, and constraints are employed. The computational methods for the three extended nucleoli are developed by repeatedly solving linear programming problems.

Ehtamo, Verkama and Hamalainen describe an interactive approach to computing Pareto optimal agreements in two-party negotiations under circumstances in which the participants do not know each other's goals. The approach is based on so-called proposals that link the decision variables together. The proposals also provide a way of enforcing the resulting agreement, meaning that no party has an incentive to break loose from it. An interactive method will be described with which the decision makers can solve Pareto optimal agreements without completely revealing their private preferences. The method also provides a self-enforcing mechanism (motivated through game theoretic arguments) that guarantees the sustainability of the agreements.

Kersten and Noronha analyze the circumstances under which rational agents make inefficient compromises and resist improvements. They do this by describing and interpreting various negotiation situations and by developing formal constructs and theorems for determining the character of a negotiation situation, using the key notion of opposition defined in terms of utility functions. The effects of various rationality assumptions on efficiency and their implications for negotiation support systems are discussed.

Support for Group Decisions and Negotiations An Overview *

Gregory E. Kersten
School of Business, Carleton University, Ottawa, ON, K1S 5B6, Canada

Abstract. This paper discusses methods for the analysis and support of group decisions and negotiations from three perspectives. First, the continuation and outreach strategies are introduced; then methods and models are positioned within the modified process of negotiation proposed by Gulliver; and subsequently methods and systems developed within five areas of study are outlined.
Keywords. Group decisions, negotiations, negotiation/group analysis and support

1 Introduction

There is a significant and growing stream of research on analysis and support of group decisions and negotiations. It is usually published in general MS/OR/IS journals often with dedicated special issues (for example, *Annals of Operations Research, Control and Cybernetics, Decision Support Systems, EJOR, Management Sciences, Theory and Decisions*), area journals *Group Decision and Negotiation* and *Negotiation Journal*, and edited books (Jessup and Valacich, 1993; Munier and Shakun, 1988; Nagel, 1993; Young, 1991; Zartman, 1994). Recent reviews of the area concentrate on negotiation modelling and negotiation support systems (NSS), (Anson and Jelassi, 1990; Jelassi and Foroughi, 1989; Teich et al., 1994), MCDM approaches to group support (Iz and Gardiner, 1993), and a taxonomy of group decision support systems (GDSS), (Teng and Ramamurthy, 1993).

In this overview, we focus on methods and models of group decision and negotiation processes and on GDSS and NSS. Three hypotheses behind this review are that there is a need for:

- an analysis of the role of formal methods in GDSS/NSS,
- a review of the relationships between paradigms underlying approaches from different areas, and
- a comprehensive perspective of the role that social and behavioural studies can and should play in model and system development.

Comparative studies and detailed reviews should lead to formulation of unified frameworks and new avenues for applied and theoretical research in the area. This overview does not offer either. Instead it makes an attempt to present models and systems from two viewpoints:

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- the areas of study within which the models and systems are being developed, and
- the activities and phases of the group decision and negotiation process which can be analyzed and supported with the existing models and systems.

While only some models and systems are mentioned, the expectation is that this article will contribute to further and more comprehensive discussion on what can be supported and with what approaches and tools. Similarly, the review of the work conducted within specific areas is far from complete and more detailed comparative studies are required.

Within the MCDM community, there is an ongoing discussion on the roles of MCDM and its future, as exemplified by several articles contained in this book. A contribution to this discussion is attempted here by raising the questions of assumption falsification and strategies for GDSS/NSS development. These questions are considered in Section 2. Key issues of group decision and negotiation process and positioning of models and systems within this process are given in Section 3. In Section 4 MCDM approaches and areas most closely related to MCDM are discussed.

2 Two strategies

"The mathematical model is a set of assumptions. We know that every assumption is false. Nevertheless we make them, for our purpose at this point is not to make true assertions about human behaviour but to investigate consequences of assumptions, as in any simulation or experimental game."

A. Rapoport *Strategy and Conscience*, 1964, (p. 147)

The above quote, while controversial, suggests that formal models and support systems in which they are embedded may suffer from false assumptions or from assumptions that either seem unreasonable or are difficult to accept. This is a criticism that many social scientists as well as decision makers (including negotiators and mediators) voice with respect to both formal approaches and model-based GDSS/NSS. It may also be the main reason for many of the model-based GDSS/NSS never being applied to real-life situations. One possible response to this criticism is the continuation of studies on modelling and the development of support tools with only superficial consideration of the behavioural and social aspects of decision making and the needs and demands of decision makers. While this *continuation strategy* may contribute to maintaining a vibrant and lively area of study, the results will have very limited impact on both research in other areas and on real-life problem solving.

The continuation strategy will lead to isolation of the MCDM area. Notwithstanding its significant achievements which expanded applications domains and fostered research on representation of preferences, values and criteria, and on decision aids, its roots are deeply established in MS/OR with its traditional unifying perspective on modelling and aiding human endeavours. Although MCDM grew from discontent with certain assumptions underlying decision modelling and support, its mainstream remained firmly established within the MS/OR quantitative and analytical framework.

Decision making is about thinking of new situations and making choices among them. It involves the decision maker and other people. The decision process is about

changing the current situation to a new situation. This aspect may be more important than the decision outcome itself. Decision makers need to be able to determine what they can change, why they want to make a change, and how it may be introduced. Fisher, Kopelman and Schneider (1994, p. 9) wrote that:

> *Research driven by hard data ... implicitly suggests that the value of research lies in the extent to which it enables us to predict something that is going to happen, or to show why a past event was inevitable. But, as the best experts in international relations have learned, the more that research focuses our attention on what is predictable, the more it diverts our attention from those things we can affect. If we want knowledge in order to improve the world, then predictability is the wrong standard. We need to turn from what is inevitable to those things we can change.*

This leads us to the *outreach strategy* in building models and developing systems. This strategy calls for challenging each and every assumption, multiple theoretical perspectives, and user and problem oriented research. The assumptions and simplifications are necessary but they need to be questioned. Accepting this, we can aid decision makers in the investigation of their own assumptions or those the analysts make for them. This will focus support on the analysis of consequences of assumptions rather than on providing a solution to the problem.

Methods for problem formulation, solution and analysis together with support tools ,and design methodologies need to be developed from the users' and not the researchers' perspective. There are examples of systems that have successfully been used in real-life applications by analysts or decision makers themselves. These approaches yypically address one specific aspect of the problem and/or process such as problem formulation, evaluation of issues, communication, or presentation. The challenges that researchers and developers face include research on approaches with weakened assumptions or assumptions that may be occasionally violated, and on ways to combine or integrate different approaches. This is one way to address decision makers' different and varying viewpoints and needs.

Negotiation and group decision processes are studied from multiple perspectives. We are convinced that in the development of analytical methods and support tools, social and behavioural theories must be considered to a larger extent and not as sporadically as they have been to-date. This includes organization theory with its focus on the organizational structure, cultural aspects, procedures and processes; small group theory focusing on group dynamics including interaction and process; political science with its research on power and forming coalitions; and management theory and leadership. That said, the space and the nature of this article does not allow us to discuss these diverse but complementary perspectives; we refer the reader to representative works of leading researchers in the respective theories (Fisher and Ury, 1983; Zartman, 1994; Kremenuk, 1991; Pruit, 1991; Lax and Sebenius, 1986).

Many analytical methods have their roots in economics where the first formal models of bargaining and negotiations were developed. We do not discuss the contributions of economics to bargaining and negotiations which has been widely discussed in the literature. While the contribution of economics is well recognized, it is often the case that developments in areas close to MCDM in which the researchers have

similar goals of developing methods for analysis and support are ignored. On the other hand, the scope of these methods is often too narrow or the underlying assumptions too restrictive to be effectively used in the actions undertaken in group decision and negotiation processes. Our argument is that coupling different methods based on similar paradigms will result in their enrichment and better adaptability to complex and dynamic social processes. Further, their enhancement with the results of the social and behavioural studies will lead to a comprehensive theory of analysis and support of group decisions and negotiations.

3 Negotiated decisions

3.1 Key aspects

Group decisions and negotiations involve two or more participants engaged in two types of complex activities: communication and decision making. Communication, or discourse of negotiations, includes passing, accepting and understanding (decoding) messages (Firth, 1995). While these activities play a significant role in processes involving multiple participants and efforts are made to construct computer systems for forming, representing and analyzing arguments (Binbasioglu et al, 1995; Chang and Woo, 1994) here the focus is on the decision making activities.

In group decisions there may be a single decision maker who has the power to decide while the other participants (e.g., analysts, experts) provide her with advice, interpretation, analysis, and so on. These types of groups are called *teams* (Holsapple, 1991) and they can be supported by meeting and conferencing systems and systems designed for cooperative work (Lewis, 1994; Nunamaker, 1991). In addition, the analysts and experts may use specialized group and individual DSS.

If the power to decide is shared among two or more participants, then decisions need to be *negotiated*. This does not imply that all such decisions are made through negotiation, but that they involve activities that are typical to negotiation processes. Therefore, we concentrate here on negotiations and their characteristics and features, indicating those activities which are similar to other types of group processes.

Two key aspects of negotiated decisions are *conflict* and *interdependence*. Conflict occurs because people have separate and conflicting interests (objectives, goals). This situation is well recognized in the MCDM community. Negotiation introduces an additional level of complexity to the conflicting objectives. This is because not only may one participant have conflicting objectives, but there are multiple participants each with their own objectives and only limited knowledge about the objectives and individual conflicts of the others.

Conflict also arises because the decision makers are interdependent. Realization of the objectives of one depends on the others and conversely, she can influence their outcomes. Interdependence can arise directly at the objective level as is often the case in bargaining. It can also be at the resource (constraints) level when different individuals control complementary resources or both at objective and resource levels.

Conflict and interdependence often lead to *calculated behaviour* which is based on the premise that the more information one has about the others, the better the calcu-

lations are and thus the better her position can be. This type of behaviour is directly related to another key aspect of negotiation and other decisions made by groups, namely *formulation of messages*. This involves definition of the issues or decision attributes, constraints and possibly certain objectives and goals. Though joint problem definition is critical in negotiations, it is unusual that a complete problem representation can be obtained. This is because of the use of strategies and some degree of concealment and mistrust. In less competitive or conflicting situations, a significant effort may be on the joint problem definition. Recently, several support systems have been introduced that focus on this aspect of the decision process (Eden, 1992; Hitchcock et al., 1994; Friend, 1989).

The exchange of information also involves exchange of offers or compromise proposals. These can be a complete proposal, that is a decision alternative, or a partial proposal in which values of only some attributes are given. The latter type of exchange is typical in, for example, intra-organizational decisions involving individuals and groups from different functional areas. Formulation of the partial proposals and their integration into a complete alternative can be effectively supported with the same above-mentioned brainstorming and meeting systems that are used for joint problem formulation.

3.2 Process

In outlining the main aspects of group decisions and negotiations, we mentioned several types of systems which can be used for support. To further delineate the possibilities of support in making negotiated decisions and to position the support systems in the decision process with multiple participants, we outline phases and activities in which the decision makers are involved. They are based on the eight-phase process model of negotiations proposed by Gulliver (1979). The model is modified to include activities typical for group decisions, to allow for a wider range of negotiated decisions than those discussed by Gulliver (1979), and to allow for the present and future use of computing and communication technologies.

1. *Search for arena and selection of the communication mode.* The participants select and agree on the location where the decision process may occur. Communication technologies allow for the use of the wide area networks bringing together participants from remote locations and local area networks for inter-office exchange of information. Thus, the location may be a physical or a virtual setting. Selection of the communication mode includes choice of synchronous or asynchronous exchange of information, discussions and negotiation on partial or complete offers, and joint use of experts, mediators and facilitators. This phase is often neglected by researchers and GDSS/NSS developers despite the fact that any formal method or support tool imposes a particular mode of communication and defines information requirements.

2. *Agenda setting.* Decision makers discuss and agree on the terminology and the issues to be decided upon. These include discussion on decision attributes and hard which are acceptable by all and possibly on certain goals and objectives. The activities of this phase set the stage for subsequent phases. They involve the construction of at least a partial problem representation and are considered key to "correct" decision processes (Buchanan and Henig, 1995). Recently, interesting techniques borrowed

from Artificial Intelligence have been proposed to establish similarities between concepts (terms) presented by different decision makers and to categorize these concepts into consistent and meaningful groups.

3. *Exploring the field.* This phase involves further problem specification and its analysis. In negotiation, the parties try to establish limits to the issues, formulate their best alternatives to the negotiating agreement (BATNA), establish reservation prices and aspiration levels for specific objectives (Raiffa, 1982), assess the opponent and the nature (strength) of opposition, and decide on the initial strategies to use. In group decisions, the limits and aspiration levels may be jointly formulated. The differences in problem understanding and specification of the areas of contribution of particular decision makers takes place here. Computer-based support, in addition to the meeting and brainstorming systems mentioned above, has a significant potential here. In fact, early support systems were used in this phase for the purpose of simulation and analysis of the implications of decision alternatives. Analytical and simulation models allow for the assessment of the decision situation, the participants' behaviour and possible process evolution. This analysis includes determination of efficient solutions and the critical constraints and objectives.

4. *Narrowing the differences and search for integration.*[1] Learning of the others' limitations, aspirations and objectives, and knowledge about efficient solutions and their outcomes, is refined in this phase through intensive exchange of information. In negotiations, the parties realize the potential of a compromise and can assess its main features. This allows them to identify the key issues, critical areas of disagreement and exchange specific and substantive proposals. Analysis of negotiation may focus on the selection and verification of strategies, the determination of concessions and revision of aspiration levels, and on the restriction of efficient solutions to these which may be acceptable to the parties. This can be supported with systems using MCDM-based methods (for example, Korhonen and Wallenius, 1988; Bui, 1994; Ethamo et al. 1996) and those utilizing MAUT (Rangaswamy and Shell, 1994).

In group decisions, common threads are sought for partial decisions and analysis of the implications of these decisions is performed. This allows for the identification of the critical issues for all members of the group. During this phase, the negotiating parties and group members may also identify the critical constraints or interrelationships. Softening the limitations may allow expansion of the set of feasible alternatives or new alternatives may be explicitly formulated and subjected to discussion. Analysis and support tools may be used to identify the critical constraints, demands, and degree of opposition between parties (Kersten and Noronha, 1996).

5. *Search for agreement and improvements.* Successful identification of the critical issues and areas of disagreement, development of joint proposals or joint softening of

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[1] We integrated two phases proposed by Gulliver ("Narrowing the differences" and "Final bargaining") into one because in many negotiations these phases are indisguishable and because the use of analytic and support tools should allow for the exchange of substantive proposals already in the "Narrowing the differences" phase. Also, the preceeding phase has been expanded to include assessment of the feasible and efficient alternatives which allows the parties to move into the phase of the exchange of proposals.

limitations, leads the parties or group members to identification of a limited number of possible compromises. At this stage, the parties or group members may have already agreed on some issues with few issues remaining outstanding.

If the achieved compromise is non-efficient, then the analytical tools may be used to identify efficient ones that benefit all the parties. The same methods which are used in phase 4 can be used to determine an efficient alternative to the achieved compromise, that is, "post-settlement settlement".

4 Models and systems

4.1 Game theory

Many theories of negotiation and group decision have a common heritage in game theory which focuses on the strategic interactions of rational decision makers. The game theoretic assumptions of perfect rationality and perfect or near perfect knowledge of all parties, led to the prescriptive/prescriptive approach (dictating what all the parties should do to achieve a stable compromise).

Game models assume that the number and identity of players, and alternatives and preferences are fixed and known, and that communication takes place only within the model and does not affect either the form or the content of a game's payoff matrix. The usual division of games is between cooperative and non-cooperative games. Cooperative games assume that a binding agreement can be imposed on the parties and posit the question of the division of the value of the game. Cooperative game theory is axiomatic and concentrates its efforts on the search for efficient solutions and the rules for choice from among different efficient solutions (e.g., stability, fairness, equity). The difficulty is, however, that cooperative games assume some form of collective (social) rationality and lack a coherent, unifying framework (Siebe, 1992).

The theory of non-cooperative games assumes that an agreement can be achieved only if it is in the parties' interests and the question is one of the rational strategies that parties choose in the situation of conflict. The focus is on a solution which is self-enforcing in the sense that a party cannot get an advantage by unilaterally defecting from this solution. That is, the agreement among parties should possess a self-stabilizing character or self-enforcing elements. This leads to the concept of the equilibrium point as stipulated by Nash (1954) within the context of non-cooperative game theory. Further research in game theory expanded the solution concepts to other types of stability that differ in their foresight and disimprovement ability (for a review of different solution concepts, see Fang, Hipel and Kilgour, 1993). The difficulty here is that efficient solutions seldom have an equilibrium property, in other words, a stable solution may often be improved for all the parties.

Specification and analysis of the equilibria lies at the roots of the Graph Model for Conflict Resolution (GMCR) developed by Fang, Hipel and Kilgour (1993) who also developed a computer-based system for conflict analysis based on the GMCR method. Other refinements of the Nash equilibria have been proposed within the context of dynamic games (Hämäläinen, 1995).

Game theory is the most rigorous approach to conflict resolution. It allows for

formal problem analysis and the specification of well-defined solutions and can therefore be used for an extensive evaluation of the scenarios and specific moves of the parties, their strategies, and the determination of the characteristics of the potential compromise solutions (Sakawa and Nishizaki, 1996).

The weaknesses of game-theoretic approaches include the treatment of the process and its impact on the game itself, and strict rationality assumptions which, for numerous reasons, rarely hold (e.g., imperfect information, parties' cognitive limitations, deception). Game theorists recognize these weaknesses and interesting extensions have been proposed. However, they often regard communication as neither central nor problematic. Negotiations may be characterized as communication superimposed on a game or a new "transcended" game where communicative acts are moves of the game (Jönsson, 1991). Thus, while game-theoretic methods have a significant role to play in the prior or posterior analysis of the group decision or negotiation problems, their usefulness as a support tool during the process is limited.

4.2 Decision analysis and MCDM

Decision analysis has its roots in economics and game theory. Extensions of decision analytic methods to group decisions and negotiations are based on either explicit or implicit creation of a supra decision maker whose aggregate utility is constructed from group members or the parties' individual utilities (Keeney and Raiffa, 1976). This allows the introduction an additional step to methods based on multiattribute utility theory (MAUT) which is the construction of an aggregate utility (value) function. Several support tools adapting the existing methods to group situations have been proposed. These include the Negotiation Assistant (Rangaswamy and Shell, 1994) and ICANS (Thiessen et al., 1996), extensions of the analytic hierarchy process (AHP), (Saaty, 1989; Saaty and Alexander, 1989) to multiplicative AHP (Van den Honert and Lootsma, 1995), interval AHP (IAHP) which is the core of the HIPRE-3+ Group Link system (Hämäläinen and Kettunen, 1994), use of AHP together with the Tchebycheff distance measure (Iz, 1992) and the simple multiattribute rating technique (SMART), (Edwards, 1977; Lootsma, 1995).

The difficulty with the specification of one compromise decision lies in the assessment of weights of the aggregate utility function which reflects the parties' power, intensity or convictions. One possibility is to resort to a voting procedure. Another is to focus on the set of possible compromises or identify the contract curve. For two-party negotiations, Angur and Lofti (1994) use a bi-criteria optimization problem to develop the efficient set.

Specification of the utility (value) functions is not necessary to determine the efficient set or a contract curve. RAMONA and NORA (Teich, 1991; Korhonen et al., 1994) are systems that allow to identify the contract curve either directly or within an envelope formed by the decision makers' preferences. Methods that allow to determine and analyze the efficient frontier are also proposed by Korhonen and Wallenius (1988) and Teich et al., 1995). These methods either guide users in their concession making actions so that they are getting progressively closer to the efficient set, or provide a single negotiation text which the users can modify in order to increase their joint gains.

Another extension of MCDM methods to group decisions and negotiations is the use of goal programming (Iz and Krajewski, 1992; Kersten, 1988). The role of the method is to provide users with feasible compromise proposals determined through minimization of distance between solutions selected by individual users. A similar approach, but based on user defined reservation and aspiration levels, has been proposed by Reeves and Bordetski (1995) and by Bronisz et al. (1988) in MCBARG system.

The main contributions of MCDM to negotiation analysis and support are: consideration of the negotiation process in addition to negotiation outcome which is the predominant focus of game theory; the use of criteria which is more meaningful for decision makers than utilities; search for the set of efficient compromise solutions rather than one solution; and, interactivity which allows decision makers to participate in and control the decision process. Many MCDM approaches are more intuitive and easier to use than those based on MAUT, and thus they can be used directly by the decision makers themselves. On the other hand, the process of finding a compromise decision is often prolonged and requires many iterations. As in decision analysis, the axiomatic rationality is also assumed in MCDM although, due to interactivity and generation of complete decisions (compromise proposals), the decision makers may terminate the process selecting an inefficient compromise.

4.3 Negotiation analysis

Negotiation analysis describes a stream of studies on conflict resolution based on decision analysis and MAUT. While this area is not well defined and some of the MCDM approaches can be positioned here, there are several underlying principles following from the overly restrictive assumptions and requirements of decision analysis and, to a lesser extent, MCDM. Both decision analysis and MCDM focused on supporting all decision makers. In education and training, involvement of all parties can be achieved and provide them with relevant knowledge and skills. In real-life negotiations, having all parties use the same system is rarely possible. Therefore, negotiation analysis seeks to advise one party given predictions of the other party's behaviour.

As a result of complexity of negotiation processes and mix of political, psychological, sociological and organizational aspects, which influence the decision process but often cannot be formally represented, causes the axiomatic rationality assumptions rarely hold. Negotiation analysis, in the attempt to provide advice and support, is based on principles of "asymmetrically prescriptive/descriptive" orientation, introduced by Raiffa (1982). This is in contrast with game theory which uses the symmetrically prescriptive approach with fully rational players being analyzed in terms of optimal strategies. That is the decision makers are assumed to be intelligent and goal seeking individuals, rather than exhibiting full game-theoretical rationality (Sebenius, 1992).

One of the consequences of the rejection of the axiomatic rationality is that the decision makers may not achieve an efficient compromise. Negotiation analysis attempts to focus on the "zone of feasible agreements" without restriction as to its efficiency. The presumption is that parties can assess the attractiveness of the no-agreement situation and select those possible agreements which yield higher utility. This

is achieved with the specification of the best alternative to negotiated agreement (BATNA) which serves as a benchmark for compromise decisions.

Agreement on an inefficient compromise does not terminate the process. The prescriptive aspect of negotiation analysis is seen in the concept of "post-settlement settlement" in which the analyst or mediator suggests one or more alternatives which all the parties prefer more than the achieved compromise.

Most of the existing analytical approaches assume a well-defined and fixed situation (issues, parties, feasible alternatives) that is being modelled and analyzed. This allows construction of mappings between the structure of the problem, preferences and criteria, and the ultimate outcome which is a compromise or a set of efficient solutions. However, purposive behaviour of the parties can change the problem structure and other elements of the situation and hence the outcomes. The situation has similar effects on the outcomes when the problem is not completely specified. Negotiation analysis attempts to take the evolving nature of the process into account through "negotiation arithmetics, adding and subtracting issues and parties, attitudinal restructuring and investigation of the changes in the parties' perceptions" (Lax and Sebenius, 1986).

Negotiation analysis is more of a framework of thought in which some analytical methods are used than a unified formal approach to negotiations. Its contribution lies in the consideration of aspects and features which were disregarded in game theory and decision analysis but which take place in real-life situations. Some of these aspects were introduced in formal models and support systems. For example, the evolutionary aspect has been incorporated into NEGO (Kersten, 1988) and MEDIATOR systems (Shakun, 1988); and management of the process has been analyzed by Nyhard and Samarasan (1987).

4.4 Cognitive theory

Cognitive processes, among decision makers, assume special significance when the choice situation is characterized by "structural uncertainty". This is the case when the situation is: (i) new and without familiar clues; (ii) complex with many clues to be taken into account; or (iii) contradictory and with different elements suggesting different interpretations. Many group decision and negotiation situations are eminent examples of structural uncertainty.

Applying a cognitive approach focuses analysis on perception and interpretation leading to the construction of representations and the use of heuristics. Problems are recognized and interpreted through references to pre-existing knowledge structures (schemata or belief systems). Specific issues and behaviours of people are categorized and interpreted with the use of judgmental heuristics that reduce complex inferential tasks to simple operations. Thus formation of an innate representation, which for complex and difficult problems can only be partial, has its roots in the individual's past experience and her knowledge of similar situations (Bazerman and Neale, 1991),.

Group decisions and negotiations are characterized by the co-existence of both cooperative and conflicting issues. A prerequisite for success is that the interests which are common outweigh those which are divergent. Cognitive theory may contribute to a better understanding of the issues, contradictions and negative perceptions (Bonham,

1993). For example, it may be used to explain the persistence of negative images of the adversaries and perception of conflicts suggesting possible modifications of conflictual perceptions.

Tools that are based on cognitive theory are used for the development of structural problem representations, and in that sense, their main contribution is to reduce the structural uncertainty. COPE developed by Eden and his associates (1992) and Strategic Choice (Friend, 1992) are examples of systems that allow for the development of cognitive maps reflecting problem perception of one or more participants. The ability to formulate a problem in a form of a map which can be easily reviewed and analyzed, already provides a significant support. In addition, these systems allow for map manipulation, analysis of its consistency and focus on specific aspects of the problem, its implications and determinants.

The difficulty in the integration of techniques based on cognitive maps and normative or prescriptive methods, lies in the main difference between intuitive and scientific theories. Intuitive theories are used to make maps but they are implicit and often used below the level of awareness. Therefore, they are both user and problem context specific and are not easily amenable to formal models derived from scientific theories which are generic in that they can be applied to a wide range of problems. One of the challenges for the developers of analytical GDSS/NSS is to determine ways of using cognitive mapping techniques to build and manipulate representations which can then be analyzed and solved.

4.5 Artificial intelligence

Cognitive mapping techniques involve symbol manipulation which is defined by the map constructor. Symbol manipulation controlled by reasoning mechanisms embedded in a computer system is conducted by knowledge-based systems (expert systems). Classical knowledge-based systems contain destilled and compiled knowledge about a specific problem or a class of problems and are used to provide expertise to their users. For negotiation support, Vedder and Mason (1987) developed a Hostage-taking Information and Tactics (HIT) system to facilitate the assessment of the hostage taking situation and suggest negotiation tactics for police negotiators. Eliasberg et al. (1992) developed NEGOTEX, a rule-based system for preparation of international negotiation including the cultural aspects of the negotiating parties. These systems use knowledge obtained from the experts and academic and practitioner publications.

Another approach to analyze a specific negotiation situation and provide recommendations is through the use of past cases. Sycara (1990, 1991) introduces case-based reasoning to planning and support of labor negotiations. Through the manipulation and merger of case elements, representation of a given negotiation problem is obtained. The construction of a new case which represents the negotiation that the user faces is guided by predefined utility values assigned to elements of past cases. The cases are also used to obtain argumentation for particular proposals, to determine interrelationships between elements of representations, and to specify options for problem restructuring.

Knowledge-based systems rarely allow for the generation and selection of alternatives or for the evolving nature of problem solving and decision making. Restruc-

turable modelling is an attempt to incorporate these two characteristics (Kersten et al., 1991). The Negoplan system which is based on restructurable modelling, has been applied in trade negotiations, labour negotiations (Matwin et al., 1989), and negotiations with a hostage taker (Kersten and Michalowski, 1989).

An interesting approach to determine and evaluate negotiation tactics has been proposed by Matwin et al. (1991) with the use of genetic-based learning algorithms. The prototype system GBML allows the user to specify the relevant issues which are used for simulation of the negotiation problem and generation of negotiation rules with genetic operations of reproduction, crossover and activation. GBML and Negoplan are examples of systems that combine a symbolic and quantitative representation, and aim at merging analytical and knowledge-based approaches.

5 Future work

We reviewed models and systems for group decision and negotiation analysis and support from two perspectives: the process and the areas of study. In the first perspective, the main activities of decision makers are considered. In that way, we took into account the users' needs but only partially. Further studies are needed to define the requirements of specific activities, to delineate the role of a support system and consider decision makers with different abilities and needs.

The second perspective is the continuation of the work of Teich et al. (1994) aiming at presentation of different studies about the same social phenomenon. It is far from complete. Many models and systems are not mentioned and behavioural and social studies are not discussed. More detailed and more comprehensive discussion is obviously needed. It should set the grounds for cross-fertilization of the MCDM field and development of systems in which different perspectives and paradigms can be included. If one accepts that the outreach strategy should be chosen, then one should follow the MCDM pioneers who went outside of their own fields to expand their decision modelling paradigms. They succeeded and developed formal decision models based on multiple criteria which were discussed in the context of accounting, organizational bahaviour and marketing. This led them and their students to conduct research on conflicting objectives, efficient solutions, interactive methods and, more recently, group decisions and negotiations. To continue their "outreach" efforts is to challenge our assumptions, study new paradigms and enrich the MCDM methodologies.

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N-Person Cooperative Games with Multiple Scenarios

Masatoshi Sakawa[1] and Ichiro Nishizaki[2]

[1] Department of Industrial and Systems Engineering, Faculty of Engineering, Hiroshima University, 1-4-1 Kagamiyama, Higashi-Hiroshima 739 Japan
[2] Faculty of Business Administration and Informatics, The Setsunan University, 17-8 Ikeda-Nakamachi, Neyagawa, Osaka 572 Japan

Abstract: In this paper, solution concepts for an n-person cooperative game with multiple scenarios are considered. A characteristic function of the n-person cooperative game with multiple scenarios associates a subset of the set of all players with its real vector value. We consider the nucleolus, which is one of solution concepts based on the lexicographical framework in an n-person cooperative game, in the game with multiple scenarios. To define extended nucleoli, three aggregation methods using weighting coefficients, a minimum component, and constraints are employed. The computational methods for the three extended nucleoli are developed by repeatedly solving linear programming problems. Finally, a numerical example illustrates the proposed methods.

1. Introduction

We intend to consider an n-person cooperative game with possible multiple scenarios. An n-person cooperative game is represented by the set of all players and a characteristic function. The characteristic function in a conventional game associates a subset of players with its real scalar value. In contrast, a characteristic function in a game with multiple scenarios associates a subset of players with its real vector value. Such a game were considered by Bergstresser and Yu [1] in the study on multiobjective games. They called them multiobjective cooperative games but we think it would be fitting to call them cooperative games with multiple scenarios or cooperative games under uncertainty.

Bergstresser and Yu mainly considered the core defined by domination structures and referred to a couple of solution concepts which yield a unique solution in n-person cooperative games. However such solution concepts have never been fully researched.

The nucleolus is a solution concept yielding a unique solution defined by Schmeidler [8] in a conventional n-person cooperative game. In this paper, we consider the nucleolus in games with multiple scenarios. To define extended nucleoli, three aggregation methods using weighting coefficients, a minimum component, and constraints are employed. Computational methods for the three extended nucleoli are developed by repeatedly solving linear programming problems. Finally, a numerical example illustrates the proposed methods.

2. Game theoretic approaches to decision making problems

In this section, we briefly review applications of conventional cooperative games to managerial and public decision making problems, and then consider the necessity of studying cooperative games with multiple scenarios.

In managerial and public decision making problems, we can find a lot of projects that multiple decision makers cooperatively carry out. One of the most important problems in such a project is how to share the costs of the project among the decision makers (participants) or how to distribute the revenues.

Owen [7] studied a linear production problem, which is a decision making problem yielding a revenue allocation problem, by utilizing the cooperative game theory. The problem can be regarded as a production planning problem with multiple subsidiaries or subcontractors. All of the subsidiaries gather with each resources to produce some products. If a coalition of subsidiaries constructs a smaller project to produce a port of all the products, a maximal resulting profit by producing them is thought to be a value of the coalition. The problem of revenue allocation is to provide a rule for allocating the value of the grand coalition among the member of the grand coalition (all of the subsidiaries), and Owen presented an allocation of the value as a solution of the cooperative game. Kalai and Zemel [4] considered a network flow problem which is a problem of transporting a commodity from the source to the sink. Arcs in the network are owned by different participants of the transportation project. They examined the problem which was to allocate the resulting costs or profits among the participants by a cooperative game theoretic approach when the maximal flow of the commodity is accomplished. Bird [2] treated a cost allocation problem for a spanning tree. One of the nodes of the graph is a common supplier and the others are multiple users. The allocation of the transportation cost of the supplier was proposed as an imputation in the core of a cooperative game. Dubey and Shapley [3] considered cost or profit allocation problems in optimization problems with multiple decision makers in a cooperative game theoretic approach similar to the study of Kalai and Zemel, and such optimization problems were called controlled programming problems by them.

Each of these studies treats a cost or profit allocation problem yielding accomplishment of a project. The management is supposed to investigate the problem in the early period of the project and in the latter period. In the early period, the problem is examined to decide whether the project is realized or not, or whether a decision maker participates in the project or not. In the latter period, it is done to allocate the cost or profit yielding accomplishment of the project. Uncertainty or risk should be introduced in the examination of the problem in the early period of the project because there is uncertainty with respect to what is going to happen before the completion of the project. The approaches in the above mentioned studies are not well-timed for the decision making of the management in the early period of the project because they do not take uncertainty into consideration. From this viewpoint, a cooperative game model with uncertainty or with multiple scenarios is required, and this paper is intended to treat such a problem.

3. A game with multiple scenarios and the nucleolus in a conventional game

A conventional game is represented by a pair (N, v) of the set of all the players $N = \{1, 2, \ldots, n\}$ and a characteristic function $v : 2^N \to R$. We assume that the characteristic function v is zero-one normalized. Namely, given a game (N, v'), let $v(S) = (v'(S) - \sum_{i \in S} v'(\{i\}))/(v'(N) - \sum_{i \in S} v'(\{i\}))$, $\forall S \subseteq N$. Then the game (N, v) is said to be zero-one normalized. The function v is a real-valued function which associates any coalition $S \subset N$ with its real value $v(S)$. von Neumann and Morgenstern [6] considered the value $v(S)$ as the min-max value in a two-person game with the coalitions S and $N - S$ as two players. Thus, it can be interpreted as the total amount of a side-payment (transferable utility) that a member of S could earn without any help from the players outside of S.

We consider an n-person cooperative game with multiple scenarios. We assume that ℓ kinds of scenarios in the game will be expected in future, a value of coalition should be considered as a vector value. Therefore, a characteristic function V associates any coalition S with its real vector value $V(S) = (v^1(S), v^2(S), \ldots, v^\ell(S)) \in R^\ell$. Thus, the game with multiple scenarios can be represented by (N, V). We apply solution concepts defined in a conventional game (N, v) to the game (N, V). A kind of lexicographical solution, the nucleolus defined by Schmeidler [8] is related to the bargaining set, which is obtained by considering the discussion that may take place during a play of the game. The nucleolus is based on the idea of an excess and, in its definition, the order relation named the lexicographical order is used.

For a game (N, v), let S denote a coalition and let $x \in R^n$ denote a payoff vector. Then an excess of the coalition S with respect to x is

$$e(S, x) = v(S) - \sum_{i \in S} x_i. \tag{1}$$

Let $H_{2^n-2} : R^{2^n-2} \to R^{2^n-2}$ denote a mapping which arranges entries of a $2^n - 2$-dimensional vector in order of decreasing magnitude. The nucleolus, which is a solution minimizing the vector of excesses $H_{2^n-2}(e(S_1, x), e(S_2, x), \ldots, e(S_{2^n-2}, x))$ in lexicographical order, is defined as

$$\begin{aligned} \mathbf{N}(N, v) \quad = \quad & \{x \in X \mid H_{2^n-2}(e(S_1, x), e(S_2, x), \ldots, e(S_{2^n-2}, x)) \\ & \leq_L H_{2^n-2}(e(S_1, y), e(S_2, y), \ldots, e(S_{2^n-2}, y)), \forall y \in X\}, \end{aligned} \tag{2}$$

where X is a set of all the imputations:

$$X = \left\{x \in R^n \;\middle|\; \sum_{i \in N} x_i = v(N) = 1 \text{ and } x_i \geq v(\{i\}) = 0, i = 1, 2, \ldots, n\right\}, \tag{3}$$

and the lexicographical order $\leq_L$ is defined in the following. Let $r(x)$ be a vector arranged in order of decreasing magnitude, i.e., if $i < j$, $r_i(x) \geq r_j(x)$. Then, for any pair of payoff vectors x and y, if $x = y$ or, for the first entry h in which they differ, $r_h(x) < r_h(y)$, x is smaller than or equal to y in lexicographical order. We consider $2^n - 2$ excesses of coalitions except for $\emptyset$ and N in the definition of the

nucleolus because $e(\emptyset, x) = e(N, x) = 0$, where $\emptyset$ is the empty set. When the game (N, v) is zero-one normalized, the set of all the imputations is independent of the characteristic function v, i.e., the set of all the imputations is the same irrespective of v.

4. The nucleoli in a game with multiple scenarios

We consider the nucleolus in a game (N, V) with multiple scenarios. For the game (N, V), excesses of a coalition S with respect to x to ℓ scenarios are

$$e^j(S, x) = v^j(S) - \sum_{i \in S} x_i, \quad j = 1, 2, \ldots, \ell, \tag{4}$$

and in a vector style, they are represented as

$$E(S, x) = (e^1(S, x), e^2(S, x), \ldots, e^\ell(S, x)). \tag{5}$$

We also assume that characteristic functions v^j, $j = 1, 2, \ldots, \ell$ are zero-one normalized and then the set X of all the imputations is the same irrespective of V. Using the excesses $E(S, x)$, we can define three kinds of the nucleolus according to aggregation methods in the game (N, V). We employ three aggregation methods: 1) An aggregation by weighting coefficients $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_\ell)$ is employed. 2) An aggregation by a maximum component $\max_j e^j(S, x)$ is employed. 3) Constraints for excesses are made by specifying admissible levels to them, and then the ℓ scenarios can be aggregated by the excess to the remaining single scenario subject to the constraints. These aggregation methods are well-known in multiobjective decision makings, and we defined the nucleoli in the game (N, V) utilizing them as follows:

For a game (N, V), three kinds of the nucleoli are defined as follows:

1) Let $\lambda \in \{\lambda \in R^\ell \mid \lambda_j \geq 0, \ j = 1, 2, \ldots, \ell, \sum_{j=1}^{\ell} \lambda_j = 1\}$ be a vector of weighting coefficients and an excess of a coalition S be defined as

$$e_I(S, x; \lambda) = \sum_{j=1}^{\ell} \lambda_j e^j(S, x). \tag{6}$$

Then the nucleolus based on the excess $e_I(S, x; \lambda)$ is defined as

$$\begin{aligned} \mathbf{N}_I(N, V; \lambda) = \{x \in X \mid\ & H_{2^n-2}(e_I(S_1, x; \lambda), e_I(S_2, x; \lambda), \ldots, e_I(S_{2^n-2}, x; \lambda)) \\ & \leq_L H_{2^n-2}(e_I(S_1, y; \lambda), e_I(S_2, y; \lambda), \ldots, e_I(S_{2^n-2}, y; \lambda)), \forall y \in X\}. \end{aligned} \tag{7}$$

2) Let an excess of a coalition S be defined as

$$e_{II}(S, x) = \max_j e^j(S, x), \tag{8}$$

and then the nucleolus based on the excess $e_{II}(S, x)$ is defined as

$$\begin{aligned} \mathbf{N}_{II}(N, V) = \{x \in X \mid\ & H_{2^n-2}(e_{II}(S_1, x), e_{II}(S_2, x), \ldots, e_{II}(S_{2^n-2}, x)) \\ & \leq_L H_{2^n-2}(e_{II}(S_1, y), e_{II}(S_2, y), \ldots, e_{II}(S_{2^n-2}, y)), \forall y \in X\}. \end{aligned} \tag{9}$$

3) Let admissible levels to $\ell - 1$ excesses for $\ell - 1$ scenarios be $\bar{\varepsilon}_S^j$, $j \neq \hat{j}_S$, i.e.,

$$e^j(S, x) \leq \bar{\varepsilon}_S^j, \quad j \neq \hat{j}_S. \tag{10}$$

Then the nucleolus with admissible levels for $\ell - 1$ scenarios is defined as

$$\begin{aligned}\mathbf{N}_{III}(N, V; \bar{\varepsilon}) = \{x \in Y(\bar{\varepsilon}) \mid\ & H_{2^n-2}(e^{\hat{j}_{S_1}}(S_1, x), e^{\hat{j}_{S_2}}(S_2, x), \ldots, e^{\hat{j}_{S_{2^n-2}}}(S_{2^n-2}, x)) \\ \leq_L\ & H_{2^n-2}(e^{\hat{j}_{S_1}}(S_1, y), e^{\hat{j}_{S_2}}(S_2, y), \ldots, e^{\hat{j}_{S_{2^n-2}}}(S_{2^n-2}, y)), \forall y \in Y(\bar{\varepsilon})\},\end{aligned} \tag{11}$$

where

$$Y(\bar{\varepsilon}) = \left\{x \in X \;\middle|\; e^j(S, x) \leq \bar{\varepsilon}_S^j,\ j \neq \hat{j}_S\right\}, \tag{12}$$

and $\hat{j}_S$ is the scenario for which an admissible level is not specified.

Before considering computational methods for the nucleoli in a game (N, V), we review the computational method for the nucleolus in a conventional game (N, v). For the game (N, v), the following linear programming problem is formulated:

$$\left.\begin{array}{ll} \text{minimize} & \varepsilon \\ \text{subject to} & v(S) - \displaystyle\sum_{i \in S} x_i \leq \varepsilon, \ \forall S \neq \emptyset, N \\ & x_1 + x_2 + \cdots + x_n = v(N) = 1 \\ & x_i \geq 0, \ \ i = 1, 2, \ldots, n, \end{array}\right\} \tag{13}$$

where $\emptyset$ is the empty set, and the problem (13) is solved. If an optimal solution (x^*, ε^*) to (13) is unique, the solution x^* is the nucleolus. Otherwise the active inequality constraints are replaced with the equality constraints $v(S) - \sum_{i \in S} x_i = \varepsilon^*$ and the renewed linear programming problem is solved. By repeating the above procedure, the unique optimal solution x^* can be obtained and it is the nucleolus in the game (N, v).

We present algorithms for computing the nucleoli in a game (N, V) in a way similar to the algorithm for the nucleolus in the game (N, v). For the first nucleolus $\mathbf{N}_I(N, V; \lambda)$, the following linear programming problem is formulated:

$$\left.\begin{array}{ll} \text{minimize} & \varepsilon \\ \text{subject to} & \displaystyle\sum_{j=1}^{\ell} \lambda_j v^j(S) - \sum_{i \in S} x_i \leq \varepsilon, \ \forall S \neq \emptyset, N \\ & x_1 + x_2 + \cdots + x_n = 1 \\ & x_i \geq 0, \ \ i = 1, 2, \ldots, n. \end{array}\right\} \tag{14}$$

The nucleolus $\mathbf{N}_I(N, V; \lambda)$ can be obtained by repeatedly solving the above linear programming problem and renewed problems.

For the second nucleolus $\mathbf{N}_{II}(N, V)$, the following linear programming problem

is formulated:

$$\left.\begin{array}{ll} \text{minimize} & \varepsilon \\ \text{subject to} & v^1(S) - \sum_{i \in S} x_i \leq \varepsilon, \ \forall S \neq \emptyset, N \\ & v^2(S) - \sum_{i \in S} x_i \leq \varepsilon, \ \forall S \neq \emptyset, N \\ & \quad \vdots \\ & v^\ell(S) - \sum_{i \in S} x_i \leq \varepsilon, \ \forall S \neq \emptyset, N \\ & x_1 + x_2 + \cdots + x_n = 1 \\ & x_i \geq 0, \ \ i = 1, 2, \ldots, n. \end{array}\right\} \tag{15}$$

The nucleolus $\mathbf{N}_{II}(N, V)$ can be also obtained by repeatedly solving the above linear programming problem and renewed problems.

For the third nucleolus $\mathbf{N}_{III}(N, V; \bar{\varepsilon})$, admissible levels $\bar{\varepsilon}_S^j$ to excesses must be specified at first. The minimum excess is $e_{S,min}^j = \min_{x \in X} e^j(S, x) = v^j(S) - 1$, and the maximum excess is $e_{S,max}^j = \max_{x \in X} e^j(S, x) = v^j(S)$. Each of the levels $\bar{\varepsilon}_S^j$, $j \neq \hat{j}_S$ must satisfy the condition

$$e_{S,min}^j \leq \bar{\varepsilon}_S^j \leq e_{S,max}^j, \tag{16}$$

i.e.,

$$v^j(S) - 1 \leq \bar{\varepsilon}_S^j \leq v^j(S) \tag{17}$$

After admissible levels $\bar{\varepsilon}$ satisfying (16) are specified, the following linear programming problem is formulated:

$$\left.\begin{array}{ll} \text{minimize} & \varepsilon \\ \text{subject to} & v^{\hat{j}_S}(S) - \sum_{i \in S} x_i \leq \varepsilon, \ \forall S \neq \emptyset, N \\ & v^j(S) - \sum_{i \in S} x_i \leq \bar{\varepsilon}_S^j, \ \forall S \neq \emptyset, N, \ \forall j \neq \hat{j}_S \\ & x_1 + x_2 + \cdots + x_n = 1 \\ & x_i \geq 0, \ \ i = 1, 2, \ldots, n. \end{array}\right\} \tag{18}$$

If an optimal solution (x^*, ε^*) to (18) is unique, the solution x^* is the third nucleolus $\mathbf{N}_{III}(N, V; \bar{\varepsilon})$. Otherwise the active inequality constraints on the first scenario are replaced with the equality constraints $v^{\hat{j}_S}(S) - \sum_{i \in S} x_i = \varepsilon^*$ and the renewed linear programming problem is solved. By repeating the above procedure, the unique optimal solution x^* can be obtained and it is the third nucleolus $\mathbf{N}_{III}(N, V; \bar{\varepsilon})$.

5. A numerical example

We consider a three-person cooperative game with three scenarios. For all the scenarios, values of one-person coalitions are zero, and the value of the grand coalition is one, because we consider a zero-one normalized game. Values of the rest of coalitions are shown in Table 1.

Table 1. Values of coalitions.

	Scenario I	Scenario II	Scenario III
{1, 2}	0.3	0.3	0.1
{1, 3}	0.5	0.3	0.5
{2, 3}	0.4	0.7	0.7

For the first nucleolus $\mathbf{N}_I(N, V; \lambda)$, let weighting coefficients λ be (0.5, 0.2, 0.3). Then the following linear programming problem for $\mathbf{N}_I(N, V; \lambda)$ is formulated:

$$
\begin{aligned}
&\text{minimize} && \varepsilon \\
&\text{subject to} && x_1 + \varepsilon \geq 0 \\
& && x_2 + \varepsilon \geq 0 \\
& && x_3 + \varepsilon \geq 0 \\
& && x_1 + x_2 + \varepsilon \geq 0.24 \\
& && x_1 + x_3 + \varepsilon \geq 0.46 \\
& && x_2 + x_3 + \varepsilon \geq 0.55 \\
& && x_1 + x_2 + x_3 = 1 \\
& && x_i \geq 0, \quad i = 1, 2, 3.
\end{aligned}
$$

For the second nucleolus $\mathbf{N}_{II}(N, V)$, the following linear programming problem is formulated:

$$
\begin{aligned}
&\text{minimize} && \varepsilon \\
&\text{subject to} && x_1 + \varepsilon \geq 0 \\
& && x_2 + \varepsilon \geq 0 \\
& && x_3 + \varepsilon \geq 0 \\
& && x_1 + x_2 + \varepsilon \geq 0.3 \\
& && x_1 + x_2 + \varepsilon \geq 0.1 \\
& && x_1 + x_3 + \varepsilon \geq 0.5 \\
& && x_1 + x_3 + \varepsilon \geq 0.3 \\
& && x_2 + x_3 + \varepsilon \geq 0.4 \\
& && x_2 + x_3 + \varepsilon \geq 0.7 \\
& && x_1 + x_2 + x_3 = 1 \\
& && x_i \geq 0, \quad i = 1, 2, 3.
\end{aligned}
$$

For the third nucleolus $\mathbf{N}_{III}(N, V; \bar{\varepsilon})$, admissible levels $\bar{\varepsilon}_S^j$ are specified in Table 2.

Table 2. Admissible levels.

	Scenario I	Scenario II	Scenario III
{1}	-0.1	–	-0.1
{2}	-0.1	–	-0.1
{3}	-0.1	–	-0.1
{1, 2}	-0.3	–	-0.05
{1, 3}	-0.1	–	0.0
{2, 3}	-0.1	–	-0.05

Then the following linear programming problem for $\mathbf{N}_{III}(N, V; \bar{\varepsilon})$ is formulated:

$$
\begin{aligned}
\text{minimize} \quad & \varepsilon \\
\text{subject to} \quad & x_1 + \varepsilon \geq 0 \\
& x_1 \geq 0.1 \\
& x_2 + \varepsilon \geq 0 \\
& x_2 \geq 0.1 \\
& x_3 + \varepsilon \geq 0 \\
& x_3 \geq 0.1 \\
& x_1 + x_2 + \varepsilon \geq 0.3 \\
& x_1 + x_2 \geq 0.6 \\
& x_1 + x_2 \geq 0.15 \\
& x_1 + x_3 + \varepsilon \geq 0.1 \\
& x_1 + x_3 \geq 0.6 \\
& x_1 + x_3 \geq 0.5 \\
& x_2 + x_3 + \varepsilon \geq 0.4 \\
& x_2 + x_3 \geq 0.5 \\
& x_2 + x_3 \geq 0.75 \\
& x_1 + x_2 + x_3 = 1 \\
& x_i \geq 0, \quad i = 1, 2, 3.
\end{aligned}
$$

The three nucleoli can be computed, and the results are shown in Table 3.

Table 3. Payoff vectors.

	Player 1	player 2	player 3
$\mathbf{N}_I(N, V; \lambda)$	0.2250	0.2775	0.4975
$\mathbf{N}_{II}(N, V)$	0.1500	0.3250	0.5250
$\mathbf{N}_{III}(N, V; \bar{\varepsilon})$	0.2500	0.4000	0.3500

6. Concluding remarks

We have defined three nucleoli in a game (N, V) with multiple scenarios by employing three aggregation methods based on weighting coefficients, a minimum component, and constraints. Computational methods for the proposed solutions have been developed, and a numerical example has illustrated the methods.

Finally, we briefly discuss the proposed solution concepts in n-person cooperative multiobjective games. When there are ℓ objectives in the game, a value of a coalition $V(S)$ can be considered as a subset in R^ℓ, and a payoff of each player i is not a scalar x_i but a vector $X_i = (x_i^1, x_i^2, \ldots, x_i^\ell)$.

When each of $V(S)$ is a one point set, the multiobjective game can be reduced to ℓ independent single-objective games (N, v^j), $j = 1, 2, \ldots, \ell$, which is a trivial case. In general, $V(S)$ is a subset in R^ℓ. In a multiobjective game, aggregations of objectives for each coalition or each player can be considered and the nucleoli can be also extended in a similar procedure in this paper.

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Successive Proposals: An Approach to the Computation and Enforcement of Efficient Agreements

Harri Ehtamo, Markku Verkama, and Raimo P. Hämäläinen

Systems Analysis Laboratory, Helsinki University of Technology
Otakaari 1, FIN-02150 Espoo, Finland

Abstract: Negotiations are often characterized by the fact that the participants do not know each others' goals. This paper describes an interactive approach to computing Pareto optimal agreements under such circumstances in two-party negotiations. The approach is based on so called proposals that link the decision variables together. The proposals also provide a way of enforcing the resulting agreement, meaning that no party has an incentive to break loose from it.

1 INTRODUCTION

There is currently growing interest towards group decision and negotiation processes. The topic is especially interesting from the viewpoint of multiple criteria decision aid research, since group decision problems are closely related to multiple criteria decision problems. In conclusion the MCDA methodology provides one approach to aiding decision makers and/or mediators facing a negotiation or a group decision (e.g., [7], [8], [9]). Generally speaking, the key issue in multiple criteria decision problems is the lack of explicit information about the decision maker's preferences, and the focus in MCDA is thus on eliciting that information interactively. When it comes to group problems, the traditional decision analytic approach has so far focused on studying ways of aggregating or combining the individual group members' preferences into a group utility function or into an interval MCDA model (e.g., [5], [6], [8]).

However, certain complications are likely to arise in many group problems, especially in negotiations, and here we would like to draw attention to two of them that are quite important. First, when there are several decision makers involved, we cannot assume that everybody knows everybody else's preferences or goals. In fact, it has been argued that most negotiations are characterized by the participants' unwillingness to disclose private information. Second, if the decision problem requires coordination of the individual decision makers' actions, the participants face an enforcement problem as well. The essential question is how to make sure that everybody is commit-

ted to the agreement. This question cannot be bypassed altogether by just stating that the decision makers, say, sign a binding contract. Evidently, there has been no need to consider these issues in the context of multiple criteria decision making with one decision maker. Our presentation puts the above mentioned questions in the forefront, and we propose ways of dealing with them.

We shall consider a group decision making situation with two parties, both of whom have control over one continuous decision variable. This can be thought of, for example, as a two-party negotiation over two continuous issues. An interactive method will be described with which the decision makers can solve Pareto optimal agreements without completely revealing their private preferences. The method also provides a self-enforcing mechanism that guarantees the sustainability of the agreements. This property will be motivated through game theoretic arguments.

The key element of our approach is the concept of a linear proposal. Here we define a linear proposal so that it links the decision variables of the negotiation problem together in a linear manner. Proposals can thus be represented as lines in the decision (action) space, see figure 1. In the figure we have also depicted a decision pair (x_1^*, x_2^*) such that the isovalue curve of one of the decision makers, DM1, is tangent to the proposal line. Suppose that DM1 and DM2 control x_1 and x_2, respectively. Then we see that DM2 could use the proposal to enforce DM1 to choose x_1^*. This is simply because if DM2 were to act so that the joint decision would always stay on the proposal line, the best DM1 could do would be to choose x_1^*. Now, because Pareto efficient decision pairs are characterized by a joint tangent of the isovalue curves, both decision makers can use the same proposal to enforce a Pareto efficient decision. This has been depicted in figure 2. The method we shall present in this paper solves these joint tangents by iterating with proposals. Hence a Pareto solution is obtained simultaneously with a mechanism to support it.

Computational methods and theoretical results pertaining to the joint tangency problem have been presented in our previous paper [3]. An extension of the method of proposals for more than two DMs is considered in [15]. The results in [15] are relevant to negotiations with as many issues as there are decision makers. In [4] we have examined a different kind of a negotiation procedure for two-party negotiations over continuous issues. That procedure does not require the DMs to disclose private information either, but no attention is paid to the enforcement problem.

A related approach has been developed by Teich [13] (for an application see [14]). The so called RAMONA method can be used to solve Pareto optimal settlements in two-party negotiations over multiple issues. Conceptually RAMONA is based on the same geometric approach as the method of proposals in the sense that it iterates with hyperplanes to find points of joint tangency in the decision space. However, RAMONA uses heuristic rules to adjust the planes, whereas we end up in a fixed point problem that can be solved with

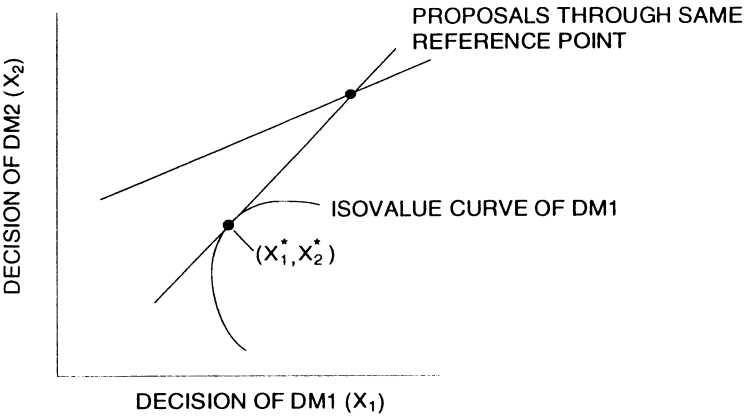

Figure 1: Proposals are represented as lines in the decision space.

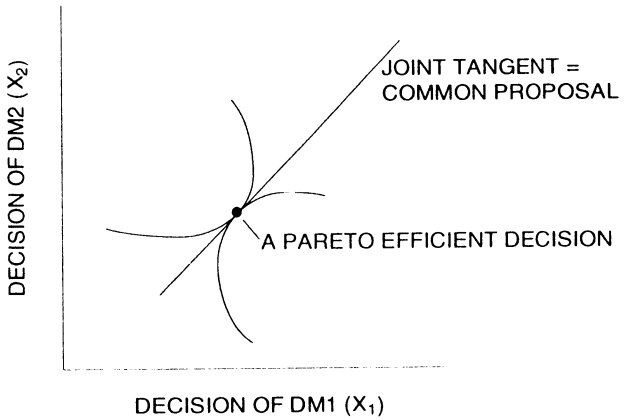

Figure 2: A proposal that supports Pareto optimal decisions.

standard procedures.

2 JOINT TANGENTS AND ENFORCEMENT OF AGREEMENTS

The situation we consider is the following. There are two decision makers, DM1 and DM2, both of whom can take a real valued action x_i $(i = 1, 2)$; we denote the joint decision (or action) by $x = (x_1, x_2)$. The actions can be chosen from a compact and convex set X, and the DMs' goals are described by payoff functions $\pi_i : X \mapsto R$. We assume that π_i are continuously differentiable and strictly concave.

Let us first show how the joint tangent at a given Pareto optimal point could be used to deter deviations. Let $x^* = (x_1^*, x_2^*) \in X$ be a Pareto optimal point that maximizes $\mu\pi_1(x) + (1-\mu)\pi_2(x)$ for some $\mu \in (0,1)$. Suppose x^* is an interior point of X, and denote the tangent of DM2's isovalue curve at x^* by $L(x^*)$. The equation of this tangent can be written as

$$x_1 - x_1^* = \eta(x_2 - x_2^*), \quad (x_1, x_2) \in R^2, \tag{1}$$

where $\eta = -(\partial\pi_2/\partial x_2)/(\partial\pi_2/\partial x_1)(x^*)$. Now, $L(x^*)$ is also tangent to DM1's isovalue curve at x^* so that its equation can equally well be written as

$$x_2 - x_2^* = \eta'(x_1 - x_1^*), \quad (x_1, x_2) \in R^2, \tag{2}$$

where $\eta' = -(\partial\pi_1/\partial x_1)/(\partial\pi_1/\partial x_2)(x^*)$. Here we have $\eta' = 1/\eta$.

Let us consider the situation in figure 3 where the DMs have jointly agreed upon the solution x^*. We notice that DM2 has an incentive to cheat by taking the action $\hat{x}_2$ instead of x_2^* because $\pi_2(x_1^*, \hat{x}_2) > \pi_2(x_1^*, x_2^*)$. However, if DM1, upon discovering that DM2 cheats, uses equation (1) and changes his action to $\hat{x}_1 = x_1^* + \eta(\hat{x}_2 - x_2^*)$, then $\pi_2(\hat{x}_1, \hat{x}_2) < \pi_2(x_1^*, x_2^*)$; that is, the action taken by DM1 punishes DM2. Hence, it does not pay for DM2 to deviate if he knows that DM1 is going to act according to (1). Likewise, DM2 can use equation (2) to make DM1 to adhere to x_1^*. Equations (1) and (2) thus define linear equilibrium strategies that support the Pareto point in question. These incentive strategies also incorporate a credible threat in the sense that if DMj cheats, DMi will lose less by following his equilibrium strategy than by adhering to x_i^* (this holds for small deviations, see [2], [11]).

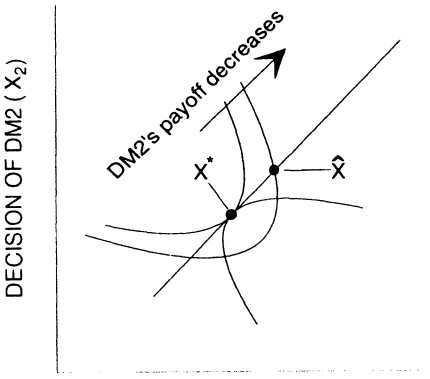

Figure 3: DM1 can use a proposal to deter deviations by DM2.

3 SOLVING JOINT TANGENTS

Next we show how DMi can solve Pareto optimal points by making linear proposals to DMj, and by knowing the (optimal) replies of DMj to these proposals. In the following let $r = (r_1, r_2) \in X$ be a reference point in the decision space, such as a prevailing status quo or a threat point. For a given r the proposals $l(\eta)$ made by DMi are straight lines with the slope η, $\eta \in [\underline{\eta}, \bar{\eta}]$, and passing through r.

Next, suppose that DM1, say, makes a proposal $l(\eta)$ to DM2. The latter responds by maximizing $\pi_2(x_1, x_2)$ subject to

$$x_1 = r_1 + \eta(x_2 - r_2), \; x \in X, \tag{3}$$

and by letting DM1 know the optimal solution $x_2(\eta)$. Let $x_1(\eta)$ be the corresponding action as given by (3), and suppose that $x(\eta) = (x_1(\eta), x_2(\eta))$ is an interior point of X. By first-order conditions, $l(\eta)$ is tangent to the isovalue curve of π_2 at $x(\eta)$. From equations (1) and (2) we see that if the equation

$$\eta = -[\frac{\partial \pi_1}{\partial x_2} / \frac{\partial \pi_1}{\partial x_1}](x(\eta)) \tag{4}$$

holds, then the proposal is also tangent to the isovalue curve of π_1 at $x(\eta)$. In other words, $x(\eta)$ is potentially Pareto optimal (for exact conditions see [3]).

This suggests an iterative interactive procedure for finding joint tangents and Pareto solutions. For a given proposal $l(\eta)$, DM1 can calculate the right hand side of (4) as soon as DM2 has announced $x_2(\eta)$. If we define the function $\chi(\eta) : (\underline{\eta}, \bar{\eta}) \mapsto R$ by

$$\chi(\eta) = -[\frac{\partial \pi_1}{\partial x_2} / \frac{\partial \pi_1}{\partial x_1}](x(\eta)) \tag{5}$$

when $(\partial \pi_1 / \partial x_1)(x(\eta)) \neq 0$, we see that the problem of locating Pareto solutions is mathematically reformulated into the problem of finding the fixed points of $\chi(\eta)$. The decision makers can solve this fixed point problem interactively by making and answering proposals. The whole Pareto efficient set can be generated by varying the reference point suitably.

Example. Let us illustrate the method with an example where two Cournot oligopolists search for joint gains. The actions of the DMs (firms) are their outputs that are sold on the market. Suppose that the market price of the goods is given by the inverse demand function $p(Q) = (5000/Q)^{1/\alpha}$ and that the production costs are

$$f_i(x_i) = \gamma_i x_i + \frac{\beta_i}{\beta_i + 1} L_i^{-1/\beta_i} x_i^{(\beta_i+1)/\beta_i} \tag{6}$$

(for more details see [10]). The DMs then aim to maximize their profits given by

$$\pi_i(x) = p(x_1 + x_2)x_i - f_i(x_i), \quad i = 1, 2. \tag{7}$$

It is interesting to note that many environmental and resource management applications take the form of a Cournot oligopoly as well [12, pp. 198–206].

We suppose that the status quo is the so called Cournot-Nash equilibrium of the market [1]. Joint gains can then be realized by cutting down outputs. For illustration purpose, let us use the parameter values $\alpha = 1.1$, $\gamma_1 = 10$, $L_1 = 0.2$, $\beta_1 = 1.2$, $\gamma_2 = 8$, $L_2 = 0.2$, and $\beta_2 = 1.1$. Now the status quo is $x^{CN} \approx (17.1, 14.7)$, and we consider proposals $x_1 - 17.1 = \eta(x_2 - 14.7)$. Table 1 shows a simulation where DM1 uses the secant method to solve the equation $\eta - \chi(\eta) = 0$ from the initial values $\eta_0 = 0.5$ and $\eta_1 = 0.6$. Convergence is very fast, and the solution turns out to be $\eta^* = 1.1658$, which corresponds to the joint decision $x^* = (5.4, 4.7)$. Figure 4a illustrates the isovalue contours of the payoff functions, while three different iteration steps are illustrated in figures 4b-d.

Table 1. $\eta_0 = 0.5, \eta_1 = 0.6$

k	2	3	4	5	6	7	8
η_k	0.8013	0.9789	1.1207	1.1682	1.1654	1.1658	1.1658

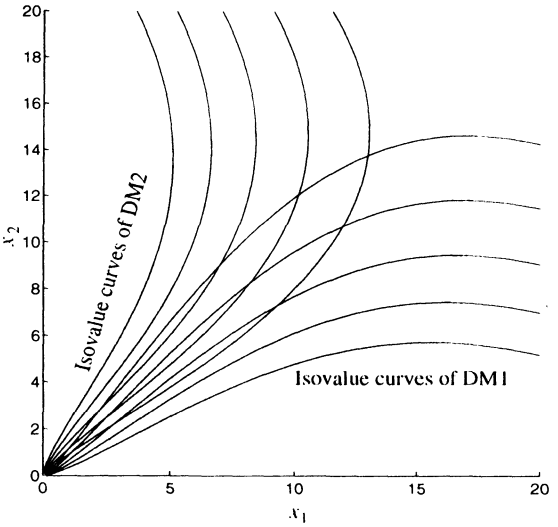

Figure 4a: Isovalue curves of DM1 and DM2 in the decision space (x_1, x_2).

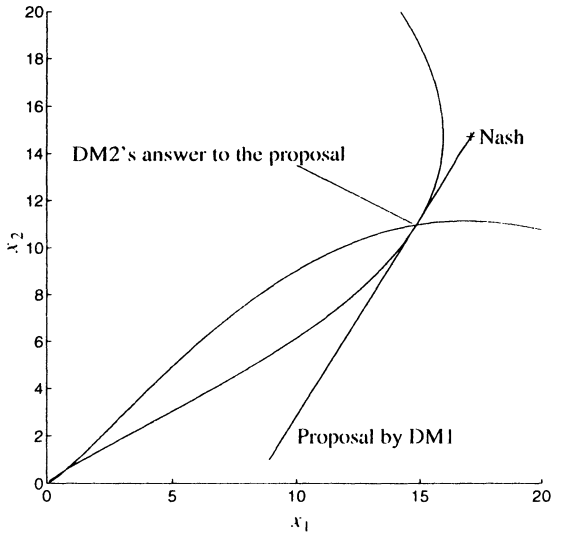

Figure 4b: Isovalue curves through DM2's optimal answer at step 1 ($\eta_1 = 0.6$).

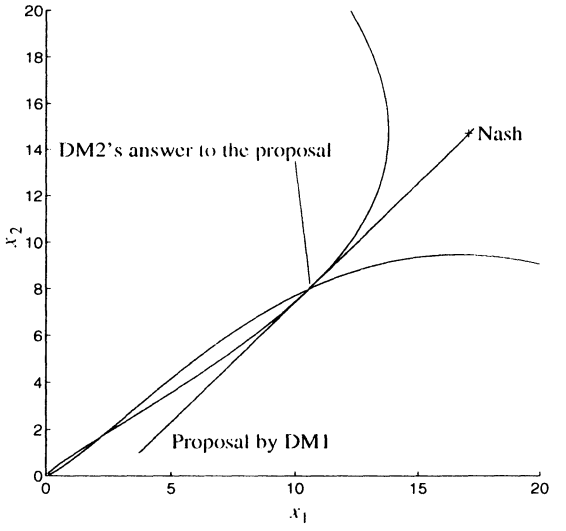

Figure 4c: Isovalue curves through DM2's optimal answer at step 3 ($\eta_3 = 0.9789$).

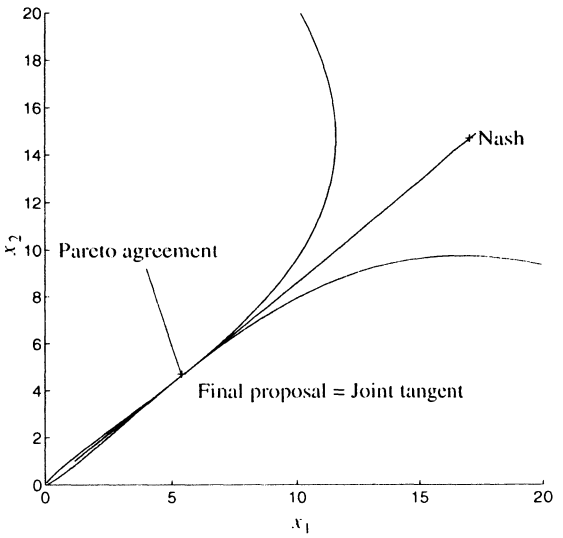

Figure 4d: In the end, a Pareto optimal agreement is found when the proposal becomes the joint tangent of the isovalue curves.

4 CONCLUSION

We have described an interactive iterative approach to solving Pareto optimal agreements in two-party group decision problems. The important advantage of this approach is that the decision makers do not have to reveal their private preferences, and furthermore, an incentive mechanism is provided to support the Pareto optimal agreements. Thus far, little attention has been paid to information and enforcement of agreements, even though they often play a significant role in negotiations.

Avenues for future research are the generalization and practical implementation of the approach. Of interest are also processes that result when several decision makers make proposals to each other and react to these [16].

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Opposition and Inefficiency in Negotiated Compromises*

Gregory E. Kersten[1‡] and Sunil J. Noronha[2]

[1] Department of Information and Systems Management, The Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong

[2] Negoplan Group, School of Business, Carleton University, Ottawa, Ontario K1S 5B6, Canada

Abstract. Several studies have shown that negotiators often do not reach efficient compromises. We analyze the circumstances under which rational agents make inefficient compromises and resist improvements. We do this by describing and interpreting various negotiation situations and by developing formal constructs and theorems for determining the character of a negotiation situation. Key among these concepts is the notion of opposition. The effects of various rationality assumptions on efficiency and their implications for negotiation support systems are discussed.

1 Introduction

Alemi, Fos and Lacorte (1990) conducted experiments with six professionals some of whom were professional negotiators or were often involved in negotiations. These experiments showed that *negotiators more often than not reach inefficient compromises*. They suggest that "If physicians and managers can arrive at Pareto optimal contracts, then their negotiations are effective ....". Also, Teich et al. (1995) conducted experiments with students using two negotiation support systems. They also often did not achieve an efficient compromise. Moreover, when the students were presented with an efficient compromise they did not want to move from the non-efficient to the efficient compromise.

In a discussion on rational choice in games McClennen (1990, p. 259) says:

> "When one turns to consider non-strictly competitive games, the assumption that rational players are bound by the principle of maximization of expected utility proves to be quite paradoxical. Except under very special circumstances, it ensures that rational interaction will fail to satisfy the criterion of Pareto optimality."

This appears to be an unusual remark and one that has no support in the literature on prescriptive decision making, rational agents and the application of

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‡ Visiting Scholar; permanent address: School of Business, Carleton University.

decision analysis to negotiations (Young, 1975; Raiffa, 1982). On the other hand this remark is substantiated by real-life negotiation and by laboratory experiments.

The objective of the paper is to introduce constructs based on the notion of "opposition" for describing and interpreting different negotiation situations, and formal relationships which allow negotiators to reason about the efficiency of their compromises, and thus provide support in the process of offer formulation and concession making. Although opposition is defined in terms of utility functions, we show that it is more fundamental in the sense that it is more intuitive to decision makers and can be used in contexts in which the parties' utilities are unknown or are partially known.

The paper is organized as follows. In Section 2 assumptions and definitions of the nature of opposition are formulated. The formal conditions that must hold for efficient compromises are obtained in Section 3. Behavioral reasons that may cause the negotiating agents to resist accepting an efficient compromise in the situation when a non-efficient one was negotiated are given in Section 4. Implications for negotiation analysis and support are presented in Section 5, which concludes the paper.

2 Preliminaries

We consider *two party negotiations* over issues on which certain constraints, which both parties must obey, are imposed. The negotiations are conducted in the *offer space* $\Re$ which is an n-dimensional space of real numbers. The set $X \subseteq \Re$ of feasible offers is defined by constraints and it is assumed convex. Utilities $U_1(\mathbf{x})$ and $U_2(\mathbf{x})$ of the two parties are functions defined over X and its "neighborhood"; they are in general convex (or concave) functions. Equality and inequality constraints on these functions therefore define convex curves and regions. We assume that each party knows his or her own utility function, but not the other party's.

Definition 1. The *contract curve* (CC) is the subset of the feasible set $X \subseteq \Re$ which contains only efficient solutions.

The set X and the contract curve are illustrated in Fig. 1.

We characterize the nature of the relationships in a negotiation case by defining the *opposition* between two negotiating parties. Since the relationship between the two parties can vary considerably over the offer space (with one pair of offers considered highly antagonistic and another pair being highly cooperative) we need to describe the *local* properties of the relationship. We first begin with the opposition at any *point* of $\Re$ at which the two utility functions are defined.

Definition 2. The parties are in *local strict opposition* at a point $\mathbf{x}^* \in \Re$ iff for all points $\mathbf{x}' \in \Re$ that are sufficiently close to $\mathbf{x}^*$ (i.e., for some ε, $\varepsilon > 0$, such that $\forall \mathbf{x}' \; \|\mathbf{x}' - \mathbf{x}\| < \varepsilon$), an increase of one utility can be achieved only at the expense of a decrease of the other utility.

The above definition can be usefully restated using vector calculus as follows (see Fig. 1). Recall that the *indifference curves* of a utility function $U(\mathbf{x})$ are curves of constant utility value $U(\mathbf{x}) = c$, for some c. The gradient $\nabla U(\mathbf{x})$ at a point $\mathbf{x}$

is therefore perpendicular to the indifference curve of $U(\mathbf{x})$ that goes through $\mathbf{x}$. Further, $U(\mathbf{x})$ increases most rapidly at $\mathbf{x}$ in the direction of the gradient, and it decreases most rapidly in the opposite direction. The rate of increase in any direction $\hat{1}_p$ is given by the dot product $\hat{1}_p \bullet \nabla U(\mathbf{x})$.

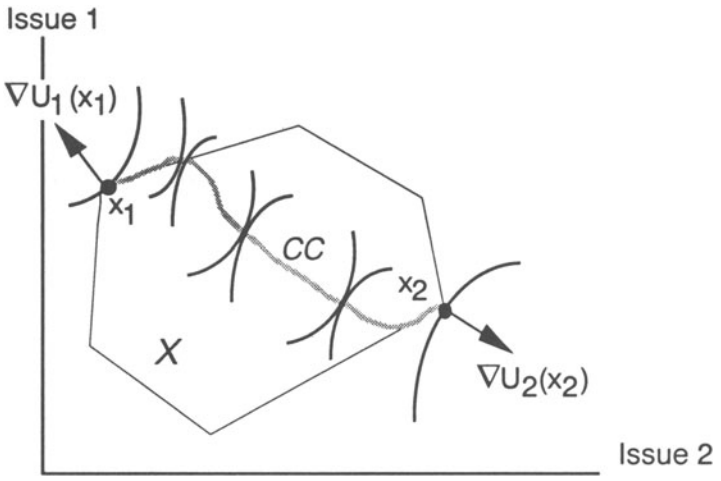

Figure 1. Contract curve and local strict opposition.

Definition 3. The parties are in *local nonstrict opposition* at a point $\mathbf{x} \in \mathfrak{R}$ iff they are not in local strict opposition at $\mathbf{x}$, that is, iff it is possible for both parties to raise their utilities by moving to a point an infinitesimal distance from $\mathbf{x}$.

We also introduce two additional types of opposition based on the tension between the two parties at an offer in terms of the cooperativeness or conflict between the interests (utilities) of the two sides. Since each utility gradient represents the "central interests" or most preferred direction in which a party would like to move, if the two directions are close to each other, the two parties have more scope for cooperatively improving upon the compromise than if the two gradients were in largely opposing directions.

Definition 4. The parties are in *local weak opposition* at a point $\mathbf{x} \in \mathfrak{R}$ iff $\nabla U_1(\mathbf{x}) \bullet \nabla U_2(\mathbf{x}) \geq 0$, i.e., iff the gradients at $\mathbf{x}$ of the two utility functions form an acute or right angle.

The case of weak opposition is depicted in Fig. 2. The fact that at no point of X the parties are in strict opposition causes that the contract curve CC is on the boundary (piecewise linear line joining $\mathbf{x}_1$ and $\mathbf{x}_2$). This is further discussed in Section 3. The tangential points of the indifference curve are outside of X and they constitute the *tangential point curve* (*TPC*). For the unconstrained case, i.e., $X = \mathfrak{R}$ we have $CC = TPC$.

Definition 5. The parties are in *local strong opposition* at a point $\mathbf{x} \in \mathfrak{R}$ iff

$\nabla U_1(\mathbf{x}) \bullet \nabla U_2(\mathbf{x}) < 0$, i.e., iff the gradients at $\mathbf{x}$ form an obtuse angle.

While local opposition allows us to discuss the nature of the opposition for an individual position—which is in or out of the feasible set X—such as an initial offer, a counter-offer and a compromise, we can define global opposition which characterizes—and is restricted to—the entire set of feasible offers.

Definition 6. The parties are in *global strict (nonstrict, weak, strong) opposition* iff for every $\mathbf{x} \in X$ they are in local strict (nonstrict, weak, strong) opposition.

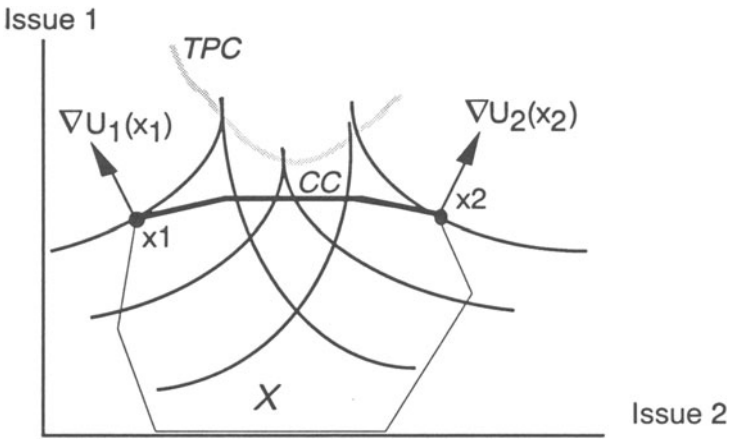

Figure 2. Weak opposition, contract curve and tangential point curve

Another type of opposition that is intermediate in scale between local and global opposition is *offer-pair opposition*. It is defined on a pair of points in offer space. Given that the two parties have made offers $\mathbf{x}_1$ and $\mathbf{x}_2$ respectively, the offer-pair opposition between the two parties is $\nabla U_1(\mathbf{x}_1) \bullet \nabla U_2(\mathbf{x}_2)$, and its nature can be characterized as strong or weak as before, based on whether its value is negative or positive. Offer-pair opposition is useful in modelling the relationship between the two parties at early stages of the negotiation when there is no single point that is considered as a compromise. In such situations, the gradients at the offers indicate which directions are most preferred for each party, and they may choose to take this into account in concession making.

3 Opposition and Compromises

3.1 Conditions for efficient compromises

Given a pair of analytic utility functions, the following theorem establishes purely local criteria that can be applied to determine whether a given position is an efficient point or not.

Let $B(\mathbf{x}) = 0$ denote the equation of the boundary of the set X, defining $\mathbf{x} \in X$ iff $B(\mathbf{x}) \leq 0$. Then $-\nabla B(\mathbf{x})$ is the outward normal direction to the boundary of X at $\mathbf{x}$.

Theorem 1. Offer $\mathbf{x}^* \in X$ is efficient iff, either

(A) $\mathbf{x}^*$ is in the interior of X and the *local* opposition at $\mathbf{x}^*$ is strict, i.e.,

$$\nabla U_1(\mathbf{x}^*) = -k \nabla U_2(\mathbf{x}^*), \tag{1}$$

where k is some positive constant, or

(B) $\mathbf{x}^*$ is on the boundary of X, and for some $\alpha, \beta \geq 0$

$$\alpha \nabla U_1(\mathbf{x}^*) + \beta \nabla U_2(\mathbf{x}^*) = -\nabla B(\mathbf{x}^*). \tag{2}$$

given convex/concave utility functions and a convex feasible set.

Proof: (A) Every point $\mathbf{x}^*$ in X, must lie on some unique indifference curve of each utility function, and the indifference curve of $U_1(\mathbf{x})$ that passes through $\mathbf{x}^*$ is described by the equation $U_1(\mathbf{x}) = U_1(\mathbf{x}^*)$. This curve defines the boundary of a set $C = \{\mathbf{x} \in X \text{ s.t. } U_1(\mathbf{x}) \geq U_1(\mathbf{x}^*)\}$. Clearly, $\mathbf{x}^*$ is efficient iff $U_2(\mathbf{x})$ is maximized within C at $\mathbf{x}^*$ (points outside C are eliminated because those points are dominated with respect to the U_1 criterion, and points at which U_2 is not maximum are dominated by points at which it is). Thus we convert the problem of maximizing both $U_1(\mathbf{x})$ and $U_2(\mathbf{x})$ over X to one of maximizing $U_2(\mathbf{x})$ given $U_1(\mathbf{x}) \geq U_1(\mathbf{x}^*)$.

This constrained optimization problem is easily solved by the standard method of Lagrangian relaxation (e.g., see Papalambros and Wilde, 1988), and immediately gives us the equation $\nabla U_2(\mathbf{x}^*) + \lambda \nabla U_1(\mathbf{x}^*) = \mathbf{0}$, $\lambda \geq 0$, as the general form of the solution, where λ is the Lagrange multiplier. This equation is rewritten as (1) to convey the geometric properties of the solution.

(B) Since the conversion of the two-criterion maximization problem to the constrained single-criterion maximization problem is general, we have the identical requirement, to maximize $U_2(\mathbf{x})$ given $U_1(\mathbf{x}) \geq U_1(\mathbf{x}^*)$ now, however, with an additional constraint, $B(\mathbf{x}) = 0$. Once again applying the Lagrangian relaxation method, we get the general form of the solution as $\nabla U_2(\mathbf{x}^*) + \lambda \nabla U_1(\mathbf{x}^*) + \mu \nabla B(\mathbf{x}^*) = \mathbf{0}$, $\lambda \geq 0$, $\mu \neq 0$. μ is positive or negative according to whether the inner or outer normal direction of $B(\mathbf{x})$ is considered the gradient $\nabla B(\mathbf{x})$; in our case, with $B(\mathbf{x}) \leq 0$ defining the region X, μ is positive. A simple rewriting of this expression obtains (2). ♦

Case A is illustrated in Fig. 1 and case B in Fig. 2.

3.2 Nonstrict opposition

We will not discuss here the case of strict opposition because it has been extensively studied in the economic literature beginning with Edgeworth (1881). We note however, that in strict opposition CC is defined by the tangential points of the indifference curves (utilities).

In the case of global nonstrict opposition, that is, if $\neg \exists \mathbf{x} \in X : \nabla U_1(\mathbf{x}) = -k \nabla U_2(\mathbf{x})$, the contract curve lies on the boundary of X because condition (1) of Theorem 1 does not apply in X and thus (2) holds. Global nonstrict opposition covers cases of weak opposition and the illustration of an example of nonstrict opposition is given in Fig. 2.

The strength of opposition depends both on objectives and preferences. Even a small change in the preferences may move negotiations from local strict opposition at some points within X to global nonstrict opposition. This phenomenon has been observed in both real-life situations and in laboratory experiments (Darling and Mumpower, 1990). Since this change may not be known to the other party, the sequence of offers and counter-offers may be the same.

3.3 Weak and strong opposition

To consider the conditions for the weak and strong forms of opposition we make two assumptions. One, often used in economics and utility theory, is that the utilities are monotonic functions. The second assumption is that rational agents will not negotiate issues on which they are both indifferent. The implication of this assumption is that:

$$\forall\, \mathbf{x} \in X : dU_1(\mathbf{x}) / dx_j = 0 \longrightarrow dU_2(\mathbf{x}) / dx_j \neq 0,\ i=1,2,\ \ldots,\ n \qquad (3)$$

where n is the number of issues. Similarly (3) holds with the utilities switched.

Theorem 3. Given $U_1(\mathbf{x})$ and $U_2(\mathbf{x})$ monotonic, and no irrelevant issues, if there is local weak opposition at any one point in X, then the two parties are in global nonstrict opposition over X.

It is not possible to prove a stronger result: e.g., local weak opposition does not imply global weak opposition; the strength of opposition can vary across X, even under the restrictive monotonicity assumption. However, since the above theorem concerns efficiency (global nonstrict opposition implies that efficient points are located only on the boundary, due to theorem 1), it is quite useful for the purposes of this paper.

Theorem 4 (complementary to 3) Given monotonic $U_1(\mathbf{x})$ and $U_2(\mathbf{x})$ and no irrelevant issues, if we have local strict opposition at any point in X, then the two parties are in global strong opposition.

Proofs of 3 and 4: Monotonicity implies that the derivatives do not change sign, i.e., each derivative is always nonnegative or nonpositive. Thus each of the components of the gradient vectors maintains the same sign throughout X. Since $\nabla U_1(\mathbf{x}^*) = -k\nabla U_2(\mathbf{x}^*)$ at an efficient point $\mathbf{x}^*$, it follows that the corresponding components of the two gradient vectors must have opposite signs or be both zero at $\mathbf{x}^*$. The latter possibility is ruled out by the assumption of no irrelevant issues. By monotonicity, the signs of the components at X must hold throughout X. Consequently if a tangential point exists within X the dot product of the two vectors must be negative throughout X (i.e., the opposition is globally strong). Since local weak opposition at a point $\mathbf{x}' \in X$ is defined by

$$\nabla U_1(\mathbf{x}') \bullet \nabla U_2(\mathbf{x}') > 0 \qquad (4)$$

it follows that it is not possible to have an efficient point within X and simultaneously have weak opposition anywhere in X. That is, local weak opposition at any point in X implies that there are no efficient points in the interior of X (tangential points). ♦

4 Rationalities and Efficiency

In Section 3 we showed that any feasible offer might be an efficient solution, not just on the boundary of X. The conditions are specified for utility functions defined on the *problem attributes* and the parties' preferences. The discussion of inefficient compromises thus far has been based on the structure of the underlying problem and preferences. In this section we consider both problem-structural and behavioural reasons for inefficiency and present several arguments as to why rational agents don't, and often should not, select an efficient compromise.

4.1 Opposition and efficiency

The results obtained in Section 3.3 also hold for offer-pair opposition. Therefore, knowledge of the opposition between the initial pair of offers provides information about the possible location of the efficient points. For example, given weak opposition between this pair, all the efficient points are on the boundaries of feasible set X. If the boundaries are known to both parties then "rational" agents can guess at and approximate the efficient set. If, however, the boundaries are not known or $X = \Re$, then rational, i.e., utility maximizing parties cannot achieve an efficient compromise. This is because in weak opposition any offer can be improved and there are no efficient compromises. Thus, *feasibility* plays a critical role in weakly opposing parties.

In the case of strong opposition, efficient points are likely to be in the interior of X. This implies that the constraints defining X may be insignificant and only the parties' utilities define efficient compromises. Thus, the *acceptability* of the offer to the parties becomes critical. The compromise's efficiency depends on accurate assessment of one's own party's and one's opponent's utilities.

Weak opposition allows for win-win negotiations while strong opposition suggests win-lose negotiations. A significant effort is often made to move the parties from the win-lose situation to win-win. This requires change in the opposition and in their utilities. Attempts to modify utilities are rational if they allow one to move from win-lose to win-win negotiations and expand the "pie". Thus, an initially efficient solution may become inefficient.

4.2 Attributes of the agent

The attributes of the agent are the characteristics that describe the decision maker and her behavior. These include knowledge and cognitive capability, risk attitude, ability and willingness to cooperate, sense of responsibility, power, and leadership. Preferences describe the values and tradeoffs of the agent but they are defined on the problem attributes and do not take into account the agent's attributes.

Attributes of the agent may modify her choice of offers. Perception of power, for example, may impact the scope of concessions but it may also cause selection of different offers for a given utility value. For a given concession level (measured with the utility function), the party may select an offer which yields the lowest utility of the opponent. This may lead to a different sequence of offers and an inefficient compromise. If such a compromise is achieved then rational negotiators

should make a joint attempt to improve it.

In decision analysis, risk and risk attitude is associated with the outcomes and used in estimation of the expected utility. Post-settlement settlement, however, introduces additional risk which negotiators may not be willing to accept. Opening the negotiation process anew may endanger what had already been achieved and, therefore, the negotiators may knowingly accept an inefficient compromise.

4.3 Process attributes

Negotiation is a process that has its own *process attributes* such as time, effort, communication and monetary requirements (Kersten and Noronha, 1995). For example, the parties may achieved a compromise based on the "split the difference" principle because they perceived that the differences between the two last offers do not warrant spending more time and effort on further discussions. This is an example of tradeoffs between the utility of a decision and the agent and process attribute values. Obviously, the split the difference approach may result in an inefficient compromise.

Consideration of the phases of negotiations introduce a qualitative difference between the initial offers, intermediary offers and the final offers (Gulliver, 1979). The first offer is made only after the agent secured sufficient process resources in the pre-negotiation phase. Offers made during the phases of differences exploration and their reduction deplete these resources. Thus, a potential offer is evaluated not only in its contribution to the final compromise but also in its resource implications for both parties.

In the final bargaining phase the parties realize that the compromise is within reach. This may significantly change their perspective as the parties realize that the remaining resources can be used elsewhere. Since the process resources are often very expensive and generic because they can be used elsewhere, there is strong competition for their distribution among different activities.

Negotiators, with the achievement of a compromise become able to use their time and effort on other issues and solve other problems. Students, in the experiments conducted by Teich et al. (1995), completed the exercise and saw no reason to move from the achieved compromises. It may be that they thought that the time they saved in accepting the compromises would be better used elsewhere. The key issue here is that with the achievement of a compromise the trade-off analysis changes from the comparison of potential gains and losses between offers to comparison of these offers with expected gains from solving *other* problems using the available resources.

5 Conclusions

One may argue that the deficiency of the above discussion is its restriction to the utility defined only on the offer space, that is decision attributes. When all the attributes are taken into account then the rational agents will always achieve efficient compromises providing that such a treatment of utility does not lead to self-contradictions or inconsistencies (like Russell's paradox) or to undecidable

problems. We accept this argument; on the philosophical grounds the utilitarian approach will indeed result in efficiency. The question is how it can be translated into formal analysis and support mechanisms in the situation when some of the attributes are initially unknown and may never be known while other attributes are the result of the process which is to be controlled with the utility considerations.

An attempt to apply the utilitarian approach introduces a significant complexity to problem representation and solution. There are no grounds to assume that there is only one utility. Nozick (1993) argues that at least three different utilities need be considered, the evidentially expected utility (a deterministic case is considered here), the causally expected utility and the symbolic utility. While efficiency can be determined with respect to one or two utilities, its absence cannot suggest that the decision maker is not rational. Due to the additional dimensions defining other attributes than those of the problem, the existence of competitive options and other decision problems, and different utilities one can assume that the rational agents will achieve an efficient compromise only in exceptional situations. Unless they disregard a number of aspects and features of the overall negotiation situation they may have important reasons not to follow the efficient frontier.

We derived the concept of the opposition from that of utility and formulated theorems which allow negotiators to determine the set of efficient compromises. We also presented arguments against basing offer formulation on utility and the limitations of utility-based approaches to negotiation analysis and support. This apparent contradiction can be resolved if opposition and its role are viewed in more general terms.

We have introduced four types of opposition and their implications. These are clearly more basic and therefore of wider applicability than utilities; for any type of opposition we have many different utilities. We also think that opposition is a social phenomenon which can be easily understood because it is closely related to conflict or consonance.

Opposition can be approximated or modified with elements (attributes) that are difficult to measure and embedded in utility. The question which the negotiator has to answer is whether an additional issue or attribute weakens or strengthens the opposition and to what degree. The "degree" need not be precise as it is sufficient to know whether the two parties remain strictly opposed or whether the opposition moves from strong to weak or vice versa.

Specification of the nature of the opposition allows for approximation of the efficient set. In the case of weak opposition this set may be approximated with some accuracy because even without knowing the utility functions some boundaries will clearly be efficient. However, this seems less relevant than the knowledge of the critical boundaries. This knowledge allows the parties to focus their efforts on softening of the binding constraints. In the pre-negotiation phase the parties assess the strength of the opposition. If it is weak then they identify the binding constraints and during the phase of difference exploration discuss on the possibility of their softening. Then the parties may concentrate on the offers which are defined by the agreed upon constraints or in their neighborhood.

A significant difference between the use of utility and opposition is the

consideration of the process and joint gains. Joint gains cannot be achieved if the parties are in strict opposition and do not wish to modify it. The efficiency of win–lose negotiations is defined by parties' utilities. Although this type of negotiation cannot be ruled out, experienced negotiators and mediators make an effort to change the nature of the opposition from strict to strong or even weak. Here, also the analysis of the opposition facilitates the process and may help to "increase the pie".

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VIII

New Computer Science Developments and Multicriteria Analysis

An invited leading paper by Antunes and Tsoukiàs presents an introduction to the subject of new computer science developments and their implications to multicriteria analysis. Antunes and Tsoukiàs provide a (non-exhaustive) survey of some new technological achievements (fuzzy sets, multimedia, distributed computing, expert systems, object oriented programming, neural networks, the world wide web) and attempt to explore their specific relevance for the MCDA field. They claim that while "modern" and "new" technologies may be positively used in creating decision support systems integrating MCDA methods, very few improvements can be obtained as far as the theoretical core of MCDA is concerned. The brief analysis of each of these keywords includes an informal introduction to the domain, some issues about their relevant and not relevant application in MCDA as well as some open questions for the future.

Matos and Borges describe a preliminary implementation of the Anchor model (a theoretical basis for decision maker/decision-aid interface), which is capable of producing meaningful sentences and "understanding" a limited set of declarations. This is aimed at making easier the communication between the decision maker and the decision-aid system by constructing a protocol using natural language labels and modifiers to characterize the alternatives. This allows the system to present, instead of numerical values, a natural language label of each solution.

Hanne presents the basic ideas, the implementation, and the application of a neural network-based decision support system, devoted to provide decision support in multiple criteria problems. A key idea is to use the features of different MCDM methods by integrating them. The author discusses the utilization of historical data (especially from former decisions) for learning. By doing so the parameters of a method, some methods, or a network of methods can tuned to adequate the method to the decision situation.

Against Fashion: A Travel Survival Kit In "Modern" MCDA

C. Antunes(*), A Tsoukiàs(+)
(*) INESC - Coimbra, Rua Antero de Quental, 199
3000 COIMBRA, Portugal, e-mail: cantunes@inescc.pt
(+) LAMSADE, Université Paris Dauphine,
75775 Paris Cedex 16, France, e-mail: tsoukias@lamsade.dauphine.fr

Abstract. Computer Science is going through a period of exciting new technological achievements which appear with increasing frequency. Multi-Criteria Decision Aid (MCDA), as any decision support activity, is heavily affected by such evolution, since MCDA methods are implemented and/or integrated in complex computer based information systems.

The paper provides an eclectic (by no mean exhaustive) survey of some new technological achievements (fuzzy sets, multimedia, distributed computing, expert systems, object oriented programming, neural networks, the world wide web) and tries to explore their specific relevance for the MCDA field. The basic thesis claimed in the paper is that while we consider that "modern" and "new" technologies may be positively used in creating decision support systems integrating MCDA methods, very few improvements can be obtained as far as the theoretical core of MCDA is concerned.

1 Introduction

When the editor asked us to contribute a leading paper on the subject of multicriteria decision analysis and new technologies we initially felt embarrassed since "new technologies" is a very large concept perceived in different ways by different people due to different backgrounds. It is therefore difficult to establish which are the relevant areas of this subject for our research field. After some reflection we opted for a "fashionable" presentation. In other words we decided to choose some "keywords" which are fascinating and "modern". Keywords that statistically are easy to find in the titles of papers and in the keyword section of abstracts.

We identified seven such key-words which are:
- fuzzy sets;
- multimedia;

- distributed computing;
- expert systems;
- object oriented programming;
- neural networks;
- the world wide web.

Such keywords correspond to specific areas of computer science (therefore adopting the equivalence new technology = computer science), with the partial exception of the first one, which have been more or less recently adopted by the MCDA area despite their effective scientific age. Normally there is a relevant time gap between the appearance of the specific technological achievement and its use in the MCDA area.

Our brief analysis of each of these keywords includes an informal introduction to the domain, some issues about their relevant and NOT relevant application in MCDA and some open questions for the future.

We definitely do not consider exhaustive our presentation neither of the "new technology" nor of the seven specific subjects. Our aim is to help to clarify the use of these tools and ideas, to argue against their trivial or indiscrimimate use, justified only by fashion reasons, and to provoke discussion about them since we do not claim that our point of view has to be shared by the MCDA community.

2 Fuzzy sets

The idea of a fuzzy set (see Zadeh 1965) is a very simple one and therefore a very powerful one. Considered any set A we can define a membership function $\mu : A \mapsto [0, 1]$ which associates to each element of A a number of the real interval $[0, 1]$ representing the degree by which an object could be in A (membership degree). Such a degree can be viewed under different points of view, but there are two basic approaches.

1. The degree is seen as a measure of uncertainty due to the imperfect knowledge about the elements of A. The best known theory associated to this approach is possibility theory (see Dubois and Prade, 1988) which introduces the concept of "possibility distribution" which is an ordinal measure of uncertainty with an axiomatization weaker that the one of probability.

2. The degree is seen as the result of the vague character of the set A and it measures the "credibility" of the membership of any element in

A. Such a degree can be therefore seen as a truth value in a system with infinite completely ordered truth values (see Goguen, 1969).

The application of such ideas in multicriteria analysis is straightforward. Decision problems are normally affected by uncertainty (much more when information is multidimensional), the relevant information may be poor or ambiguous, the decision objectives may be more or less vague. However we want to emphasize some points.

- When a fuzzy set approach is used it should be clarified if it is an uncertainty representation or a vagueness representation. Since the calculus could be significantly different in the two cases it is not allowed any arbitrary use.

  For instance, an uncertain preference is not the same as a vague preference. In the first case we have a membership degree due to the imperfect knowledge of the decision maker, while in the second case we have a membership degree due to the vague nature of the preference relation. In the latter if we adopt a "preference intensity" point of view we need a cardinal representation which is impossible using a possibility distribution.

- The use of fuzzy sets should not be trivial. For instance, to associate a membership function to a set A and then "cut" the function to a certain value obtaining immediately a crisp set is trivial.

- The use of "valued" binary relations (that is binary relations seen as fuzzy sets) for preference modeling and aggregation purposes should be done in a coherent way. There exist now sufficient axiomatizations in literature (see for example Perny and Roy, 1992 and Fodor and Roubens, 1994)) which outline the foundation of this approach.

- The use of a fuzzy set as an uncertainty distribution instead of a probability one should be justified and argued. The two approaches have a different axiomatization (and therefore a different calculus) which should be compared with the information provided and with the kind of uncertainty present in the decision process (see also Slowinski and Teghem, 1990).

Some open questions about the use of fuzzy sets in MCDA include, but are not limited to:

1. the extension of the expected utility theory under qualitative uncertainty distributions like the possibility one (see Dubois and Prade, 1995);

2. the enhancement and improvement of the "valued binary relations" approach;

3. the analysis of the links between the concept of "preference intensity" and its fuzzy representation.

3 Multimedia

The evolution of multimedia equipment is a result of the development of new computer devices and software. The increasing storage capacity of computer memory, the high performance of new processors, combined with appropriate operating software enables to store and manipulate any kind and very large amounts of numerical data including images, sound etc., besides the usual texts. Therefore it has been possible to conceive workstations able to manipulate and integrate different kinds of information sources. The development and improvement of (tele)communication systems enabled also to establish different size and nature networks (last, but not least the internet). This is not really a revolution since these are all achievements inside the genetic structure of conventional computer systems. It is a result of a quasi optimal usage of their potentialities. From a decision aid point of view there are different levels of integration of multimedia equipment in decision support systems (see Chang and Holsapple 1994, Maybury, 1994, Bieber, 1995).

- Conceive individual decision support systems that integrate visual modeling facilities improving the user interface and the man-machine interaction. The integration of different information sources may also improve decision support systems using data and knowledge under synthetic representations and/or visual ones.

- Conceive collective decision support systems (groupware, negotiation support systems etc.) which use such facilities in order to improve communication among the participants of the decision process. Moreover it is possible to integrate in the system, geographical information systems, large data bases, monitoring networks etc..

Concerning the use of multimedia equipment in the MCDA field we can make the following observations.

1. There is nothing specific in the development of such facilities concerning multicriteria decision analysis. The technological advantages offered by such equipment are the same for any decision support theory branch and although specific technical solutions have to be used when a multicriteria model is at stake these concern the hardware and software and not the model itself.

2. The use of multimedia equipment does not alterate the weakness or strength of the decision aid approach included in the decision support system created. In other words when a specific decision model is chosen the use of such facilities may improve the way information is collected and results are presented, but it will not affect the contents and quality of the result neither from a theoretical point of view nor from an operational point of view. A weighted sum is always a weighted sum under any kind of computer dress.

3. The most promising direction, to our point of view, in the use of multimedia equipment is to explore the possibility to provide a decision support during the whole decision aid process (enhancing interactivity, improving learning etc.), while presently decision support is conceived in a rather static manner.

4 Distributed computing

This section is strongly linked to the previous one as it is a pure technological achievement. The idea of distributed computing is the one to substitute the old centralized edp systems with networks of workstations enabling users to work autonomously sharing only some facilities. The development of multimedia equipment has been a further improvement in this direction.

An interesting extension of this idea is to conceive decision support systems as a network of distributed autonomous agents each of them facing and solving a specific problem. Under this point of view different multicriteria models and methods can be integrated in the same system.

Our observations are practically the same ones as in the previous section. Last, but not least, a confusion that may occur is between distributed and parallel computing. The latter consists in an alternative hardware architecture using parallel processors for the execution (in a parallel way) of

algorithms (namely large scale problems can be decomposed is parts that can be solved in parallel). It is well known however that the computational improvement obtained by this technology does not affect the computational complexity of the algorithms.

5 Expert systems

The development of expert systems is a field of Artificial Intelligence (AI). Expert systems use knowledge representation in an explicit qualitative form about a problem domain, concerning a specific area of human expertise, to support decision maker's (DM) when solving particular problem instances. Expert systems are designed to replicate the problem solving behaviour of an expert in that domain, thus acting as an intelligent assistant to the DMs. Even though in theory expert systems and DSS are different, in that the former are aimed at explicitly or implicitly replacing the DM by simulating human reasoning, in practice expert systems have been used in a decision aid role similar to that of DSS. Once knowledge (facts and rules) is elicited from an expert (knowledge acquisition), it is represented under a specific form such as production rules, frames, semantic nets (knowledge representation). The inference engine uses both types of declarative and procedural knowledge to derive conclusions (ideally the same that the expert would reach). The quality of these conclusions strongly depends on the knowledge acquisition process. However, elicitation of knowledge from experts is not a straightforward task, because experts generally have difficulty to explain their decision process as these become automatic. To this purpose Pomerol (1995) claims that "expert systems paradoxically are not suitable for simulating human expert judgment in discrete decisions" (see also Hammond 1987, Mumford 1987). Expert systems can also use case-based reasoning rather than a set of rules, that is a process of deriving conclusions based on specific examples of what has occurred in the past.

We want to point out the following observations.

- The literature is full of presentations of prototypes of trivial "expert systems" which in practice are very simple multi-criteria methods where the well known weighted sum is replaced by some heuristic decision rule which does the preference aggregation job. Obviously there is no expert knowledge represented as the heuristic is confused with knowledge.

- A severe limitation of traditional expert systems is that they perform only pure classification tasks where the situation at hand (the facts) have to be matched to a situation already described by the knowledge. In other words expert systems are essentially diagnosis machines. Often the problem is not only to classify, but also to rank, to choose, to compare and generally under different (and conflictual) points of view (which is a typical MCDA task).

- The idea that expert systems could solve any kind of problem (provided that an expert exists for any kind of problem) has been very soon rejected. Organizational decision making and other complex decision processes seem to escape from the expert systems approach (see Hatchuel and Weil, 1992 for a very interesting discussion).

The non trivial coupling of AI/expert systems techniques and MCDA methodologies has revealed, at least in theory, to be a promising research direction. It may be used to help in the choice of aggregation rules, as a tool of qualitative analysis of the results produced by the procedural components, to guide the interactive decision process by combining expert and preferential knowledge, to support the knowledge elicitation process etc. A mutual benefit can be viewed exactly in the analysis of the cognitive dimension of the decision aid process which is a problem solving process also. Finally the treatment of uncertainty is a common ground of research.

6 Object oriented programming

Object oriented programming (OOP) is one of the most important trends in programming methodology. It introduces a different style of programming by using the key features of encapsulation (of state and methods together) and inheritance. An object has a set of operation and a local shared memory. Encapsulation refers to the concept of combining the state of an object with the methods that manipulate it. That is, data is packaged with the operations and procedures which may access the data elements. Encapsulation means information hiding in the sense that the data structure users do not see the representation of data (which is the principle of abstract data types). Users will operate on an object using the messages provided at the implementation stage.

Inheritance is the ability to derive new objects from existing ones, by allowing the child object, besides sharing the properties and operation of the

parent object, to modify or add new properties and operations. This enables code sharing, reuse and development of generic functions, encouraging a differential programming style (by successive modifications). Because of historical reasons OOP has been confused with graphical user interfaces and windowing systems. Also some dialects of conventional programming languages exist (such as C or Pascal).

Since MCDA methods are implemented as software it is reasonable to question whether OOP could be relevant or specific for the purpose of creating multicriteria based decision support systems. In fact the structure of a multicriteria decision aid method presents elements of inheritance and makes large use of abstract data types (a criterion, a preference structure etc.). We may emphasize however that:

1. the gains obtained using OOP in implementing a MCDA method do not affect the method itself, but only the way the user perceives it;

2. it is relevant to study OOP as an efficient programming language for decision support systems in general (independently of the methods used);

As pointed out by Brooks (1987), changing programming language does not solve the "core" (or essential issues) of the algorithm to be implemented. Shifting to OOP may enhance our possibilities in using software for MCDA, but it will not modify our problems.

7 Neural networks

An (artificial) neural network is a massively parallel distributed processor that has a natural propensity for storing experiential knowledge and making it available for use (see Hertz et al., 1991). Neural networks mimic the way in which the brain performs a particular task or function. The similarity with the brain (viewed as an adaptive machine with abilities to learn from and adapt to the environment) lays in two main processes:

- knowledge is acquired by the neuron (elementary information - processing unit) network through a learning process; and

- the synapses (structural and functional units that mediate the interactions between neurons) strengths (or weights) are used to store the knowledge.

Neural network architectures include single-layer feed-forward, multi-layer feed-forward, recurrent and lattice structures. The procedure used to perform the learning process modifies the synaptic weights of the neural network in order to attain a desired goal. In the same way weights are used in some MCDA methodologies to reflect, up to a certain extent, the DM's preferences, neuron inputs are weighted to represent the relative importance of each input to a processing element.The ability to learn and generalize can be interesting in MCDA for instance by using former decisions (in appropriate similar contexts) to tune a network of methods capable of replicate decisions (thus in an attempt to recognize patterns of decisions).

Our observations concern two points.

1. In order to have reliable and innovative applications of artificial neural networks in the MCDA context the question of the "weights" have to be seriously addressed. "Weights" have different meanings in MCDA, largely depending on the preference aggregation procedure adopted and on the nature of the different criteria. Either such a procedure is chosen externally and imposed to the network thus simply replicating well known MCDA methods or the preference aggregation procedure has to be chosen by the network introducing a meta level of reasoning for which neural networks do not seem (for the moment) very well suited.

2. It seems that neural networks are particularly suited for some classes of MCDA problems, namely the sorting ones. This is obvious since neural networks are essentially classification machines (in that being very similar to expert systems). However, it is not yet clear if there is a real advantage in using such a technique, while it is also questionable the confidence of the DM to a tool performing as a "black box". For further discussion see also Pomerol (1995).

8 The World Wide Web

The World Wide Web (WWW) is the "universe of network-accessible information, an embodiment of human knowledge" (as defined by the WWW Consortium), physically implemented as a (massive) "client-server" architecture. By means of a server program supporting an appropriate protocol, the information provider can offer a large range of information (such as text, pictures or sounds, and the possibility of file transfer), to which the client

has access by using a browser. The most common use of the Web has been the access to a server's home page, organized in a hypertext structure in which the information is made accessible by clicking on links. This somewhat "static" approach has now begun to change. A recent Web browser named Hot-Java, developed by Sun Micro-systems, can download its own programs, called applets, and run them embedded on a Web page. Independently of its computer architecture or operating system anyone can access the same applications as long as (s)he is using a Java-enabled browser (such as Netscape 2.0). By enabling the distributed execution of programs over a network, thus acting as a kind of "universal" interface, Java lays the foundations for a more uniform and integrated collective work environment. The potential it opens is enormous, as are security issues. The possibility of providing on the Web interactive services also has important implications to be exploited in decision aid (and decision aid business).

The same observations done in the multimedia and in the distributed computing section apply here the WWW being an advanced realization of such fields. It may be useful to notice that propositions already exist to include decision support facilities in the WWW context. (see Karacapilidis and Gordon, 1995 and Karacapilidis et al., 1995).

9 Conclusion

So-called emerging or new technologies are very broad concepts, which are perceived in different ways by different people due to different backgrounds. Our presentation focussed on some fashionable and modern keywords (which are neither exhaustive nor mutually exclusive). A brief description of each concept is made and it is then attempted to shed some light on the use of these tools and ideas, to argue against their trivial or indiscrimimate use (justified only by fashion reasons which can have harmful effects), and to provoke some discussion about them. Nevertheless whenever properly used in context these technologies can actually add value to MCDA environments.

Using the distinction of essential and accidental issues in the MCDA field we can summarize our conclusions as follows:

- new technologies cannot affect the "quality" of the "essence" of MCDA, that is the principles of preference modelling and aggregation underlying any different MCDA method;
- new technologies can improve "accidental issues" such as user inter-

faces, integration of MCDA methods in more general decision support systems, construction of wide-band DSS etc;

- there is no "modern" MCDA, as it does not exist "good" or "bad" MCDA. It exists however "correct" and "validated" MCDA (see Bouyssou et al., 1993) and this could be a challenge for new technologies. Are they useful?

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A Flexible Interface for Decision-Aid in Multicriteria Decision Problems

Manuel A. Matos[1 2] and **Pedro Borges**[3 1]

[1] INESC - Instituto de Engenharia de Sistemas e Computadores
[2] FEUP - Fac. Engenharia U. Porto (Dep. Eng. Electrotécnica e Computadores)
[3] UCP - Universidade Católica Portuguesa

Abstract. Communication between the Decision Maker (DM) and decision-aid (DA) methodologies is a main concern in Multicriteria Decision Problems, particularly when a large set of nondominated solutions exists. In a previous MCDM conference, Matos and Miranda (1989) presented the theoretical basis for a DM/DA interface (the "Anchor model"), using fuzzy sets to "translate" numerical values of the attributes into linguistic labels more meaningful to the DM. The interpretation of linguistic declarations about the DM goals was also outlined, and a new concept of "qualitative domination" was introduced.

This paper describes a preliminary implementation of the Anchor model, capable of producing meaningful sentences and "understanding" a limited set of declarations of the DM. For instance, the system can produce, in an environmental problem, a list of requested "cheap and very safe alternatives".

The prototype was designed using an object oriented methodology. This implied a modular design and the preservation of independence between interaction details and the Anchor model itself. This modularity and flexibility allow easy shaping of the program to different scenarios.

The paper includes an illustrative example of a possible "dialog" between a DM and the system, in the framework of a specific decision problem.

1 Introduction

Communication between the Decision Maker (DM) and the decision-aid system (DA) is widely recognized as important. Different strategies have been proposed to deal with this, some based on the use of graphic information about parameters and results, others by reducing the amount and structuring the data presented to the DM. Nevertheless, the usual paradigm is essentially numeric:

a) The DM gives data to the DA: attributes or objective functions and constraints
b) The DA requires some parameters or intermediate decisions
c) The DM gives numerical answers (weights, trade-offs, etc.) or yes/no answers
d) A final result is produced by the DA (eventually not an *unique* final result)

Obviously, the steps are not so simple as stated, but the general picture is

essentially this. Treatment of qualitative attributes is mainly done with some correspondence scale, such as the one proposed by Saaty (1980). In short: models are numeric, so input data must be numeric and results are, consequently, also numeric, except for graphics.

When the problem has a small limited number of possible solutions, numerical results may be meaningful for the DM, who is always aware of the globallity of the alternatives. If the DM deals with a great number of alternatives (in a large multiattribute problem or in a multiobjective problem), numerical information is not generally sufficient. The difficulties depend on the type of decision-aid, the main situations being:

a) *A priori* incorporation of the DM preferences: in this case, the dimension of the alternative set is not important. Weights, trade-offs or equivalent information is required, or a value function is constructed after some interrogation scheme. It is widely reported in the literature that the reliability of those values is not very high, due to the lack of global information regarding scales, individual optima, etc. In these approaches, results are simply *the* result, so no difficulties arise for the DM regarding interpretation: he just takes it or leaves it.
b) Interactive methods: these methods are more common in multiobjective problems, and again only numeric (or yes/no) information is required to the DM. Generally, some overall information in presented to the DM in order to help him make partial decisions. That information is numeric or graphic (bar charts, nomograms, etc.)
c) *A posteriori* incorporation of preferences: after some screening (elimination of dominated alternatives, selection of extreme efficient solutions, etc.), the DM has to decide between a number (possibly large) of remaining alternatives. This is a difficult task, due to information processing limitations of the DM, which may be "lost" in a huge amount of numbers.

A common characteristic of the different approaches is the non existence of a DM to DA way of dialog. The DM "speaks" only when he is requested, in a predefined way, and he never has the opportunity to express his wishes in a familiar language. Definition of aspiration levels in goal programming type models is not an answer to this question, because lack of knowledge about the problem limits may lead even to suboptimal choices.

The approach presented in this paper tries to ease communication between the DM and the DA system by constructing a protocol using natural language labels and modifiers to characterize the alternatives. This allows the system to present, instead of numerical values, a natural language label of each solution, such as "cheap and not very safe alternative", where "cheap" and "safe" are criteria labels, and "not very" is a linguist modifier reflecting the degree of fulfillment of the safe criterion in the problem. This is more meaningful to the DM than only numeric values for the attributes. The main improvement, however, lies on the possibility of the DM to communicate with the system, requiring to see, for instance, every "mostly safe and at least medium cheap" solution. As the system answers with a limited list of solutions, or no solutions, or almost all the solutions, the DM *learns* about the problem, the feasibility of his initial aspirations, etc. In

subsequent queries, the DM can further limit or enlarge the previous choice set, until he gets his preferred solution.

The selection process becomes transparent to the DM, in the sense that he "feels" what he is doing and the options he takes, without being influenced by crude numerical values of the attributes. The actions of the user along the process are of his own responsibility and he is always able to go back or try different approaches, until he can improve his overall view of the problem and hence facilitate his decisions. The subjective opinion is used to advantage, without the need of translation to numerical values.

The basis for this approach was presented by Matos e Miranda (1989), following initial work of Matos (1988). Translation of numbers into complex labels and interpretation of natural language declarations uses modifiers that are fuzzy sets of degrees of fulfillment. The general framework is described in the next sections.

Once defined the communication protocol, some technical questions remain. The present work addresses some of them, by describing the prototype of a line interface for the anchor model, programmed in an object-oriented language. An important issue is the flexibility of the interface: as it will be shown later, the basic structure can be augmented or modified without code modification.

There is an attempt to make the prototype able to answer the decision maker in several ways and in different situations, to avoid the feeling of "talking" with a hermetic machine. To this respect other things should be done, namely the integration of this model in decision support systems (DSS) that use natural language interfaces. All the interaction is done by means of language. The connection of the prototype with other systems is referred ahead.

2 The Anchor Model

The Anchor Model was proposed by Matos (1988) in a general form. Matos and Miranda (1989) presented the main characteristics of a possible implementation of the model in a previous MCDM conference. We review here the main concepts and formulas involved.

The model addresses multicriteria decision problems, departing from a limited (although large) set Z of n efficient solutions $\mathbf{z_k}=(z_{k1},..,z_{kc})$, k=1..n, defined in an attribute space with dimension c. This set Z may be directly the nondominated solutions set of a multiattribute problem, or a representative set the nondominated solutions of a multiobjective problem, after some generation phase.

The model is conversational, that is, it establishes a framework that allows the DM to exchange information with the Decision Aid system about the alternatives, the DM preferences and some "pruning" operations, but it has not a prescriptive characteristic. The DM may restrict the original set according to his interests, and use the system to arrive at a final decision, but he also may only "navigate" in the problem to gain insight of the decision environment.

2.1 Macro-solutions

The main concept of the model is that of Macro-Solutions, defining the global

alternatives of decision in fuzzy way. Macro-solutions are linked to anchor points of the problem, defined by the DM. Natural anchor points are the alternatives in Z that optimize each of the attributes, and also the Ideal solution, a non feasible solution that groups the individual optima of all the attributes, but other significant points (for the DM) may be defined. Mathematically, each macro-solution $\tilde{M}_i$, i=1..s, is a fuzzy set of real alternatives in Z, related to an anchor point $s_i=(s_{i1},..,s_{ic})$:

$$\tilde{M}_i = \{(z_k, u(z_k, \tilde{M}_i)) \mid z_k \in Z\}$$

where u: Z→[0,1] is the degree of membership of z_k to $\tilde{M}_i$, defined by:

$$u(z_k, \tilde{M}_i) = \left(\sum_j w_{ij} \cdot u_j(z_k, \tilde{M}_i)^p \Big/ \sum_j w_{ij} \right)^{\frac{1}{p}}$$

In this expression, p∈[1,∞) is a parameter (generally p=1), and u_j and w_{ij} are, respectively, partial degrees of membership and weights for scaling:

$$u_j\left(z_k, \tilde{M}_i\right) = \min\left\{1 ; \frac{z_{kj} - z_j^{min}}{s_{ij} - z_j^{min}}\right\} \text{ and } w_{ij} = \frac{s_{ij} - z_j^{min}}{z_j^{max} - z_j^{min}} \quad \text{(maximizing attribute j)}$$

$$u_j\left(z_k, \tilde{M}_i\right) = \min\left\{1 ; \frac{z_j^{max} - z_{kj}}{z_j^{max} - s_{ij}}\right\} \text{ and } w_{ij} = \frac{z_j^{max} - s_{ij}}{z_j^{max} - z_j^{min}} \quad \text{(minimizing attribute j)}$$

An important consequence of the given formulas is that a particular alternative is not penalized for being better than the anchor point in some criteria. Thus, the degree of membership of a given alternative to a macro-solution is not just a similarity measure between the solution and the corresponding anchor point, but it is a measure of the fulfillment of the objective expressed by the macro-solution. As it is evident from the formulas, it is also not a simple change of scale in the attribute.

Intersection of macro-solutions uses standard rules of fuzzy sets theory, as stated by Zadeh (1965), to obtain the degrees of membership of any alternative z_k to more complex references, involving, for example, both $\tilde{M}_i$ and $\tilde{M}_j$:

$$u\left(z_k, \tilde{M}_i \cap \tilde{M}_j\right) = \min\left\{u\left(z_k, \tilde{M}_i\right) ; u\left(z_k, \tilde{M}_j\right)\right\}$$

2.2 Labels, Modifiers and Sentences

Derivation of simple natural language sentences about a multicriteria problem bases on the combined use of labels of the macro-solutions and predefined

qualifiers such as "most", "little", etc., that express the degree of membership in a non numeric way. Labels are defined by the DM when he chooses the macro-solutions, and qualifiers are grading modifiers of the labels, defined as fuzzy sets of degrees of membership, $\tilde{G}_m=\{(u,g(u,\tilde{G}_m))|u \in [0,1]\}$. This allows two-way communication between the DM and the DA system: (a) classificative when the system describes a particular alternative by a sentence like "A medium cheap and little polluted alternative", and (b) interpretative when the DM requires, for instance, a listing of "at most medium polluted options".

Figure 1 shows the membership functions g(u) for five modifiers. Similar definitions for three or seven modifiers can be used in the interface.

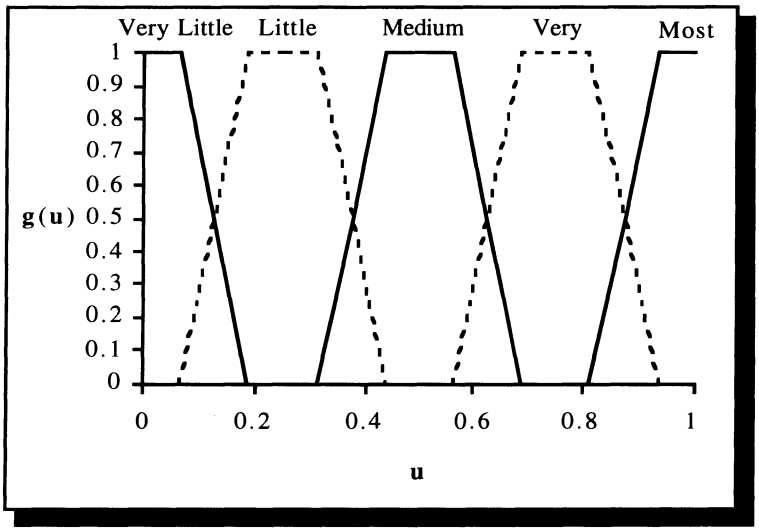

Figure 1 - Membership functions for five modifiers

In the model, no restriction exists for the number and general shape of the modifiers, which should be meaningful to the DM and adapted to him, possibly by interactive *mutual* training (the DM trains the system and trains himself at the same time). The only requirement is that the set of modifiers constitutes a fuzzy partition of the [0,1] interval, which implies that $\Sigma_m\, g(u,\tilde{G}_m)=1$. This requirement combines with the definition of the union of modifiers with fuzzy partition rules, that is $g(u,\tilde{G}_m \cup \tilde{G}_p) = g(u,\tilde{G}_m)+g(u,\tilde{G}_p)$. For details about fuzzy partitions see, for instance, Bezdek (1981).

Once defined the modifiers and some aggregation rules, sentences can be produced and "understood". An elementary statement $\tilde{D}_{mi}$ uses a macro-solution $\tilde{M}_i$ and a grading modifier $\tilde{G}_m$, and corresponds to the fuzzy set:

$$\tilde{D}_{mi} = \{(\mathbf{z_k}, d(\mathbf{z_k}, \tilde{D}_{mi})) \mid \mathbf{z_k} \in Z\} \quad \text{where} \quad d(\mathbf{z_k}, \tilde{D}_{mi}) = g(u(\mathbf{z_k}, \tilde{M}_i), \tilde{G}_m)$$

Label of $\tilde{D}_{mi}$ is obtained simply by applying the modifier to the macro-solution label. Examples are "very cheap" or "medium polluted" labels.

To construct more elaborated statements envolving several attributes, general *max* and *min* operations of fuzzy sets theory are used. Labels of complex statements result from the application of semantic operators "or" and "and", respectively when union and intersection of elementary statements are performed:.

$$d(\mathbf{z_k}, \tilde{D}_{mi} \cup \tilde{D}_{nj}) = \max\left\{d(\mathbf{z_k}, \tilde{D}_{mi}); d(\mathbf{z_k}, \tilde{D}_{nj})\right\} \quad \text{("very cheap or most clean")}$$

$$d(\mathbf{z_k}, \tilde{D}_{mi} \cup \tilde{D}_{ni}) = d(\mathbf{z_k}, \tilde{D}_{mi}) + d(\mathbf{z_k}, \tilde{D}_{ni}) \quad \text{("very cheap or most cheap")}$$

$$d(\mathbf{z_k}, \tilde{D}_{mi} \cap \tilde{D}_{nj}) = \min\left\{d(\mathbf{z_k}, \tilde{D}_{mi}); d(\mathbf{z_k}, \tilde{D}_{nj})\right\} \quad \text{("very cheap and most clean")}$$

$$d(\mathbf{z_k}, \bigcup_{m \geq q} \tilde{D}_{mi}) = \sum_{m \geq q} d(\mathbf{z_k}, \tilde{D}_{mi}) \quad \text{("at least medium cheap")}$$

$$d(\mathbf{z_k}, \bigcup_{m \leq q} \tilde{D}_{mi}) = \sum_{m \leq q} d(\mathbf{z_k}, \tilde{D}_{mi}) \quad \text{("at most little polluted")}$$

Classification of an alternative can be achieved by selecting the elementary statement (for each attribute) to which the alternative has the greatest degree of membership. Interpretation consists of selecting a chosen number of alternatives with the greatest degree of membership to the designation.

2.3 Qualitative Domination

The concept of qualitative domination was introduced by Matos and Miranda (1989), and is an extension of the basic concept of domination used in multicriteria analysis. The main idea is to make use of the order relations of the linguistic labels in each criterion to reduce the dimension of the original set.

The role of qualitative domination is intended to be only descriptive, because it is not a question of preference, as in the primitive concept of domination, but of representation. A selected alternative (by qualitative dominance) represents a rough tendency of decision that includes, as actual hypothesis, some of the rejected solutions. Operationally, one expects that the DM shall use the qualitative domination to have a general picture with fewer alternatives, backtracking to the original decision set when he wishes to take his final decisions.

3 Implementation

An applicable decision support system should not be constituted only by a model like the anchor model or a procedure alone and by itself. This should act as a component of more global systems. By its generality, the anchor approach can be implemented as a general component, fitting easily and playing a relevant role in complex systems designed for different purposes, particularly the ones that interface the decision agent through natural language approaches.

Figure 2 represents a possible structure of a DSS, where the anchor module appears as an independent module. The other modules displayed might be independent from each other, and from the interface and core of the system, or not, without loss of generality as to what the anchor module is concerned.

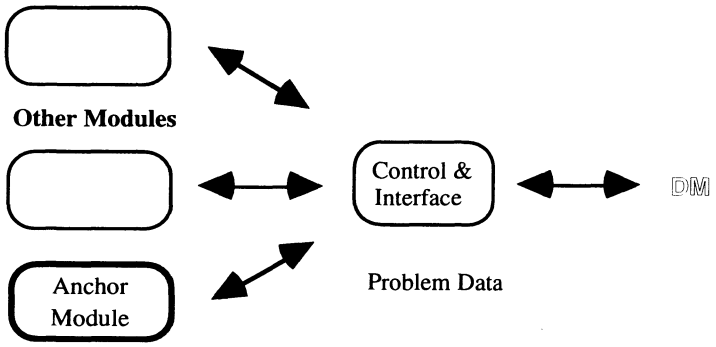

Figure 2: Possible architecture of a DSS using the Anchor Model

Being its structure well defined and bounded, the anchor model can be implemented as a separate module easily applicable to any system with this kind of configuration. The previous definition of the interface with the control module can avoid all tailoring when adding anchor to a system. Under other philosophy, the necessary modifications or additions to allow fitting the module to different applications could be in the interface between problem data and the anchor model, so as to access data directly in its existing format. The knowledge for this is usually elementary to the DSS developer, that in this way is hidden the anchor module details.

This prototype was developed for demonstration purposes, the aim being to see how easy it would be to implement and test this approach and mainly to obtain hands-on user reaction to the model. Details on the implementation, including file format descriptions can be found in Borges (1992). The prototype was written in an object oriented language so as to reuse code already available when implementing parts of the prototype and to obtain a class that acts as an independent module, having straightforward information hiding mechanisms. Figure 3 illustrates the structure of the prototype.

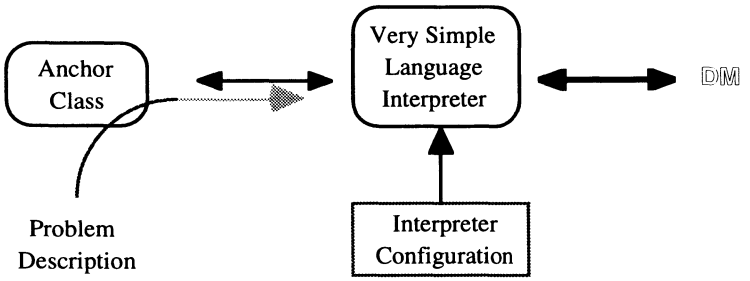

Figure 3: Prototype overview

As seen in the figure, the anchor module accesses data directly. The data elements are so simple that the same code could be used to access data through the controlling module with minor changes (data is, in fact, being accessed through an interface class).

The user interface and DSS control core are based on a family of classes that deal with token and text manipulation and matching operations. With these classes, an elementary system was build to emulate natural language to an obviously limited extent. The language interpretation is based on a filtering process and matching rules' usage on the user input. The behaviour of the language interpreter is defined and/or configured through a text file that contains some rules and vocabulary. These are used together with the words from the problem description to form a dictionary and determine the system response. This has the advantage of creating some independence between language definition and the program itself and enables extensions by enlarging the text configuration file. The configuration includes mechanisms to deal with common user language and behaviour, as many users tend to communicate to the program as if they were talking to a person, so some *Eliza* style behaviour was included (In the sense that the program does not really understand what the user means, but tries to give a "correct" answer anyway). The program searches the DM command for terms and expressions that might show his state of mind and tries to answer accordingly.

Some kinds of questions are shown ahead in an example of the program usage, namely those related to the anchor model itself. There are obviously severe limitations in the language process when outside the scope of the anchor module. The program does not use a grammar in the usual sense, so some sentences badly constructed are taken as correct and answered.

The grading modifiers or descriptors are fuzzy sets that could be different in each attribute and, together with the qualitative descriptors, should be allowed definition in interaction with the DM, taking account of his Max-min variation. In this prototype, however, the membership functions are fixed. The concept of qualitative domination is supported in this version of the prototype. Also allowed are questions involving unions, as defined before, including "at least" and "at most" iterated unions.

4 Illustrative Example: A "dialog"

In this section, we illustrate the use of the interface by simulating a dialog between the DA system and a DM who wants to select a washing machine from a set of 33 nondominated solutions in four attributes: cost, time, electricity consumption and water consumption. It is a problem taken from Zeleny (1982).

Table 1: Selecting a washing machine

No.	$	min	kWh	lit.
1	509	74	1.5	114
2	425	80	1.5	110
3	446	72	1.6	135
4	564	65	1.6	118
5	547	53	1.8	140
6	450	68	1.6	135
7	473	65	1.6	130
8	484	56	1.7	115
9	456	68	1.6	130
10	488	72	1.6	114
11	530	55	1.7	135
12	477	76	1.5	110
13	589	53	1.6	130
14	534	61	1.4	122
15	536	57	1.7	110
16	494	71	1.5	135
17	425	65	1.8	120
18	555	53	1.7	125
19	543	57	1.6	120
20	515	68	1.5	130
21	452	76	1.5	112
22	547	68	1.5	120
23	421	76	1.4	130
24	498	68	1.6	120
25	467	65	1.7	130
26	595	50	1.8	135
27	414	68	1.7	125
28	431	66	1.7	110
29	452	72	1.5	115
30	408	77	1.6	119
31	478	59	1.8	110
32	395	76	1.5	120
33	543	57	1.5	135

The anchor points selected are alternatives 32, 26, 14 and 2 (in the later case, alternatives 12, 15, 28 or 31 could also be chosen instead of 2), which correspond to the individual minima of the attributes, and the ideal point, in this case (395, 50, 1.4, 110). Labels of the macro-solutions were defined in the following way. Solution 32, the anchor point of MS_1, is the cheapest alternative, so the label for MS_1 is "cheap solution". Different alternatives will have degrees of membership to MS_1 that reflect their compatibility to the concept expressed by the label. In a similar way, labels for MS_2 through MS_5 were set as "fast", "power saver", "water saver" and "ideal". The dialogue reproduced below illustrates some features of the system.

```
: What is the number of alternatives?
The problem: Choosing a Washing Machine
Has 33 alternatives.

: Display the criteria
The criteria you will use to decide are:
cheap       fast        power saver      water saver      ideal
```

```
: Show the descriptors
The descriptors you may use are:
very little; little; medium; very; most;

: describe option m32
 Alternative 32 is most cheap, little fast, very power saver, very
to most water saver, and very to medium ideal.

: show most fast alternatives
      Alternative :m26        - 1
      Alternative :m13        - 0.814286
      Alternative :m18        - 0.814286
      Alternative :m11        - 0.357143
      Alternative : m8        - 0.128572

: display most cheap options please.
It's my pleasure
      Alternative :m32        - 1
      Alternative : m2        - 0.611111
      Alternative :m21        - 0.605882
      Alternative :m29        - 0.605882
      Alternative :m30        - 0.407189
      Alternative :m12        - 0.213725
      Alternative :m23        - 0.0464053

: Let me see solutions at least very fast and at least very cheap.
      Alternative :m14        - 0.985714
      Alternative : m8        - 0.535294
      Alternative :m19        - 0.394117
      Alternative : m7        - 0.0714288
      Alternative :m17        - 0.0714288
      Alternative : m4        - 0.0647058

: Let me see alternatives at least very water saver, at least very
fast, at least very cheap and at least very power saver.
      Alternative :m14        - 0.592308
      Alternative : m8        - 0.535294

: Use qualitative domination.
Alternative 3 dominated by 1.
Alternative 10 dominated by 1.
(a list of similar statements follows)

: list all alternatives
Criteria: cheap  fast  power saver  water saver

Alternative : m2        -       425     80      1.5     110
Alternative : m8        -       484     56      1.7     115
Alternative :m13        -       589     53      1.6     130
Alternative :m14        -       534     61      1.4     122
```

Alternative :m26	-	595	50	1.8	135
Alternative :m28	-	431	66	1.7	110
Alternative :m29	-	452	72	1.5	115
Alternative :m32	-	395	76	1.5	120

As stated before, the system doesn't prescribe any solution. It merely aids the DM to gain insight about the problem, always maintaining a global vision about the decision alternatives, by means of using linguistic descriptions of the alternatives. This is more friendly than just numbers, although some numeric graduating information was also delivered.

5 Conclusions

Interfaces with linguistic capabilities seem to be interesting for constructing decision-aid environments, since they provide easily understandable holistic information about the decision alternatives in problems with large data sets. The fuzzy sets based Anchor Model provides a framework for constructing and interpreting a limited range of sentences that can be used for that purpose. The prototype presented in the paper was developed using object-oriented programming, enabling the design of a flexible conversational interface that may be enriched with new terms and capabilities without further programming.

In its present form, the prototype has only demonstration interest. Obviously, many more features must be introduced to turn it into a useful tool for decision-aid. Among them, we can mention line editing facilities, interactive modification of the descriptors' shape, matching mechanisms and graphic representation abilities.

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Decision Support for MCDM That Is Neural Network-Based and Can Learn

Thomas Hanne

Department of Economics, Operations Research, FernUniversität Hagen, Lütkenheider Str. 2, 58084 Hagen, Germany, E-mail: thomas.hanne@fernuni-hagen.de

Abstract. In this paper we consider the basic ideas, the implementation, and the application of an object-oriented decision support system (DSS), especially for multiple criteria decision making (MCDM). One main idea of the DSS is to utilize the features of different MCDM methods by integration. We discuss several approaches of integration esp. the combination of methods, the use of neural networks, and a generalization of network-like structures. Information acquisition is an essential part for any DSS. Beside aspects of interactive data entry, we discuss the utilization of historical data (especially from former decisions) for learning. By doing so the parameters of a method, some methods, or a network of methods can be tuned and we can answer questions like: Which method should be used? How can the parameters be adjusted? Is it useful to apply different methods and to aggregate their results to build a compromise solution? And how should this be done?

1 Introduction: Goals for a Decision Support System

Decision support systems (DSS) have gained a substantial interest in the area of multiple criteria decision making (MCDM). 'System', in this context, means some computer-based device. 'Decision support' concerns the system's position in the decision making process. The computer becomes a tool for the decision maker. It does not decide by itself. This implies that the decision maker, somehow, interacts with the system. The system aids the decision maker.

The reasons why we require DSS for MCDM can themselves be formulated as an MCDM problem: Reduce the costs and improve the results of decision making. In the next sections we discuss some strategies to achieve these goals.

In the next subsections some specific motives for constructing a new DSS are discussed. In section 2 we consider structural concepts for the DSS based on the combination of methods and neural networks. Section 3 presents approaches for realizing learning within the DSS and section 4 gives an example application. Finally we present some conclusions.

1.1 Integrating Several Methods

During the last two decades, MCDM research has led to the development of a lot of methods. Many of them are so complex that some computer support

is required for their regular use. This motivation led to the first generation of MCDM software: One program for one method (Lotfi and Teich, 1991). Since there are so many methods with quite different concepts, especially when they follow divergent schools of MCDM, an integration of some of these methods is an essential feature of our new MCDSS. We do not think that different philosophies of MCDM exclude the joined use of the corresponding methods in the DSS.

Such a system should have a user-friendly interface. This facilitates the usage by nonspecialists of MCDM, makes MCDM methods more acceptable in practice, and supports the application of more than one method to a given MCDM problem.

1.2 Interactivity

Above, we have emphasized the importance of man-machine interactions. There are two ways of achieving this information exchange. First, a decision maker has a dialogue with the computer. He or she chooses a method and assigns parameters. Then the DSS applies the method to the problem. If the method itself is 'interactiv' there is additional information exchange between man and machine. When the results are calculated the user analyzes them, and if he or she is not satisfied the process is repeated with changed parameters or another method.

1.3 Learning

The other approach of applying an MCDSS is to use given data, especially from historical decisions, and let the system learn how to make good decisions. Learning is particularly essential for recurring decisions where the same type of problem has to be solved again and again. With the help of historical information a method can be chosen or parameters can be tuned. In the extreme case no decision maker need to be present. The automation of decision processes reduces costs while the quality of the decision process is retained. Additionally, if the former decision making was not method based it can be analyzed (e. g. whether it was rational) and future decisions should be improved.

2 Structural Concepts for an MCDSS

Now, let us discuss how methods can be integrated in a decision support system. The simpliest way is just to provide a homogeneous user interface for accessing the different methods. For example, the user formulates the problem, chooses a method via a special language or a menu system, and applies it to the problem. This approach suffices to implement the interactive features mentioned above. In the next two subsections two approaches for integrating methods more deeply are discussed.

2.1 The Combination of Methods

Beyond the sequential application of methods new methods can be constructed by the combination of existing ones. There are several ways of combining methods: For instance, an MCDM problem is solved by a method with different parameter values or by various methods. Then, the solutions are aggregated to a compromise solution. Since the aggregation of different results can be regarded as an MCDM problem, another MCDM method can be applied to this task. In this way new, aggregated MCDM methods can be constructed. Such an aggregation does not respect the original philosophy of a method (see example below). The results of methods are considered as a quantitative means for valuating alternatives.

Let us consider an example taken from Hanne (1994a): We want to select stocks for an investment. Therefore, we consider 6 criteria concerning the chart of the stock prices (e. g. price volatilities) and the economic background of the firm (e.g. earning price ratio). The problem is analyzed with 4 multiple attribute decision making methods:

Simple additive weighting (SAW) (see Hwang and Yoon, 1981, p. 99-103),
TOPSIS (see Hwang and Yoon, 1981, p. 128-140),
PROMETHEE II (see Brans, Mareschal, and Vincke, 1984), and
conjunctive levels (C. L.) (see Hwang and Yoon, 1981, p. 68-70).

The different results which we obtain are aggregated to a compromise solution. For this purpose we again use the SAW approach. The structure of this analysis is illustrated by the following graph.

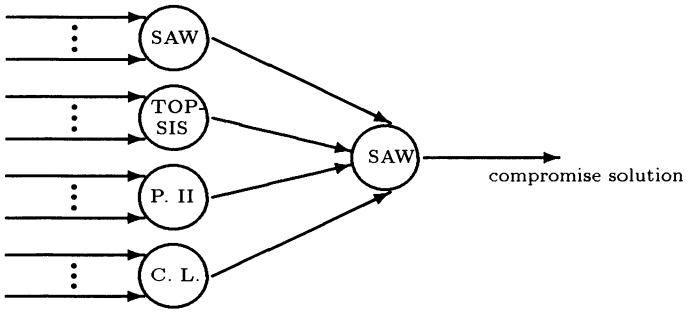

Fig. 1. A network of MCDM methods.

Here we see how, for instance, the concept of conjunctive levels is disregarded: The method is modified (or interpreted) so that it calculates the result 1 if an alternative is 'feasible' and 0 otherwise. Depending on the results of the other methods it may happen that an 'infeasible' alternative becomes the best compromise solution. Although this can be intended, when interpreting this process of combining methods one must consider that the original meaning of the single methods has been lost.

By analogy to neural networks (see section 2.2.) we call this approach of combining methods an MCDM network. It can be generalized to more complex decision making structures.

Another concept to combine methods is to decompose similar methods consisting of several steps and to assemble new methods from the parts. This is especially meaningful if the original methods produce interim solutions of the same kind. For instance, outranking methods like ELECTRE or PROMETHEE II calculate a preference matrix A in different ways but with a similar meaning. Later, the methods evaluate A differently. By splitting ELECTRE and PROMETHEE II into the two parts in which A is calculated and evaluated and by recombining them, new methods can be created. For example, we compute A as in ELECTRE but evaluate it with the PROMETHEE II approach.

2.2 Neural Networks

Beyond these approaches we use artificial intelligence (AI) techniques as a framework or a tool for our decision support system. Here, we focus on the neural network approach which is unlike 'cassical AI' not based on the physical symbol systems concept (Newell and Simon, 1976). The lack of symbolic representation may be regarded as a disadvantage, especially when for a decision problem corresponding cognitive concepts (e. g. decision rules) are easily available. Because typically many MCDM problems are ill-structured these information is often not obtainable. Neural networks are then an appropriate tool which utilizes (possibly self-organized) learning to simulate intelligent human problem solving.

Neural networks are also a natural concept for decision making because they mimic the structure of biological neural information processing. The parallelity of data processing in these networks directly corresponds to multiple criteria modelling of decision making problems (see examples below). Neural networks usually have the property that similar inputs lead to similar outputs. This quality of stability make them fault tolerant and can be interpreted as an ability to generalize. Moreover, during the last years there was a lot of research into the efficient implementation of neural network software and into the development of hardware support.

There, already, are some surveys including expert system ('classical AI') applications in MCDM to which we refer (see Ozernoy, 1992, Petrovsky, 1992). Although the neural network research has rapidly grown during the last decade there are not many publications about the combination of neural networks and MCDM. Malakooti and Zhou (1994) analyze neural networks used to determine utility functions for discrete MCDM problems and derive interesting properties of neural networks used in the context of MCDM.

Reflecting the usage of neural networks for MCDM we can think of neural networks as a simulation tool for MCDM methods or as an MCDM method itself. To explain these ideas let us consider what a neural network is. (Artificial) neural networks are models of natural neural systems. They consist of

(artificial) neurons which are somehow interconnected. Most biological neurons have some hundred inputs connected with the neuron via synapses. From the input values one output value is calculated and forwarded to other neurons via the axon. In the McCulloch-Pitts model (McCulloch and Pitts, 1943, see also Müller and Reinhardt, 1990) the state (output) S_j of a neuron j is defined by

$$S_j = \psi(\sum_{i=1}^{n} w_{ij} \cdot e_i - \Theta).$$

ψ is a sigmoid function like

$$\psi(x) = \begin{cases} 0, x < 0 \\ 1, x \geq 0 \end{cases}$$

or as in the stochastic neuron model

$$\psi(x) = \tanh(\beta x)$$

with $\beta > 0$. Θ is a threshold value and the w_{ij} are weights. The e_i are the input values. The weights w_{ij} may be positive (prohibitory inputs) or negative (inhibitory inputs). Prohibitory inputs lead to an increase of the output value of a neuron while prohibitory inputs reduce it. In a neural network the neurons receive their inputs from outside or from other neurons and forward their results to other neurons or to the outside. A neural network can have more than one output value. A very common network type is the perceptron (feed-forward layered network) where the neurons are located in layers and every layer feeds its signals only into the next layer. There are no feedbacks.

A single neuron can be regarded as a means to calculate the value function for a multicriteria alternative. A neuron has multiple inputs, weights them, and calculates an aggregated result interpretable as a goal to be optimized. In this way, a single neuron works quite similar to a scoring resp. simple additive weighting method (see Hwang and Yoon, 1981, p. 99-103): Here, positive criteria have positive weights, negative criteria have negative ones, $\Theta = 0$, and ψ is the identity function ($\psi(x) = x$).

With a little more effort, we can construct a neural network working as the conjunctive levels method (Hwang and Yoon, 1981, p. 68-70). For a problem with n criteria, a perceptron with $n+1$ neurons is necessary. In the first layer there are n neurons. The ith neuron has the ith criterion as input value with weight 1. The threshold value of the ith neuron is the ith level. ψ is the step function

$$\psi(x) = \begin{cases} 0, x < 0 \\ 1, x \geq 0. \end{cases}$$

The outputs of these neurons are the inputs of the $(n+1)$-th neuron weighted with $1/n$. The threshold value of this neuron is $\Theta_{n+1} = 1$ which means that

at all preceding neurons the threshold values must be satisfied to achieve an output of 1. Otherwise the output is 0.

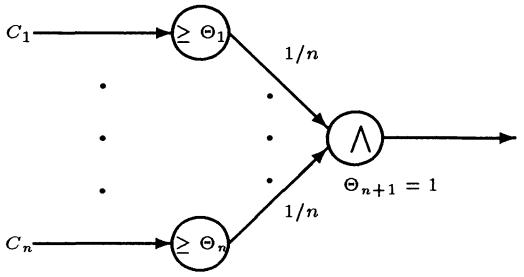

Fig. 2. A neural network for the conjunctive levels method. C_i is the ith criterion value of an alternative. The result indicates whether the alternative is admissible.

3 Learning for MCDM

Neural networks are capable of learning. Therefore, we do not need to construct a special neural network for a special purpose (for instance a special MCDM method). A sufficiently complex neural network should be able to learn to compute the function we are interested in. Primarily, learning means to modify the weights. The weights store the information about the function a neural network calculates. But there also are other parameters which can be tuned during a learning process, for instance the threshold values or even the architecture of the neural network.

Before we focus on some concrete learning concepts let us consider the nature of learning. Generally, learning effects the acquisition of knowledge and an increase of information. A method realizing the learning process can be regarded as a metamethod, a method which is applied to other methods instead of ordinary problems. The reason why we do not conceptually treat the learning procedure as a part of the method is that a method may have different learning methods and learning methods sometimes are applicable to various methods. Especially, we are interested in the use of universal concepts to generalize the idea of learning for other methods. The terminology of problems, methods, and metamethods is reflected by the object-oriented implementation concept of the DSS, which we present more detailed in Hanne (1994b).

A well-known learning method for the perceptron is error back propagation. In the simple perceptron without hidden layers a gradient approach is used to assess a modification of the weight w_{ij} (which is a measure for the connection strength from the ith neuron or input to the jth neuron or

output):

$$\Delta w_{ij} = \epsilon \sum_{\mu=1}^{z} (\zeta_j^\mu - S_j^\mu) e_i^\mu.$$

Here, $0 < \epsilon \ll 1$ is a given parameter, the $e_i^\mu, \mu \in \{1, ..., z\}$ are z input values, the $S_j^\mu, \mu \in \{1, ..., z\}$ are the corresponding output values. The $\zeta_j^\mu, \mu \in \{1, ..., z\}$ are the desired output values of the jth neuron. We start with given, possibly randomly determined weights and a sample of input and corresponding desired output values. Then we calculate the actual output values, tune the weights with the help of the above perceptron learning rule, and repeat this procedure with the same or new data samples until satisfactory results are achieved. A generalization of this concept for multilayer networks leads to the error back propagation.

Another learning concept we like to present is based on evolution strategies (Rechenberg, 1973, Schwefel, 1983). Like genetic algorithms (Holland, 1975), simulated annealing, and neural networks, evolution strategies follow the concept of 'learning from nature'. All of these algorithms are inherently parallel problem solving approaches applicable to a broad range of situations. Therefore, they can be utilized for learning by different methods. Evolution strategies simulate the main evolutionary principles which are replication, mutation, recombination, and selection. It should be mentioned that evolution strategies have already been directly applied to multicriteria optimization by Kursawe (1992).

For our application in the DSS we build up a population of the method (respectively representations (floating point numbers) of its relevant data, i. e. parameters of the method) to be improved. First, the $\mu, \mu \in N$, methods of the parental generation are replicated. The data of the $\lambda, \lambda \in N$, children methods is mutated by adding random variables (normally distributed with expected value 0). It is also useful to recombine the data so that a children method inherits its information from different parents. In this case the data of a children method is copied from different parents. In the next step the fitness values of the children methods are calculated and the μ best are selected as parents for the next generation. In the $(\mu + \lambda)$-evolution strategy children and parents are considered in this selection process whereas in (μ, λ)-evolution strategies (with $\lambda > \mu$) only the children can survive. The simulated evolution continues for a fixed number of generations or until a stop criterion is reached.

The calculation of the fitness of a method is performed similar to the perceptron learning rule approach: We apply a method to a problem and compare the result to a reference result. The distance measured by means of an l_p metric between the calculated result and the desired result is to be minimized.

An example: We try to rebuild the application of the TOPSIS method to the fighter aircraft problem presented in Hwang and Yoon (1981, p. 133f). We construct a 2 layer perceptron. Layer 1 consists of 5 (stochastic) neurons, each with 6 inputs (because the problem has 6 criteria). In the second layer

there is only one (stochastic) neuron (because TOPSIS delivers a scalar value for the decision) connected with each neuron of layer 1. We choose the weights w_{ij} and the β values randomly. Then, we define a $(5+10)$-evolution strategy for the assessment of the weights and the β values. Within 500 generations we obtain (after a simulation time of about 15 minutes on a 33 MHz 486-PC) a neural network calculating the TOPSIS results to a precision of 10^{-6}.

4 An Example Application to Share Investments

For a somewhat more sophisticated application exploring the learning features of the decision support system let us study an essential problem in the area of investment decision making. When shall one buy or sell shares of one particular firm? There are two main approaches for the selection of shares. One is based on the consideration of the economic data of the firm (fundamental analysis). Here, we follow the second approach called chart analysis and use only historical stock prices for our present decisions.

Therefore, we consider a time series. For each time t we want to predict the future success by means of k historic prices $p_{t-t_i}, t_i > 0, i \in \{1, ..., k\}$. In our example: $k = 9, \{t_i : i = 1, ..., 9\} = \{1, 2, 3, 4, 7, 10, 16, 31, 91\}$. We use a time series with 3 stock prices (of Volkswagen shares) per banking day. As a measure of success we use the relative change of the prices some time units later. In our example we try to predict the price change one day later: $(p_{t+3} - p_{t-1})/p_{t-1}$. We let our DSS learn with one part (the half) of the time series (of 1245 time units) and look how the other part, the future, can be predicted.

We use three methods to compare their applicability to this problem: a linear neuron (behaving like a simple additive weighting method), a stochastic neuron, and a perceptron consisting of 2 layers, the first one with 6 stochastic neurons, the second one with a single stochastic neuron. As a criterion of the success of these methods we use the l_2 metric between the predicted and the actual future results. The three methods are improved during a 500 generation process with a (5+10)-evolution strategy.

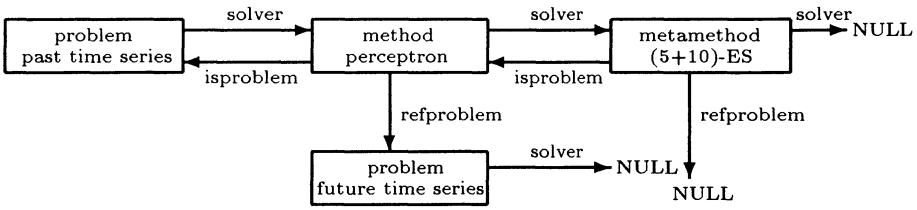

Fig. 3. The relationship between the objects *past time series, perceptron, future time series*, and *evolution strategy* in this application. For further explanations of the object-oriented concept of the DSS see Hanne (1994b).

After the completion of the optimization process, the linear neuron has a fitness value of 11.93 for the future part of the time series. This is significantly worse than the fitness value of 5.43 for the past part for which it was optimized. On the other hand this in not surprising in the light of conventional stock price theories which assume stock prices to follow a random walk so that predictions are impossible.

The stochastic neuron obtains a slightly better result of 4.9198 for the past part and 8.3116 for the future part of the time series. The perceptron gets distinctly better fitness values: 2.2174 for the past part and 2.7998 for the future part. We see that this approach does not only allow a better approximation of the function to be learned but that also the behavior is more stable and deteriorates hardly when the neural network is applied to new data. By using the perceptron the future can be predicted in a significantly better way in our example. Similar positive experience of using neural networks in time series analysis has been reported in the literature, see e. g. Refenes (1995).

5 Conclusions

In this paper we discuss the main ideas for a new decision support system for MCDM. We consider goals, structure, learning, and application issues of the DSS. Beside the completion of the implementation and the expansion of the system, further tests and real-world applications as well as theoretical considerations will be necessary to analyze the usability of such a system for decision making. Especially, we will analyze the possible advantages of the joint use and hybridization of different methods. The optimal areas for the application of MCDM methods and neural networks (and other concepts of decision making) should be clarified (if possible). As a consequence of the considerations mentioned above concerning MCDM networks and neural networks as MCDM methods we think about a hybridization of these concepts. Neural networks can be used as a supplementary part of an MCDM network (possibly consisting only of one MCDM method). In this network, they can be applied separately to standardize or rescale the data or to choose a method. More generally, one could build networks where the nodes are neurons or MCDM methods (neural MCDM networks). Up to now, we do not have any experiences with the advantages of this approach. As a case study we will continue our investigation of investment problems which are of high practical interest and imply complex and poorly defined relationships between large quantities of numerical data. Therefore, neural networks as well as multicriteria approaches are well suited for application in this field.

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IX

Behavioral Research and Multicriteria Analysis

Pekka Korhonen and Jyriki Wallenius organized and chaired a workshop on the implications of the behavioral issues in multicriteria analysis. In the following invited paper the authors do not intend to reproduce the interesting workshop discussions but evolve from them to emphasise the potential of cross-fertilization between these two areas.

The paper starts by a brief review of behavioral decision theory literature. The next chapters deal with the limitations of the traditional axiomatic approach for modelling preferences, the visualisation aspects in human/computer interfacing and the crucial classical discussion on weights and importance of criteria.

In the conclusion the authors start pointing out twelve implications of their findings for the design and development of interactive decision tools. Finally, it is stated that "systematic behavioral experimentation with human subjects is needed to clarify several of the open-ended research questions raised in this paper".

Behavioral Issues in MCDM: Neglected Research Questions

Pekka Korhonen and Jyrki Wallenius
Helsinki School of Economics and Business Administration,
Runeberginkatu 14-16, 00100 Helsinki, Finland

This paper has evolved from a Workshop organized and chaired by the authors at the *XIth International Conference on Multiple Criteria Decision Making*, Coimbra, Portugal, August 1-6, 1994.

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Abstract
Behavioral decision theorists have studied human decision making in great detail. Since the late 1960's, Einhorn, Edwards, Kahneman, Roy, Tversky, and others have developed new theories to explain choice and decision behavior. Thus far this behavioral research has had little impact on Multiple Criteria Decision Making (MCDM). Only a handful of MCDM-researchers have critically examined the behavioral underpinnings of our field. To improve the success of decision tools in practice, MCDM-researchers should pay more attention to the behavioral realities of decision making. In this paper, we discuss various behavioral issues relevant for MCDM based on our personal observations and experiments with human subjects. The spirit of our paper is to pose questions rather than provide definite answers.

1. INTRODUCTION

Multiple Criteria Decision Making (MCDM) has been an important and active research area for a quarter century. From the beginning, many of the topics in MCDM were optimization-related.[1] In the 1970's, research in MCDM focused on the theory of multiple objective mathematical programming and procedures for solving such problems. In the 1980's, there was a shift in emphasis towards multiple criteria decision support. Accordingly, much research focused on the user interface, the behavioral foundations of decision making, and on supporting the entire decision-making process in the broad sense. The 1990's have brought about an information and communication technology revolution, with powerful personal

[1] Multiattribute Utility Theory (MAUT) has a basis in decision analysis, and developed separately.

computers, networks, and exciting software. The consequences of this revolution on our field remain yet to be seen.

In MCDM, the mathematical assumptions upon which the systems/tools are based vary. Also, the nature and type of information requested from decision-makers (DMs) differs from one system/tool to another. For example, researchers frequently postulate different assumptions about the form and stability of a DM's composite value (utility) function. Researchers frequently request DMs to provide and revise their aspiration levels for different criteria -- or provide pairwise preference comparisons. Researchers also frequently request DMs to provide importance rankings for criteria. Yet, only a few are concerned about the behavioral realism of these and other assumptions or whether DMs can reliably furnish the requested information. Furthermore, even if DMs could furnish such information, we can often reasonably ask, whether we understand what their answers mean.

In this paper we discuss many of these important behavioral issues. There is an apparent need for such a discussion, as was demonstrated by the active participation in the Workshop that we organized and chaired at the *XIth International Conference on Multiple Criteria Decision Making* in Coimbra, Portugal, August 6th, 1994. In fact, we wish to thank the participants of the Workshop for their inputs, in particular Professor Oleg Larichev from VNIISI, Moscow. This paper has evolved from the Workshop.

2. A BRIEF REVIEW OF BEHAVIORAL DECISION THEORY LITERATURE[2]

The "rational man" model of classical economics and the satisficing model of Simon (1955) are two major theories of human decision making. Linear, compensatory models form the basis of the rational model. In these models tradeoffs are allowed between attributes -- a high value on one attribute can compensate for low values on other attributes. In contrast, noncompensatory, satisficing models eliminate the need to confront the DM with tradeoffs between attributes. Examples of such noncompensatory models include lexicographic ordering and Elimination By Aspects (Tversky, 1972). These and other theories seek to explain human decision making. Empirical research, however, shows that decision processes are complex and dynamic. Indeed, the following empirical studies suggest that no single explanatory model is good for all purposes and that task and context largely dictate the decision process taken.

Ford, Schmitt, Schechtman, Hults, and Doherty (1989) found that noncompensatory strategies were the dominant mode used by DMs. Task complexity was strongly related to the strategy used, increasing complexity leading to nonlinear strategies. Murtaugh (1984) found that DMs used a noncompensatory, hierarchical process. He suggests that such processes may be used in environments which are familiar to the individual. Bettman and Park (1980) noticed that strategies changed from noncompensatory to compensatory as alternatives were eliminated from consideration. Experiments reported by Houston, Sherman, and Baker (1991)

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[2] This section is based on Davey, Olson, and Wallenius (1994).

demonstrated that decision processes are dynamic. Simplifying heuristics, such as representativeness and anchoring are often used to reach a decision (Tversky and Kahneman, 1974). Individuals tend to start with simple decision rules and, if they do not lead to a decision, apply more complex rules (Montgomery and Svenson, 1976). In a recent study, Kasanen, H. Wallenius, J. Wallenius, and Zionts (1994) found that DMs consider multiple criteria in their decision tasks, but choose and often commit themselves on an alternative at an early stage. They subsequently attempt to test the feasibility of the chosen alternative and to find supporting arguments. No open comparison of alternatives necessarily takes place.

The way DMs process attribute/alternative information has also been studied by various researchers. Payne (1976), among others, found that information search patterns are characterized by alternating sequences of intra-alternative and intra-attribute search[3]. Bockenholt, Albert, Aschenbrenner, and Schmalhofer (1991) discovered that subjects used selective information processing, rather than examining all attributes of alternatives. They concluded that more information was needed when alternatives appeared equally attractive and that information processing shifted between attributewise and alternativewise. These results are similar to those found by other scholars. Rosen and Olshavsky (1987) suggested that information acquisition concerning attributes and alternatives was consciously limited when recommendation information was available. Interestingly, information presentation format has been found to affect information search patterns (Bettman and Kakkar, 1977). Subjects could be induced to search by attribute or by alternative if the information was organized in that fashion. This is what happened in Davey, Olson, and Wallenius (1994). See also Korhonen, Larichev, Mechitov, Moshkovich, and Wallenius (1994).

How do we support DMs in decision environments described above? We begin with a brief account of the traditional axiomatic approach and its limitations in Section 3. Section 4 describes our personal observations from using decision tools in interactive environments. Section 5 addresses the vexing relationship between criteria and weights. Section 6 highlights the importance of visualization in MCDM. Section 7 concludes the paper with a list of implications of our findings for the design and development of interactive decision tools.

3. TRADITIONAL AXIOMATIC APPROACH AND ITS LIMITATIONS TO MODELLING PREFERENCES

The rational model of economics postulates a set of behavioral axioms for DMs. If these axioms are satisfied, there exist preference numbers so that preferences for alternatives can be determined by calculating expected preferences. This approach is particularly popular in economics. However, it has been documented that DMs violate one or several of the axioms (notably the transitivity and independence

[3] When we investigate all the attributes for one alternative before proceeding to the next alternative, is an instance of intra-alternative search; in contrast, when we investigate one attribute for all the alternatives before proceeding to the next attribute, is an instance of intra-attribute search.

axioms) (see Von Winterfeldt and Edwards, 1986). In fact, sometimes these violations are systematic and predictable (Tversky, 1969). These results are well-known in decision theory. From a descriptive point of view, occasional intransitivity is usually not contested. If we assume a normative point of view, the situation becomes more complex. Many proposed decision tools are based on the assumption of complete rationality on the DM's part. In the normative case, the arguments pro and con can be illustrated with a classical example taken from the medical context.

> "Would you want your attending physician to be intransitive when prescribing treatments for you?"

Most people would answer: "No" (argument pro transitivity).

> "If your attending physician turns out to be wrong, would you still want him/her to be transitive? Or is he/she allowed to admit the error and behave intransitively?"

Most people would answer: "No" to the first question and "yes" to the second question (argument against transitivity).

Without engaging in a philosophical debate, we adopt a pragmatic view. We think that we can do a much better job in supporting DMs, if our decision tools are based on a solid behavioral foundation. Hence the underlying assumptions should be behaviorally plausible. Also, the information requested from DMs should be something that they can reliably furnish.

4. DECISION TOOLS IN AN INTERACTIVE ENVIRONMENT

Over the years we have accumulated extensive experience in using interactive decision tools in practice. Such experience, though valuable, is not always easy to document in a scientific manner. We have also conducted a number of systematic experiments with human subjects to attempt to understand their choice behavior in an interactive, computer-supported setting. Several of these experiments have been masters theses written at the Helsinki School of Economics under our supervision. We wish to emphasize that our empirical evidence is not always conclusive. Definitely, additional behavioral experimentation is needed to enhance our understanding of many of the underlying behavioral issues. Frequently we generate more questions than answers.

We continue with an account of our personal observations regarding the use of interactive decision tools in practice. In the text, such personal observations are marked with "®". In many cases we do have plausible explanations to accompany such observations, which we share with the reader.

® *If allowed, DMs may*[4] *wish to reconsider alternatives once discarded. In other words, individuals may exhibit cyclic behavior.*

In fact, such behavior seems natural in interactive settings, provided that the decision tool allows it (Korhonen, Moskowitz and Wallenius, 1990). The path followed by DMs, when using interactive decision tools, is not always associated with increasing utility/value. How could it be, if we don't know the mathematical representation of the DM's value/utility function. Cyclic behavior may be due to a number of reasons:

(1) Subjects compare alternatives in a relative rather than absolute sense (Additive Difference Model by Tversky, 1969; and its extension, Prospect Theory by Kahneman and Tversky, 1979) (see also Korhonen et al., 1990; and Salminen and Wallenius, 1993).

(2) Subjects exhibit exploratory behavior, in particular when approaching their most preferred solution. This is simply because of their desire to boost their confidence (Tversky, 1989).

(3) The DM's preference structure is neither known nor fixed prior to solving the problem. Learning one's preference structure is a gradual process.

(4) Subjects make mistakes due to the complexity of the decision task.

We have used the prospect theory model to explain subjects' choices in interactive decision environments, and have found it to be a reasonable model of choice for many individuals (Korhonen et al., 1990; Salminen and Wallenius, 1993). We acknowledge that in a specific context other explanations may be plausible as well.

We, along with other scholars, have also observed that:

® *Interactive procedures converge in a few iterations -- much faster than traditional single objective optimization procedures.*

Why is this true? Do people simply get tired and prematurely stop using the decision tool? Or are people smarter than machines and are able to perform "intelligent jumps" over the feasible decision space? Perhaps so, but we have developed a plausible explanation based on Prospect Theory (Kahneman and Tversky, 1979). People become easily "satisfied", if they react more strongly to negative than positive stimuli. If we make tradeoffs between different criteria (on the efficient frontier), we win in some and loose in some other criteria. If the losses weigh more heavily than the gains, the DM stops (Korhonen et al., 1990).

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[4] In the text, we occasionally use words, such as "may", "maybe". Sometimes the phenomenon in question takes place 10% of the time, sometimes 40%. However, that is not the point. The point is that our observations are not rare anomalies. If they were, it would not be so important to pay attention to them.

® *Both in single DM and group DM tasks, the DM's most preferred solution may depend on the starting point and/or the path leading to the most preferred solution.*

In an experiment, when an individual solved the same decision task three times (without realizing that the tasks were identical), the same most preferred solution was identified each time (that is, thrice) in 1/3 of the cases, twice in 1/3 of the cases, and in 1/3 of the cases the most preferred solution was each time different (Mikkonen, 1991). The results are tentative, and warrant additional experimentation. In a Single Negotiating Text type two-party mediation, we found that a biased starting position, which is not compensated for by the path, impacts the outcome of negotiations (Korhonen, Oretskin, Teich, and Wallenius, 1995). Again, we offer explanations that tie together with the Kahneman and Tversky research on framing, prospect theory, reference points, and anchoring (Kahneman and Tversky, 1979; Tversky and Kahneman, 1981).

® *In practice, the DM chooses the alternative he/she believes is his/her most preferred alternative.*

In practice, a DM's most preferred solution to a decision problem is not a solution which maximizes the individual's utility/value function over the feasible set. How could it be, if we don't know the utility/value function. How do we define the most preferred solution in practice? The question is not trivial.

We propose the following *requirements* that a solution must fulfill to qualify as a "most preferred" solution. No assumptions are made about the DM's underlying preference structure.

> "A solution can be regarded as the most preferred solution to a decision problem, if the DM is *convinced* that it is preferred to all other nondominated solutions, and he/she has a *realistic perception* of these solutions."

By *convinced* we mean that the DM believes the current solution really is the best one, which he/she can find from among the set of (nondominated) solutions. By a *realistic perception* we mean that there exists no other potential (nondominated) solutions that would "surprise" the DM (positively) (Korhonen and Halme, 1992).

® *The realism of the computer model affects the way the DM assesses and revises his/her aspiration levels with respect to different criteria.*

The computer model has to be roughly consistent with the DM's cognitive model of the problem. Otherwise, for example, if the computer model promises too much, the DM is led to believe that something which is not feasible is feasible. On the other hand, if the computer model promises too little, the DM is led to settle for less than he/she should (Neuvonen, 1989). Generally speaking, aspiration levels play an important role in many MCDM tools. Yet, we lack an understanding of how individuals set and revise aspiration levels in lieu of new information particularly in MCDM contexts.

® *The DM wants to control the search process. Also he/she wants that support is available, when needed.*

DMs dislike dictated solutions. They do not want to be "surprised". They do not want mathematics to take over. They want to control the type and level of support available. In other words, whether support is a simple spreadsheet to check for dominance or a sophisticated value analysis of all conceivable options.

® *The DM does not always want to have maximal support.*

The common saying is "the more the better". This is not true with regard to the level of decision support. In an experiment, using Korhonen's VIMDA software with different levels of support, we found that the simpler levels of support were in fact more appreciated than the more complicated visual features of the software (Pajunen, 1989).

® *The DM does not always recognize the necessity of support.*

DMs sometimes make decisions that turn out to be bad. Perhaps they did not correctly perceive all the risks involved. Perhaps they did not perform enough market analysis. Perhaps they failed to consider viable decision options. Using a simple 3-state, 4-option investment example, we have noticed that DMs make dominated choices, simply because it is not always easy to detect dominance.

5. CRITERIA AND WEIGHTS

® *DMs relatively easily rank criteria in terms of importance. Yet, DMs are typically unable to explain what they mean by their rankings.*

We conjecture that individuals have a cognitive model in their mind that would explain the apparent ease of performing the criterion ranking. The problem is to understand the model. We have attempted to investigate this question systematically (Öörni, 1995). Our tentative conclusions are the following.

(1) The lexicographic model seldom explains the DM's criterion ranking.
(2) The relationship between weights and importance of criteria cannot be explained exhaustively. Generally, DMs don't possess additive utility functions. Even if they did, dependence of criteria messes up this relationship.
(3) The criterion ranges and weights seem to have a connection. The criterion ranges are highly subjective, though. Also, there are individual differences in framing the problem.
(4) The criterion ranking is related to the individual's "expectations" regarding the values of the criteria.

Further statements require additional experimentation.

6. VISUALIZATION

Recent technological advances make it possible to visualize problems, alternatives, etc., in an unprecedented fashion. The potential of:

(1) computer graphics;
(2) animation; and
(3) digitized video, among others

pose new research challenges for our field. Research should be conducted to identify the circumstances when specific visualization techniques ought to be used, instead of representing information numerically (Kasanen, Östermark, and Zeleny, 1991; Korhonen, 1991; Peiponen, 1991). New interfaces should be developed to incorporate the above techniques.

7. CONCLUSION

We conclude with twelve implications of our findings for the design and development of interactive decision tools. See also Korhonen et al. (1990) for a discussion of implications 1-5.

1. Both in single DM and group contexts, there is evidence that the 'path' or sequence in which solutions or settlements are presented to DMs, may affect the final choice. It is therefore important to look at the problem from different perspectives, use multiple representations, multiple starting points, and so forth. This helps the DM reconcile between different solutions and finally make up one's mind.

2. We must pay attention to "framing" a problem properly so that the choice process of a DM is consistent with the model being used and its assumptions. Otherwise, severe discrepancies and biases may exist between the model results and the DM's solution.

3. Decision tools should have built-in mechanisms to deal with inconsistencies.

4. Generally speaking, the less restrictive "ad-hoc" behavioral assumptions are made, the better.

5. Behavioral convergence of interactive decision tools is more important than mathematical convergence. Therefore, they should be designed to make "good progress" in the initial iterations; the latter iterations are less important.

6. The decision tool should convince the DM that the final solution is, indeed, the best he/she can do. Otherwise the DM might not even find the solution acceptable. On the other hand, if the decision tool does not provide the DM

with a realistic perception of all nondominated solutions, he/she may erroneously believe that the final choice is the best one. Hence the DM would not be able to choose an alternative that would be preferred to all other alternatives at the moment of final choice.

7. Ideally, decision tools should have graphic and numeric presentation capabilities. Graphics can be used to provide DMs with a holistic view of the problem. Numeric representation, in turn, provides a detailed view of the problem.

8. We must let the DM be in the "driver's seat". In other words, we must let him/her control the search process.

9. It is useful to list all decision-relevant criteria. We should also check for hidden criteria. If we can, we should check for dominance.

10. We must be sensitive to the DMs' needs regarding what kind and how much support they need/want.

11. Avoid requesting DMs to furnish criterion rankings, unless you know what they mean.

12. The attributes that the DM focuses attention on do not remain the same throughout the decision process, although commonly assumed in normative MCDM research (Steuer, 1986). A more evolutionary approach, as described in Korhonen, Narula, and Wallenius (1989), allowing the DM's attention to shift from one group of attributes to another during the decision process should be considered.

Additional systematic behavioral experimentation with human subjects is needed to clarify several of the open-ended research questions raised in this paper. We hope that this paper helps generate such research. We conclude with a quote from Korhonen et al. (1990, p. 178):

> "What we have implicitly proposed is an evolution of the field of interactive multiple-criteria decision making toward paying added respect and attention to the behavioral realities of decision making, and integrating the results of behavioral decision theory into the design and development of interactive multiple-criteria methods. This is certainly an area that has been overlooked in the operations research literature."

We welcome the colleagues' input.

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X

Multicriteria Approaches in Practice — Applications and Methodological Issues

Foreword

Real-world problems and decision situations generally involve the need to consider explicitly multiple, conflicting and incommensurate evaluation aspects. This fact is generally recognized as the starting point of multiple criteria decison aid (MCDA) as a scientific discipline, although some authors consider the multiple criteria approach as essentially an "act of faith" (Bouyssou, 1993). Even though the real-world application (with the meaning: *whenever someone pays for it!*) of the large corpus of techniques and methods MCDA has been developed lags well behind the theoretical and methodological advances, reports on MCDA in practice are steadily growing in the scientific literature. For instance, White (1990) presented a survey listing more than 500 references covering the period 1955-86 (excluding those approches which used an *a-priori* explicit value function). However, one must realize that just a small number of cases actually involves an implementation phase (and most of those seems to be goal programming-based studies). The improvement of the implementation rate of MCDA studies should be a crucial concern of MCDA researchers and practitioners, namely having in mind the conceptual and operational validation of the use of MCDA techniques in real-world problems with actual decision makers (Bouyssou et al., 1993). This trend to emphasize actual studies, as well as the search for new potential areas of application, also contributes to make MCDA to receive further cross-fertilization from other scientific disciplines. Nevertheless, it is fair to recognize that real-world applications are not easy to get from organizations (both in public and private sectors) even in a free an research-oriented basis. Under these (difficult) circumstances, sound academic case-studies (clearly assumed as such) supplied with *realistic* data may have an important role to play, not just as valuable experimentation frameworks, but also as *evangelization* tools to show the potential benefits which can be harvested from an MCDA approach in complex decision situations.

This section, which includes twelve papers, is a selection, made by the referees, of several interesting communications presented at the conference, aimed at revealing the importance and potentialities of MCDA to help decision makers worldwide, in a broad diversity of application areas, to achieve better plans of action.

The paper by Alves, Antunes and Clímaco uses a regional input-output planning model, as a laboratory case study, to make an experimental comparison of classical multiple objective interactive methods (STEM and Zionts-Wallenius) and a control panel-based interactive environment (SOMMIX). SOMMIX interactive

environment is based on a control panel which offers a large set of commands for selecting different forms of scalarizing functions, strategies for reducing the scope of the search, types of information required from the user and ways of information visualization. The potentialities of the package are compared with the performance of the classical methods in the context of a regional planning case study. This is based on a static input-output model, with data from an interior region in Portugal. It considers four objective functions: private consumption, level of employment, deficit of regional trade balance and energy consumption. The constraints arise from the relations among the different sectors of the economy and bounds imposed on certain model variables.

The paper by Beinat and Rietveld attempts to show the role of multicriteria value functions in the context of environmental decision making. It focuses on the combination of laboratory data (effects of the contaminants on organisms) and expert judgement (whenever the knowledge about those effects is unavailable or insufficient, as it is the case for many relevant chemicals). Additive value functions are used, which are linked to the expected effects of the pollutants. Specific assessment techniques to assess expert judgements have been designed in the framework of environmental studies, allowing for interactive data collection and imprecise estimations. The findings indicate that communication with experts improves during the assessment process as experts become familiar with the specific way of reasoning required by the model.

Bergeron, Martel and Twarabimenye present a case study describing how a multicriteria methodology based on outranking relations can assist loan officers in evaluating loan requests. This is a very important problem for financial institutions since corporate loans represent a primary source of both income and risk. Once they have evaluated a loan application under each criterion, loan officers must classify it into one of several pre-determined risk categories (sorting problematic). In their study the authors use ELECTRE-TRI method to classify loan requests in their respective risk category. Twelve criteria, grouped in four dimensions (macroeconomic, financial, managerial and collateral) have been considered. Criteria relative importance coefficients have been computed using AHP. Indifference, strict preference and veto thresholds have been established by means of some knowledge elicitation processes from experts. The upper and lower boundaries which define each risk category have been inferred from a sample of requests previously categorised by the loan officers. A study has been carried out at a large canadian institution. It is reported that the determination of parameters needed in the multicriteria procedure have not been an easy task for the experts committee.

M. Brannback studied the strategic management process of a finish drug company. Expert Choice, a software package based on AHP, has been used in two processes: to acquire a systematic ranking of the company's competitors; and to acquire a better ranking of competitors as a result of the first process. This is aimed at finding a ranking for the critical success factors (assets and skills which the firm have to manage better than their competitors), which are necessary to creating a sustainable competitive advantage. Expert Choice helped the firm's managing director to organise complexity, support systematic analysis and include subjective considerations, in order to understand how the critical success factors (objectives) were linked to each other and affecting the firm's performance, by finding out the most important ones and how well the competitors were performing with respect to them.

The paper by Grauer and Boden provides an introduction to the OpTiX -II software environment, which supports non-sequencial solution approaches to decision problems based on nonlinear optimization models. In the problem formulation phase the user defines the analytical formulation of the decision problems, which is then translated into a machine code representation suited for parallel processing in a heterogeneous network of parallel computers. In the phase of control of the optimization problem solution, the user has to choose the optimization algorithms and allocate computer resources from the network to the individual optimization tasks. The features and application of OpTiX-II are presented in a mechanical engineering case study: designing a gear reducer with minimal weight. This is a nonlinear optimization problem consisting in minimizing a function subject to a set of constraints arising from structural mechanics.

Hodgson, Rosing and Storrier report on the use of a bicriteria model which trades-off node- and flow-based demand in real-world network traffic problems. The model combines the p-median location-allocation model, which treats demand for service as weights expressed at nodes on networks, with the flow capturing location-allocation model, which deals with demands expressed on paths by means of traffic flows between origins and destinations. The model has been used with real-world data in a system of 177 traffic zones in the town of Edmonton, Canada. The findings indicate that, with morning traffic, residence-based demand will tolerate flow capturing optimality better than would flow-based demand tolerate residence-oriented optimality.

Hurson and Zopounidis propose the use of different multicriteria decision aid methods for management of stock's portfolios. The approach is twofold: - in the first step ELECTRE TRI and MINORA decision aid methods are used to help the portfolio manager in the selection of a set of attractive stocks taking into consideration his/her preferences and experiences; - the ADELAIS multiobjective linear programming system is then employed to constitute a portfolio of the attractive stocks by determining the proportions invested in each selected stock. MINORA ranks the alternatives from the best to the worst by estimating a certain number of separable additive utility functions. ELECTRE TRI makes a classification of the alternatives in categories delimited by some reference profiles, using a valued outranking relation. In each iteration ADELAIS proceeds by assessing an additive utility model, based on the information supplied by the decision maker when evaluating an efficient compromise solution previously proposed, to compute a new reference solution. The approach is illustrated with data from Athens Stock Exchange.

Spronk and Vermeulen present a multidimensional framework for performance evaluation, which takes into account the influence of external sources of risks (beyond the decision maker's control). Uncertainty is modelled by means of the so-called multi-factor method, which relates unexpected performance changes through sensitivities to unexpected changes of a multitude of risk factors. The vector of sensitivities (risk profile) can be seen as a multi-dimensional risk measure. A distinction is made between fixed and changeable firm characteristics, these being operational characteristics which are controllable by the decision maker. The approach is based on an interfirm conditional performance review, which considers both the firm's characteristics and the realized values of external risk factors when

evaluating *expost* performance. The approach is illustrated by using data concerning 67 bakery shops.

Stewart discuss the reasons why "state scenarios" (which are neither exhaustive nor complete), widely used for strategic planning as internally consistent representations of possible future trends, should be treated as part of the objectives' hierarchy. The challenges for the MCDA field are related to the selection of good scenarios, and to the provision of methods to make comparisons between those scenarios. It is argued that when dealing with uncertainty in the scenario planning context by means of MCDA methodologies, direct aggregation across scenarios should not be done. Performance under different scenarios should rather be tackled as part of the objective's hierarchy.

The paper by Tabucanon and Zhijing investigates the relative strengths and weaknesses of industrial zones and analyzes the appropriateness of industries for promotion. First, it examines the industrial sector using data envelopment analysis (DEA) to assess the efficiency of special economic zones (decision-making units in DEA terminology) with respect to certain important factors of development. The aim is to pinpoint comparative gaps of development among the decision-making units. The findings obtained from DEA on the appropriateness of the criteria are then used to develop an AHP model to determine the type of industries suited for the conditions of the region, aimed at improving the industrial sector's efficiency. Sixteen cities in a Chinese province have been chosen as decision-making units. The study indicates, for instance, that industries which are technically-intensive are must suited for promoting the regions's development.

The paper by Wenstop, Carlsen, Bergland and Magnus introduce the use of decision panels to valuate environmental goods for public policy purposes as a supplement to more traditional contingent evaluation methods. It aims at addressing problems arising from road traffic and seeks to specify the most important environmental and health impacts (both for valuation and policy making purposes). The model focus on end-impact criteria rather than environmental load criteria. Even though the former are less operational they are considered to be more understandable. Two case studies, one at national and one at local level in Netherlands, are reported. Two weight elicitation methods have been used: pairwise ordinal comparison of criteria and pairwise numerical trade-off between criteria. The expert panels consist of members from different state authorities. The experiments indicate that considerable uncertainties within the panels about proper judgement may arise, and the differences between panels were not strong.

D. Wijnmalen reports on a problem of selecting military equipment, in which an evaluation methodology based on hierarchical multicriteria analysis has been applied. This methodology, which relates the contribution of alternative options to military objectives, consists of two approaches: an aggregate (holistic) one and a detailed one, aimed at complementing each other. The aggregate approach deals with assessing alternative options *vis-a-vis* high level criteria based on general expertise, while the detailed approach concerns setting up a hierarchy of organizational function and performance indicators and connecting them with the task hierarchy. The aim is to offer military planners a general framework that should stimulate structured thinking in order to make priorities explicit. It is reported that the comparison of the results of the holistic and detailed procedures, in the context of the selection of an air-mobilem special vehicle, revealed no significant differences, although it triggered important additional discussions.

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An Experimental Comparison of MOLP Interactive Approaches Based on a Regional Planning Model

Maria J. Alves [1,3], C. Henggeler Antunes [2,3] and João Clímaco [1,3]

(1) Faculty of Economics, University of Coimbra
(2) Dept. of Electrical Engineering, University of Coimbra
(3) INESC, Rua Antero de Quental, 199
3000 Coimbra, Portugal

Abstract. This paper reports how previous methodological studies and experiments in power system planning and telecommunication network modernization planning led to the development of interactive environments to deal with multiple objective linear programming problems (MOLP), particularly with three objective functions, paying special attention to the use of graphical displays and controls. These tools are well suited to assist decision makers (DMs) in making a progressive and selective familiarization with the efficient solution set. However, whenever the problem demand the consideration of more than three objective functions, some of those techniques could not be used, and the interactive environments had to be expanded in a coherent manner to preserve the usefulness of visual inspection as a cornerstone of computer tools to assist DMs. SOMMIX is a control panel-based interactive environment which offers a large set of commands (embodying search strategies, techniques to compute new nondominated solutions, ways to express his/her preferences, means of interaction and information presentation, etc.) to assist the user, by facilitating and motivating the sucessive actions throughout the interactive solution search process. The main aim is to shed some light on the usefulness of computer packages which are flexible, user-friendly and technically sound to accommodate the very nature of human decision making processes. Using a regional planning model as a laboratory case study this paper is aimed at presenting some computer experiments with two classical multiple objective interactive methods (STEM and Zionts-Wallenius) and the SOMMIX package.

Keywords. Interactive environments, interactive methods, MOLP, input-output analysis, regional planning

1. Introduction

The authors have been dealing with applications of multiple objective programming in energy planning, new telecommunication service planning and regional planning for the last decade. Most of these have

been MOLP models considering three objective functions. Some conclusion drawn from early studies in energy planning had revealed the weakness of classical methods in assisting DMs, namely because without some previous knowledge about the shape of the efficient region (and thus the nature of the trade-offs to be made in different sub-regions), it was very difficult to provide with confidence the local information requested by the methods (regarding trade-offs, pairwise comparisons, relaxation quantities, etc.). The conclusions of these studies laid the foundations for the development of the TRIMAP interactive environment, which is aimed at assisting DMs to perform a progressive and selective search of the efficient region, namely by providing him/her graphical information using displays of indifference regions in the weight space. TRIMAP is devoted to three-objective LP problems, which permits the use of visualization means which are suited to assist DMs in making a progressive and selective familiarization with the efficient solution set [4].

Some studies that followed in energy planning and new telecommunication service planning problems, for which the shape of the efficient region presented some distinct characteristics, showed the advantages of combining in the same computer package different types of methods - an integrated interactive method environment, called TOMMIX [2]. The aim is to provide the DM/analyst the possibility of using and combining in sucessive interactions the different techniques to compute efficient solutions, strategies to reduce the scope of the search, and ways of information communication. TOMMIX has been developed mainly for three-objective function problems, and it extends the flexible environment offered by TRIMAP to the main procedures used in some representative interactive MOLP methods.

Both interactive environments use extensively graphical displays to convey the information, in a way that could be assimilated by the DM/analyst. For instance, in TOMMIX techniques have been developed to translate into the weight space the constraints introduced on the objective function values or the contractions of the cone of the objective function gradients. However, these and other functionalities become more complicated, or even lose their appeal, when the MOLP problem demand the explicit consideration of more than three objective functions. The SOMMIX [6] interactive environment is the result of our effort to overcome some interaction difficulties, namely regarding the usefulness of visual inspection, to extend and further ellaborate in a coherent manner the environment developed for TOMMIX. The SOMMIX interactive environment is based on a control panel which offers a large set of commands making the information available to the user. The potentialities of SOMMIX are herein demonstrated and compared with the performance of two classical methods: STEM (a feasible region reduction method) and Zionts-Wallenius (a weight space reduction method), in the context of a regional planning problem.

The MOLP problem is based on a static input-output model, with data from an interior region in Portugal. Four objective functions are considered in this model, which quantify the private consumption, the level of employment (both to be maximized), the deficit of regional balance of trade, and the energy consumption (both to be minimized). The constraints are obtained from the relations among the different sectors of the economy (model balance constraints), bounds imposed on the variation of certain variables of the model, and defining constraints (the private consumption defined as a function of the household revenues).

2. A MOLP regional planning model

The definition of a macroeconomic policy is a resource allocation problem, in which limited resources must be allocated to several inter-related economic activities, in order to achieve specific goals and respond to concerns of distinct nature, not only economical but also political, environmental and social. The static input-output model is an equilibrium model which involves the disaggregation of an economy in a number of inter-related sectors (or industries), thus focussing attention on the structure and interdependence of economic activities [7]. The output of each sector may be used as an input of other sectors of the economy or to satisfy final demands (the model is an "open" one if there are both n sectors and a household sector which exogenously determines final demand). The input-output transactions table records the flows of goods and services between sectors and the final demand of an economy during a time base.

The static input-output model can be expressed as set of linear equations $x_i = \sum_{j=1}^{n} a_{ij} x_j + y_i \quad (i=1,...,n)$, where x_i is the gross total output of sector i, y_i is the final demand of sector i, and the a_{ij} are the technical (inter-industry) coefficients. Considering this system of equations as inequalities together with upper and/or lower production bounds for the industries and an objective function to be optimized (such as maximizing consumption or minimizing the deficit of the trade balance) subject to these constraints, a LP model is obtained.

The early works combining mathematical programming techniques with input-output analysis gave rise to regional economic planning models, considering the maximization of a benefit criterion or the minimization of a cost criterion as a single objective. The extension of input-output analysis by means of multiple objective programming models can be useful because it enables for the explicit consideration of different objectives and concerns of distinct nature. Besides contributing to make the mathematical model more realistic, the multiple objective approach enables decision makers and planners to analyse a broader range of alternative plans, offering him/her a better perception of the

different, and frequently conflicting, evaluation aspects in order to find a satisfactory compromise solution from the set of nondominated solutions.

This input-output regional planning model is based on the accounts for Beira Interior, an interior region in central Portugal. The structure of this small regional open economy is characterized by a strong dependence on some key sectors: agricultural products (which account for about 43% of employment), foods and textiles. The final demands are disaggregated in private consumption, collective consumption, investment in fixed capital, investment in change in stocks and exports. The total output of each sector is represented by a decision variable. All variables are considered as endogenous, except gross fixed capital formation and collective consumption. It is assumed that the structure of the region's economy is stable during the planning period.

The input-output table is organized in 12 sectors (an aggregation based on the 49 sectors of the national accounting), consisting of an inter-sector flow matrix, a column vector with the values for the final demand, and 6 row vectors with the distribution of the primary inputs. The relationships among the different sectors of the economy (coefficients of the inter-sector flow matrix) are used to develop the constraints of the MOLP model.

2.1. Model constraints

Balance equations of the model:

$$\sum_{j=1}^{n} a_{ij}\, x_j + d_i\, p + c_i + f_i + v_i + e_i = x_i \qquad i=1,\ldots,\ n \qquad (1)$$

in which

n = number of economic sectors,
x_i = total output of sector i,
a_{ij} = element (i,j) of the technical matrix A with dimension (n+k,n) where k is the number of primary inputs,
p = total private consumption,
v_i = change in stocks for sector i,
e_i = exports from sector i,
c_i = output of sector i for collective consumption,
f_i = gross fixed capital formation for sector i,
d_i = ratio between the private consumption for sector i and the total private consumption.

These equalities may be re-written as inequalities assuming that the intermediate consumption and final demands of a given good (or service) cannot exceed the total amount available of that good (or service), thus v_i is the slack variable, and c_i and f_i (i=1,...,n) are considered exogenous:

$$x_i - \sum_{j=1}^{n} a_{ij} x_j - d_i p - e_i \geq c_i + f_i \qquad i=1,\ldots, n \qquad (1a)$$

Private consumption is assumed to be a function of household income. It is also considered that the average propensity to consume is constant during the planning period, which is given by the ratio *household consumption /gross household disposable income.*

Another constraint defines the private consumption which is a decision variable of the MOLP model. The private consumption in the region is given by the residents total consumption plus the consumption made in the region by the non-residents less the consumption of the residents made outside the region. The resident's total consumption is considered to depend linearly on the gross available household income (the most significative components of which are wages and the entrepreneurial income): $p= \alpha \sum_{j=1}^{n}(a_{wj} + \beta.a_{oj}).x_j$ in which α and β are constants and a_{wj} and a_{oj} are the j elements of matrix A corresponding to the wages and operating surplus rows.

Upper (U_i) (related to the production capacity) and lower bounds (L_i) for the total output of each sector:

$$L_i \leq x_i \leq U_i \qquad i=1, \ldots, n \qquad (2)$$

Upper bounds for the exports of each sector (E_i):

$$e_i \leq E_i \qquad i=1,\ldots, n \qquad (3)$$

Defining constraint for obtaining the gross added value (GAV):

$$GAV= \sum_{j=1}^{n} (a_{wj} + a_{oj} + a_{tj}).x_j \qquad (4)$$

in which a_{wj}, a_{oj}, and a_{tj} are the j elements of the row vectors of A corresponding to some primary inputs: wages, operating surplus, and indirect taxes (linked to production) less subsidies, respectively.

2.2. Objective functions

Four objective functions are considered taking into account fundamental aspects such as economic development, external debt, environmental factors, and social concerns (namely employment): private consumption, balance of trade, employment level and global energy consumption. The objective functions are defined to guide the development of the region economy according to some strategic policy guidelines issued by national and/or local authorities.

Private consumption (surrogate measure of the citizen well-being):

$$\max \; f_1 = p$$

Balance of trade (difference between total of imports and total of exports):

$$\min f_2 = \sum_{j=1}^{n} a_{mj}.x_j + d_m.p - \sum_{i=1}^{n} e_i$$

where a_{mj} is the element j of the row of A corresponding to non-competitive imports (a primary input), and the product $d_m.p$ is the amount of imports which has the private consumption as destination.

Employment level:

$$\max f_3 = \sum_{i=1}^{n} t_i.x_i$$

where t_i is the ratio employment/output of sector i in the base year (number of workers per 10^6 currency units).

Energy consumption (surrogate for the evaluation of environmental impacts):

$$\min f_4 = x_{energy}$$

where x_{energy} ($=x_2$) is the output of the energy sector

The MOLP problem consists of 35 decision variables, 4 objective functions and 50 constraints. For further details on the model, including a discussion of its main limitations, see [1].

3. Some computer experiments

In this section the application of the decision support system to the MOLP problem is presented. The aim is to illustrate the type of decision aid the system can provide as well as to present some results of the laboratory case study. We will suppose that a hypothetical decision maker provides information about his/her preferences regarding the solutions presented by the decision support system.

The objective function values corresponding to the solutions which optimize each objective function individually are displayed in table 1. Private consumption, deficit of trade balance and energy consumption are expressed in 10^6 currency units, and employment is given in number of workers.

solution	(f_1) private consumption	(f_2) deficit of trade balance	(f_3) employment	(f_4) energy consumption
1	117 047	30 439	227 922	11 733
2	115 009	13 270	220 249	10 683
3	116 885	31 902	228 289	10 321
4	102 004	31 379	206 366	8 959

Table 1 (Pay-off table)

Some general characteristics of these solutions are:

- solution 1 (maximizes private consumption): relatively high values

for the sector outputs; the output of the construction sector is very high and cannot be absorbed by intermediate consumption or other final demands, which leads to significant inventory changes within this sector (no exports are possible in the construction sector); high imports value.

- Solution 2 (minimizes deficit of trade balance): higher values for the sector outputs with respect to solution 1, except for the construction sector whose output is much lower; high exports value and medium imports value.
- Solution 3 (maximizes employment): sector output values similar to, although lower than the ones in solution 1; construction sector output at its maximum value; medium level of exports; high imports value. It should be noted that the ratio employment/construction sector output in the base year is very high which justifies that the construction sector reaches its maximum value when f_3 is optimized.
- Solution 4 (minimizes energy consumption): low sector output values; low level of imports and exports (the sector outputs are generally low and besides the energy sector has a great impact on imports value).

3.1. Using STEM and Zionts-Wallenius' methods

STEM is a feasible region reduction method, in which in each interaction a weighted Tchebycheff distance to the ideal solution is minimized. The problem in each interaction reflects the DM choices in previous interactions through the reduction of the feasible region derived from the setting of constraints on the objective function values [3].

The initial solution computed by STEM is: f_1= 114 466; f_2= 15 337; f_3= 220 852; f_4= 10 433. This is a balanced solution with medium sector output values, and relatively high exports value. Let us suppose that the DM considers that the deficit of trade balance is a satisfactory value and is willing to relax it by 2 500. The new solution presented by STEM is: f_1= 114 585; f_2= 17 837; f_3= 221 788; f_4= 10 365. With respect to the initial solutions, the construction sector output increases and a slight increase in imports is observed together with a decrease in the exports mainly due to a lower output in the "other products" sector (products such as paper, wood, cork and rubber). By relaxing f_4 by 500 a new solution is obtained: f_1= 115 703; f_2= 14 935; f_3= 222 193; f_4= 10 715. The main sector output changes, with respect to the previous solution, are verified in the "other products" and energy sectors which increase, and both imports and exports levels increase again. In an attempt to further increase the employment level a new relaxation of f_2 by 2 900 is permitted, leading to the solution: f_1= 115 703; f_2= 17 835; f_3= 223 133; f_4= 10 526. This solution presents the best value for the employment objective, mainly because of an increase in the construction sector output.

Zionts-Wallenius is an weight space reduction method, in which in each interaction a weighted sum of the objective functions is optimized. The problem in each interaction reflects the DM choices in previous interactions through the reduction of the weight space derived from the

preferences stated regarding pairwise comparision of solutions and trade-offs corresponding to displacements along nondominated edges [8].

The initial solution, obtained with equal weights for the objective functions is: f_1= 116 400; f_2= 18 706; f_3= 224 258; f_4= 10 676. The sector output values are similar to the ones in solution 2 (see table 1) with an increase in the construction sector, and high exports value.

The first adjacent solution (sufficiently distinct for pairwise comparison) is: f_1= 116 508; f_2= 21 237; f_3= 225 198; f_4= 10 519. With respect to the initial solution, this solution improves employment by worsening the deficit of the trade balance, due to an increase in the construction sector and a decrease of exports. Let us suppose that the DM prefers this one (namely because the better value for employment), which corresponds to imposing a constraint in the weight space. The new incumbent solution computed with a central weight set in the restricted weight space is the same as above, which then becomes the new current solution.

A new iteration is performed, and the adjacent solution to the current one is : f_1= 116 180; f_2= 18 588; f_3= 223 939; f_4= 10 570. Since all objective function values are worse than in current solution, except for deficit of trade balance, let us suppose that the DM prefers the current solution to this one. Since no more adjacent solutions which are sufficiently distinct exist, the following trade-offs are presented which correspond to unit changes along nondominated edges emanating from the current vertex solution (+/- means improvement/degradation of the objective function value):

(1) f_1: -0.05147; f_2: -0.59885; f_3: +0.37573; f_4: +0.03212
is accepted

(2) f_1: -3.73764; f_2: +12.28069; f_3: -8.13854; f_4: +0.14040
is not accepted

(3) f_1: +0.07306; f_2: -1.33815; f_3: +0.51314; f_4: +0.03506
is accepted

Based on the responses to trade-offs, new constraints are imposed on the weight space and a new possible solution is computed using a central weight set in the reduced weight space: f_1= 116 880; f_2= 31 901; f_3= 228 289; f_4= 10 321. Although this solution is very good regarding employment, it has the deficit of trade balance near its worst pay-off table value (see table 1). So, this solution is not preferred to the current one, which is adopted as the final solution.

3.2. Using the SOMMIX package

The SOMMIX interactive environment is based on a control panel which offers a large set of commands making the information available to the user. Control panel-based tools need some training to learn in an integrated manner the information available by means of their different components in a way that motivates the sucessive actions in the interactive search process. The use of SOMMIX to study the regional

planning model is aimed at showing some of its main features in assisting the DM throughout the interactive decision process by invoking the appropriate control panel tools and instruments within each tool (fig. 1).

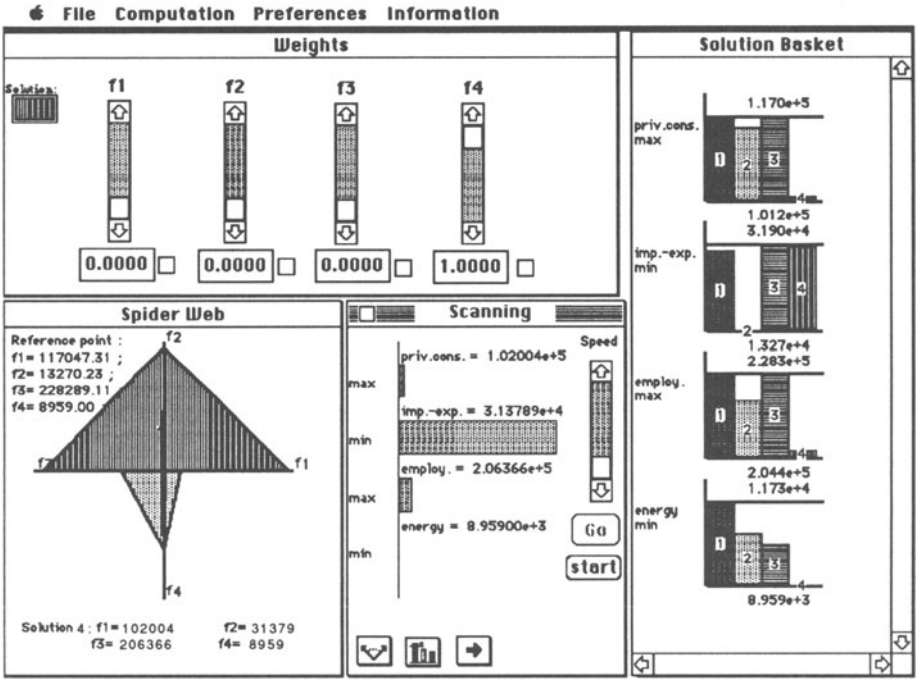

Fig. 1 - The control panel

The solutions which optimize each objective function individually are already known (table 1). Let us proceed the search by computing some more solutions by using sets of weights "well dispersed" in the weight space (table 2), in order to have a better informed overview of the nondominated solution set.

solution	(f_1) private consumption	(f_2) deficit of trade balance	(f_3) employment	(f_4) energy consumption
5	116 801	26 609	227 255	10 378
6	116 073	16 057	222 300	10 727
7	116 801	26 609	227 255	10 378
8	116 508	21 238	225 199	10 519

Table 2

The visualization of the indifference regions in the weight space corresponding to each vertex solution has revealed to be a powerful tool to enable the DM to grasp the shape and to perform a selective search of the nondominated frontier. However, with more than three objective functions its visualization is not straigthforward. SOMMIX uses user-defined cuts, with scroll bars to adjust dinamically the weights, to exploit

graphically and travel through the weight space. Fig. 2 shows a cut of the weight space helding constant the weight of the employment objective function at the levels λ_3=0.3 and λ_3=0.6.

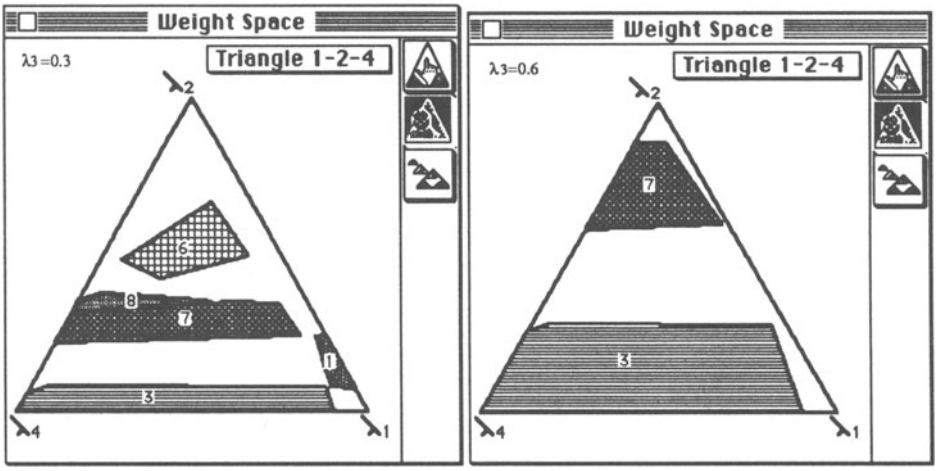

Fig. 2 - Cuts of the weight space

In order to reduce the scope of the search let us suppose that the DM decides to impose an upper bound on the trade balance deficit at a value considered acceptable: $f_2 \leq 23\,000$. This constraint can be translated into the weight space, by solving an auxiliary LP problem, and the region where it is satisfied is displayed in fig. 3. By interactively selecting a new set of weights within this region a new vertex solution satisfying the additional constraints is computed - solution 9: f_1=116 400; f_2=18 709; f_3=224 260; f_4=10 676; which very similar to the first solution computed in Zionts-Wallenius method.

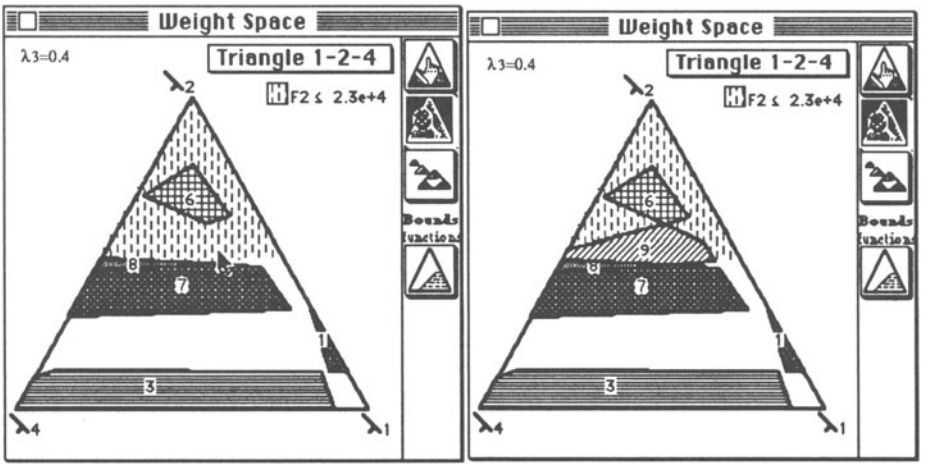

Fig. 3 - Searching for a new solution in a restricted region of the weight space

The *Scanning* tool is well suited to focus the search on the neighbourhood of solution 9 (considered a good compromise), by performing a free-style search (in the spirit of Pareto Race method). Having chosen the improvement of emplyoment as direction of motion, some solutions are scanned (fig. 4). Let us suppose that the DM stops the search at solution 10: f_1=116 606; f_2=22 474; f_3=225 703; f_4=10 579. This scanning direction over the nondominated frontier has improved employment by worsening the deficit of the trade balance (due to a cumulative effect of decrease of exports and increase of imports), although maintaining the limitation previously imposed.

The *solution basket* window displays the solutions "collected" by the user throughout the interactive decision process using bar graphs (fig. 5).

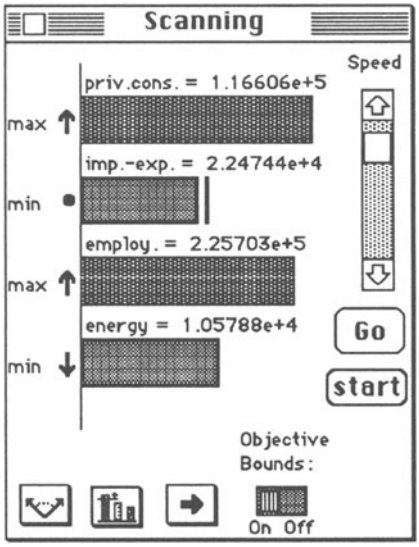

Fig. 4

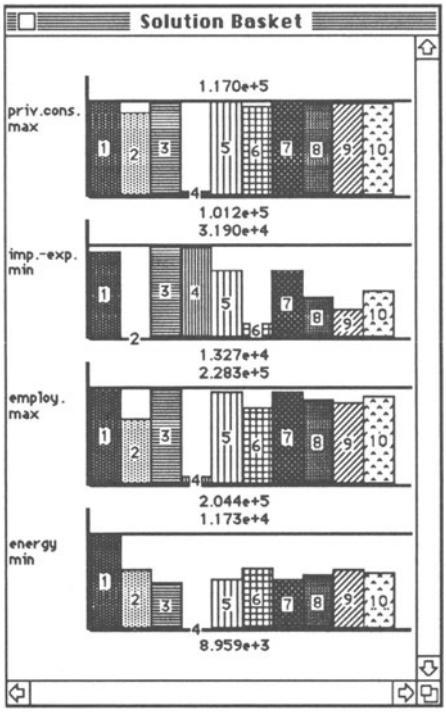

Fig. 5

4. Conclusions

As far as the STEM and Zionts-Wallenius interactive methods are concerned these experiments confirm their main advantages and criticisms. STEM is very simple both from computational and DM's understanding points of view. Some of the rigidity of the original version can be overcome if in each iteration more than one objective function can be relaxed and a given function can be relaxed more than once (enabling to proceed in small steps). In Zionts-Wallenius method, due to the local nature of the information required, the DM may feel uncomfortable

responding to pairwise comparisons or trade-offs, mainly because he/she has no information about what is being discarded.

Note that the main disadvantages generally associated with these methods show up clearly whenever the shape of the nondominated frontier is such that there are regions where the objective function values change very smoothly between adjacent vertices, while between these regions there are faces with sharp variations of those values. In these cases it is essential to use interactive tools such as SOMMIX which could provide a better understanding of the problem by helping to identify the regions where the solutions more interesting to the DM are located, while a more structured (and more informed) local search would be useful in a later phase. These conclusions are clearly illustrated in a experimental comparison of interactive methods to tackle an energy planning problem [5]. In the regional planning problem herein presented these advantages and disadvantages are not so visible, because of the shape of the nondominated region. In fact, the nondominated region does not possess regions with sharp changes of the objective function values, which change regularly and smoothly almost all over it. Under these circumstances, the main disadvantages of methods such as STEM and ZW are less important and they can be used with more confidence.

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A Multiple Criteria Approach to the Estimation of Environmental Quality

Euro Beinat[1] and Piet Rietveld[2]

[1]Institute for Environmental Studies, Free University, Amsterdam, The Netherlands
[2]Faculty of Economics, Free University, Amsterdam, The Netherlands

Abstract. This paper shows the role of multicriteria value functions in the context of environmental decision making. It focuses on the structuring of expert judgement and on the information necessary to overcome the problems related to the customary evaluation based on environmental standards. These standards, although widely used, do not provide sufficient support for the decision process and do not allow for compromises between multiple objectives. The shortcomings of the standards are highlighted, the necessary information to improve decision making is discussed and the use of value function techniques is illustrated.

1. Introduction

Decision making for environmental management is almost always based on the knowledge of environmental indices or environmental standards. One of the most common situations is that of quality indices or standards which are defined by concentrations of environmental pollutants. A key issue in the definition of these indices is the knowledge of the effects of the contaminants on living and non-living organisms. Unfortunately, this knowledge is unavailable or insufficient for many relevant chemicals. In these cases it is necessary to combine the available laboratory and toxicological information with some more informal and implicit information, such as expert judgements. This combination of laboratory data and expert knowledge is the focus of this paper.

Multicriteria value functions are commonly used to specify human preferences and judgements in a mathematical framework. These function can be used in this environmental setting, provided specific assessment techniques are designed to assess expert judgements and to adapt the appraisal procedure to the kind of information experts are able to provide.

This paper is structured as follows. Section 2 describes decision making based on environmental standards and the related problems. Section 3 extends the discussion to multiple criteria decision making and Section 4 introduces the value function technique with the most common assessment strategies. Section 5 describes the requirements for environmental applications and Section 6 shows an assessment technique tailored to these specific needs. Section 7 closes the paper with some conclusions and comments.

2. Environmental quality and environmental standards

The term environmental quality encompasses a wide range of criteria which are used to analyse the compatibility of human activities with the environment. Environmental quality may be expressed in relative terms, indicating that one solution is better than another, or in absolute terms, specifying that a solution is environmentally acceptable or not. The simplest form of absolute environmental quality assessment is based on environmental standards. They separate acceptable from unacceptable effects and provide the most elementary means of evaluating the acceptability of decisions.

Environmental standards are determined on the basis of risk assessment procedures. To determine a standard for a contaminant, it is necessary initially to set a level of acceptable risk and then the corresponding trigger concentration of pollutant which separates the acceptable and the unacceptable levels of risk (Straalen and Denneman, 1989). Dose-effect functions are at the basis of this approach. "A damage, or dose-effect, function is the quantitative expression of a relationship between exposure to specific pollutants and the type and extent of the associated effect on a target population" (Ott, 1978). The estimation of a dose-effect function for a pollutant requires significant modelling and data collection. This is due to multiple exposure routes to the contaminant, the exposure time, the transfer rates between external dose (indirect cause of effects) and internal dose (direct cause of effects), and the complexity of the models required to estimate effects (De Haan et al., 1993).

In practice, analyses of effects are often conducted to determine reference concentrations rather than to complete dose-effect curves. Examples of these reference concentrations are, for instance, the EC_{50} and the NOEC doses. The EC_{50} is the concentration at which 50% of the responding individuals show a specific effect, such as reduced reproduction rates, impaired breath or even death. The NOEC, on the other hand, is the maximum dose for which no effects are detected[1]. These concentrations are obtained under controlled laboratory conditions. A problem which usually appears is the extrapolation of these results to different conditions. For instance, results of tests for short term exposure which need to be extrapolated to estimate effects of long term exposures. Alternatively, reference effect doses for a single species which need to be extrapolated to determine threshold doses for entire ecosystems. Kooijman (1987) has pointed out that this practice often means to extrapolate "experimental findings far beyond the limits of our knowledge".

This lack of detailed and precise effect assessment information is not likely to change in the near future, partly because of economic reasons. It has been estimated that each year about five hundred new substances are introduced on the market (Tietenberg, 1994). Although the largest part of them are or are likely to be harmless, it is however not conceivable to proceed with detailed assessments of each substance due to the costs involved. As shown in Evans et al. (1985), a 2-years bioassay for carcinogenic effects of a single new substance may be as expensive as 1.25M US$. These considerations

[1] The reader is referred to, among others, Moriarty (1988) and Ariëns et al. (1976) for more precise definitions of these reference doses and for their laboratory determination.

explain why "dose-effects relationships are simply unavailable for some 90% of the chemical agents produced on a commercial scale" (Aiking et al., 1989).

From the decision making standpoint, this means that there is a significant gap between the information needed to state environmental effects and the information available. Consequently, almost any decision which concerns the effects of environmental pollution on man and ecosystems has to be based on uncertain, provisional and incomplete knowledge. Under this setting, the clear-cut separation provided by environmental standards is unquestionably unrealistic. Threshold concentrations provide valuable reference points but their step-wise distinction between acceptable and unacceptable pollution levels must always to be interpreted against the knowledge of their effects upon which they are based. As stressed by Kooijman (1981), it is often the order of magnitude of these concentrations that is meaningful rather than their numerical value.

Besides this uncertainty, decision making based on a strict standard compliance may also result in sub-optimal solutions. Almost every real decision process involves multiple conflicting aspects which, together with environmental considerations, determine the suitability of the available solutions. Compliance to environmental standards is only one of the many aspects to be considered. As shown in Figure 1, strict standard compliance would reject alternatives A_3 and A_4 since they do not respect the threshold. This also would make no distinction between A_1 and A_2, which are both acceptable in terms of the threshold fixed.

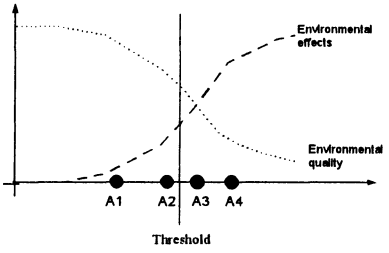

Figure 1. Evaluation through standards. The x-axis shows the contamination level for 4 hypothetical alternatives (A_1, A_2, A_3, A_4). The y-axis shows effects (dashed line) and environmental quality (dotted line).

A more articulate analysis would be most welcome. Relevant issues can then become the acceptability of A_3 if it proves desirable on other evaluation criteria or the distinction between A_1 and A_2 in terms of relative environmental performances. It is generally recognised that real decision making processes involve this kind of flexible evaluations, for which standards provide insufficient support. The most natural solution would be to go one step backwards and refer to the dose-effect curves rather than to the standards. However, since they are unavailable it is common practise to refer to expert judgement to fill the gap between available and necessary information (Aiking, 1989).

The main problem with expert judgement is that it does not always follow a precise and structured path. The outcome is not transparent and there is little

knowledge on the assumptions and information on which expert assessments are made. Experts are believed to provide some holistic estimate of effects which goes beyond the strict information conveyed by the standards. Informal and implicit extrapolation of available information and adaptation to the case at hand take place. Consequently, it is often impossible to reproduce the appraisals and generalise the outcomes, such that each new case has to be evaluated *ex novo*.

Structured expert judgement requires a frame of reference for expert assessments and a precise interviewing protocol. The objective is to provide an estimation of the effect curve around the available standard. The result is likely to be affected by the lack of information, but has the advantage of making expert judgement explicit and transparent. Reproducibility of results and generalisation to similar cases may be significantly improved, provided the assessment process makes clear the information background on which it is based. As shown in Figure 2, this means to estimate the portion of the environmental effect curve in the lightly shaded area. This approach still requires some sort of standards compliance. The dark areas beyond the levels L and H (cf. Figure 2) are not within the domain of expert judgement. Concentrations lower than L, for instance, might be considered as irrelevant since they are all equally acceptable. Concentrations higher than H, on the other hand, can be considered as unacceptable over and above any other consideration. These two separators are somewhat easier to assess, since they represent a limit condition (H) and a target (L).

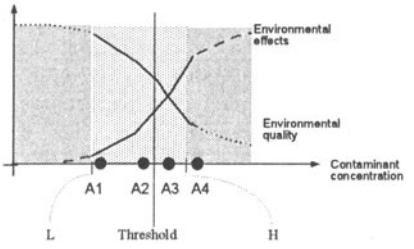

Figure 2. Expert-based estimations of effect curves. The x-axis shows the contamination level for 4 hypothetical alternatives.

Although the level H might be as imprecise as any other standard[2], its role of threshold is somewhat diminished since it only fixes a constraint not to be exceeded. Decision making under this setting has several distinctive features compared to the customary standard compliance:

- It first aims at avoiding pollution levels higher than a specified amount which cannot be extended for any reason.

[2] The definition of the H and L levels depends on the context of the decision. For instance, the level L can be unanbiguously set to zero for most contaminants. However, there are some substances which are naturally present in the environment but are dangerous at high levels. In these cases a reference L level may be computed on the basis of optimal background concentrations (see Edelman and de Bruin, 1986).

- For pollution between H and L there is no sharp separation between acceptable and unacceptable levels. Rather, a continuous transition from a positively unacceptable to a positively acceptable situation is highlighted.
- Decision making aims at selecting the option which is closest to the target level, but accommodates for compromise solutions which consider other relevant decision criteria.

3. Quality standards for multiple attributes

Decision making in many practical situations requires the evaluation of alternative actions which involve more than one contaminant and which show effects in more than one environmental compartment (e.g. air, soil, water) and in several ecosystems. Therefore, besides the above mentioned considerations, the combined effects of multiple compounds in multiple environmental compartments play a key role in determining the global acceptability of one action. Even by assuming that each compound is below its standard, the combined total pollution level might be significant and above acceptable thresholds. In this context, environmental standards for single pollutants fail almost completely and provide little support for the decision making process.

This problem could be solved, at least in principle, by extending dose-effect relationships to contaminant mixtures. The result would be represented as a dose-effect surfaces, rather than curves, and standards would become lines or surfaces. Clearly, each standard would be represented by an (infinite) set of pollutant combinations rather than by a single number (Beinat et al., 1994a). This problem is far more difficult than the single substance dose-effect estimation. At the present it is tractable only under the assumption that composite effects can be estimated from single compound effects, i.e. by combining effects of individual pollutants evaluated separately. However, as Moriarty (1988) pointed out, "our ability to predict the effects of chemicals is rudimentary, so it is inevitable that our ability to predict the effects of mixtures is also very poorly developed". Although several studies are appearing on this subject (see, for instance, Kraak et. al, 1993) a sufficiently precise answer is still lacking and decision making has again to be based both on science and pragmatism.

Along this line, Berg and Roels (1991) provide an evaluation scheme for simultaneous exposure based on expert judgement. This scheme, implemented in the Dutch guidelines for soil sanitation, is based on a linear dose-effect function for all substances and partial additivity of compound effects. Full additivity is assumed for organic compounds while partial additivity, where substances show antagonistic interaction, is adopted for metals and inorganic compounds. This additive scheme is widely accepted mainly for pragmatic reasons. However, scientific evidence is still insufficient to support or negate the validity of this rule.

By extending the above mentioned scheme for a single contaminant, expert judgement could be used to estimate substitutes for unavailable dose-effect curves. Multicriteria value functions (Beinat, 1992; French, 1988; Keeney and Raiffa,

1976) provide the necessary framework in this context. They have been especially developed to formalise in a mathematical model, preferences, priorities and knowledge necessary for the analysis of decision making

4. Value functions and assessment techniques

The simple additive formulation will be used throughout the remainder of this test. Let us denote with X_i the i-th criterion (substance), i=1,...n, and R_i the set of all possible scores (concentrations) of interest such that $R_i=[L_i, H_i]$. In this setting, a pollutant mixture is a score profile $x=(x_1,...,x_n)$, where $x_i \in R_i$. A uni-dimensional value function $v_i(.)$ translates the score x_i into the value $v_i(x_i)$ represented in an arbitrary scale, between 0 and 1 or 0 and 100. The limit points, L and H, are used as scaling anchors and assigned the values $v_i(L_i)=100$ and $v_i(H_i)=0$. The multidimensional value model is specified as:

$$v(x) = \sum_{i=1}^{n} w_i \cdot v_i(x_i) \qquad (1)$$

where the w-factors are the weights of each criterion (substance). This additive model maps score profiles (pollutant combinations) into [0,1] or [0,100] scales, where the limit scores are assigned to the limit H and L profiles.

The definition of value may differ for different problems. However, it is always linked to the expected effects of the pollutants; a description of the kind of effects being considered and the conditions assumed have to be made explicit before the assessment.

Several assessment techniques have been developed for the specification of additive value functions and weights. The reader is referred, among others, to Beinat and Janssen (1994), Winterfeldt and Edwards (1986), and Fishburn (1966). In general terms, however, the following components can be highlighted:

- Assessment strategy. Two main strategies are recognised: decomposed (direct) or holistic (indirect) scaling. Decomposed scaling requires the assessment of uni-dimensional value functions and the estimation of criteria weights. Their additive composition (Equation 1) is then used to evaluate multidimensional profiles. Holistic scaling, on the contrary is based on holistic judgement over sets of multidimensional profiles. Value functions and weights are then estimated indirectly, usually via regression or optimisation techniques.
- Information available. Human judgements are intrinsically difficult to represent with numbers. Preference and knowledge statements are imprecise in their very nature. However, there are several ways to tackle the problem and to represent uncertainty. Two main strategies can be implemented. The first assumes some precise preference representation and then checks the effects of uncertainty through sensitivity analysis techniques. This simply aims at testing the robustness of outcomes and uncertainty is neither reduced nor addressed

explicitly. Alternatively, one may start by representing uncertainty in preferences explicitly and thus tackle the problem directly with appropriate computational techniques.

- Interaction with the expert. In contextual modelling-evaluation procedures, the experts estimate the value model on the basis of the actual alternatives to be evaluated. Outcomes, therefore, are likely to be significantly dependent on the assessment context. On the other hand, by separating assessment and evaluation, the outcomes have a more general applicability, although they might not be optimally fitted for the problem at hand.
- Objective of the value model. The most common objective is the complete cardinal ranking of the alternatives under evaluation. Less demanding results are the ranking of clusters of alternatives or even the selection of one alternative out of the available set. This choice is often linked to the information available. By addressing information uncertainty explicitly it is often necessary to accept only incomplete results, such as partial rankings or only indications of the best alternative (Beinat and Janssen, 1994).

5. Environmental applications

Environmental management problems have some features which dictate the assessment requirements. First, for several reasons it is necessary to separate assessment and modelling. Experts availability is always a concern in this setting and good experts are also expensive. Additionally, a single value model has to be assessed by several experts to have the certainty that it conveys the available experience. The possibility of using these outcomes more than once thus becomes a relevant concern. Adding to this point is also the number of potential applications of the same information. As an example (Beinat et al., 1994a), the estimated number of polluted sites which need urgent clean up in the Netherlands is estimated to be around 100,000. Of this sheer number, many cases are extremely similar and show the same contaminants in similar combinations. Given this background, the same expert judgements can be used in several cases.

Regarding the assessment strategies, both direct and indirect assessment have to be used. Direct judgement aims at constructing unidimensional value functions and at estimating the weights. This information is obviously linked to factual data, although it may be insufficient or unavailable. The consequence is that experts tend to act as a plain source of information and are reluctant to provide personal judgements. It can be said that, to some extent, information from the direct assessment comes straight from eco-toxicology dossiers, papers and books and that experts simply rearrange it to the current information structure. In this process they are afraid of providing data which may be contradicted by further evidence of by future studies. This behaviour can be graphically synthesised as in Figure 3. The x-axis shows the steps for constructing the value function. The initial step is the specification of the curve shape type (e.g., convex, concave, sigmoidal), then the assessment of value ranges, the elicitation of the value points and finally the specification of the complete curve. The y-axis shows the

confidence of the experts in the assessment of their curves. As it can be seen, the confidence of the experts decreases as the information content of their assessments increases. Experts tend to stick to low levels of information and consider detailed assessments as highly unreliable.

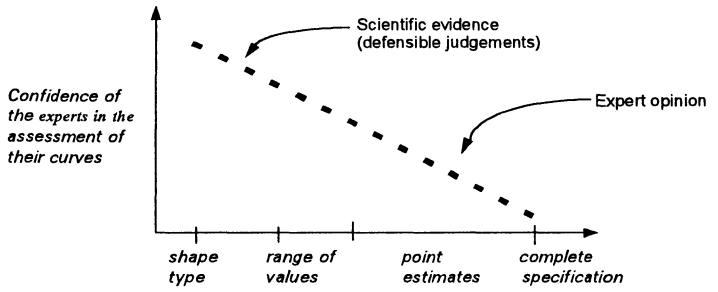

Figure 3. Available information provided by the experts and confidence of the experts in their assessments.

Indirect judgement, on the other hand, supplies information on the same value functions and weights, but from a totally different perspective. Since structured and detailed effect data on combinations of contaminants is missing, these judgements rely heavily on the experience of the assessors and on the informal knowledge and perception they have about these cases. It has to be expected, however, that the experts can reasonably evaluate simple cases, such as combinations of two pollutants.

6. The assessment technique

Following these remarks, the main requirements for an effective assessment technique can be summarised as follows:

- The procedure should allow qualitative and tentative responses for all parameters to be estimated. To attach a value to a concentration, the expert must synthesise a complex of effects and conditions. The experts often cannot comply with the requirements for precise estimations.
- Given the qualitative and imprecise nature of expert responses, it is necessary to evaluate their consistency and filter out the underlying knowledge.
- Direct and indirect assessments have to be used simultaneously in order to gather both factual data and experience. By doing so, the complete span of knowledge is grasped.
- In order to keep the procedure manageable, the evaluation scheme should be additive. Non-additivity requires long and complex sessions with experts, which are often infeasible. Even if expert responses seem to indicate non-

additive behaviour of pollutant mixtures, the additive representation should be exploited, provided the assessment includes mechanisms to guarantee that the additive model is the closest approximation to the "real" one.

- The assessment procedure should be interactive and flexible, should allow experts to decide the course of action, to modify judgements and to provide guidance on the most convenient assessment procedure.

These requirements have been fulfilled in a new assessment procedure especially tailored for these applications (Beinat, 1995; Beinat et al. 1994a, 1994b; Beinat, 1992). It includes methods for integrating direct and indirect assessment and allows for interactive data collection and for imprecise estimations. As concerns the value functions, for instance, after fixing the reference range of concentrations for each substance, the experts indicate a so-called value region, which is a range of possible values likely to include the real value function. In a similar fashion, experts estimate a qualitative ranking of importance for the substances, which determines a qualitative set of weights. Value functions and weights estimated in this way constitute the direct assessment. Figure 4 shows an example of a value region for cadmium in soil pollution. The range of concentrations selected here goes from the target level of no pollution (0 mg per kg of soil weight) up to a maximum of 40 mg per kg of soil-weight, which represents an unacceptable pollution level. Similar estimates are gathered for all other substances (criteria) involved in the process.

To check the consistency of these inputs and to refine them, the experts are asked to evaluate the same problem from a different angle, through an indirect assessment. The experts are asked to assess simple combinations of contaminant concentrations and evaluate them in a qualitative scale. Figure 5 shows an example case where nine combinations of cadmium and zinc are evaluated; the numbers shown represent priority, number 1 being the highest. The ranking in Figure 5 is the result of an implicitly used combination of weights and value functions.

Since direct and indirect assessment essentially provide information on the same types of variables, the combination of them results in a surplus of information which can be used to highlight the underlying most consistent set of value functions and weights. This is obtained by using an optimisation technique, such as linear programming modelling. The value functions and weights are selected which: i) obey the value regions; ii) obey the weight order; iii) evaluate combinations of contaminants in agreement with expert judgement. It is not always possible to achieve this result since assessment inconsistencies are likely to occur. In such a case, the objective of the optimisation procedure is to estimate the functions and weights which maximise consistency with all the expert inputs.

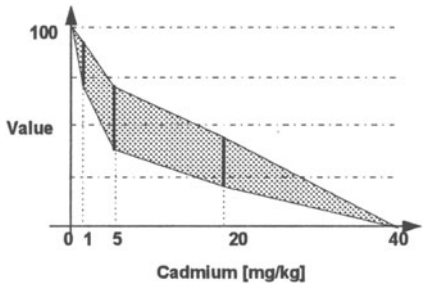

Figure 4. Example of a value region for cadmium.

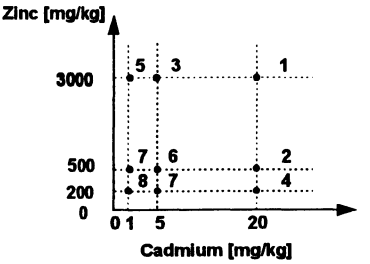

Figure 5. Cleaning-up priorities for nine different combinations of pollutants.

The result of the optimisation model is a set of reference value functions and weights, a set of uncertainty levels for value function and weights and a set of consistency errors. If the results show high consistency errors, the process can be iterated by refining or changing initial elicitation and starting a new optimisation round. This sequence of refinements of inputs, optimisation and evaluation of outputs can be repeated several times before reaching satisfactory results. Typical results of an assessment session are shown in Figure 6, where the same value function for cadmium has been estimated for five different experts.

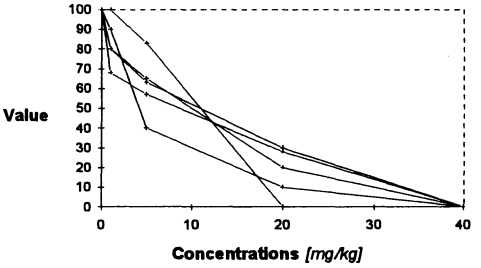

Figure 6. Cadmium value functions for five different experts.

Figure 7 shows the average of the curves in Figure 6 compared against a dose-effect function for cadmium as found in Straalen and Denneman (1989). This second curve is a proper dose-effect function based on statistical elaboration of laboratory evidence and was computed under the same conditions of the expert assessment. In terms of relative comparisons of pollution levels, the dose-effect curve does not differ substantially from the value function. Although this is no proof that expert judgement is a proper source for information, it does suggest that decision making based on any of the two curves produces basically the same outcomes.

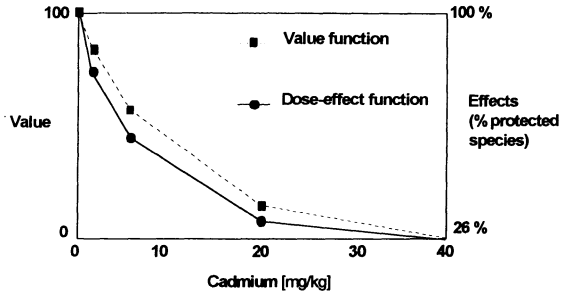

Figure 7. Comparison of a dose-effect curve with a value function. The value function translates doses into values, which indicate the environmental quality on a relative scale. The dose effect curve translates doses into the corresponding percentage of non-endangered species in the target environment.

7. Comments and conclusions

Some aspects of the expert-analyst interaction in this specific field of application are worth stressing. First, communication with experts improves during the assessment process. This is probably related to the fact that they become familiar with the specific way of reasoning required for the model and have a direct feeling about the assessment procedure. In particular, experts are usually rather sceptical at the very beginning of the process but almost always end up appreciating the technique. In all the assessment sessions we have run, experts found results an acceptable representation of their knowledge and acknowledged the fact that this approach improves the consistency of decision making and makes use of available information in an attractive way.

However, several open issues remain. One of the most relevant relates to the aggregation of different expert responses. Trial tests have been run with a series of aggregation procedure (Beinat, 1995), but a formal procedure with general applicability has still to be found. Another difficult issue relates to the problem of adapting value functions assessed under specific condition to other situations. For instance, the results in Figure 6 were obtained on the basis of a precise soil composition (the so-called standard soil). Extension to other soil compositions has still to be tested.

In this respect, it has to be made clear that results obtained in one assessment session can be generalised to other situations only provided the same assessment conditions hold. Value functions are not a panacea for substituting dose-effect functions, but rather tools for providing decision makers with more structured and effective information. The interpretation of value functions must always be careful and, in no way, should they replace the need for accurate (eco)toxicological test.

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Application of MCDA to Corporate Loan Requests Evaluation

Michel Bergeron[1], Jean-Marc Martel[2] and Pater Twarabimenye [1]
[1] École des Sciences de la gestion, Université du Québec à Montréal
[2] Faculté des Sciences de l'administration, Université Laval

Abstract: The corporate loan decision making process is a central concern in financial literature and amongst practitioners. The process, though entails many difficulties. There is, for example, no consensus in the literature on the choice or on the relative weight of the potentially relevant aspects (the macro-economic, financial, and management assessment aspects, and the aspect of the value of loan collateral) that one should take into account in a decision on a loan. The literature is also in disagreement as to how these aspects can be combined (aggregation) in order to generate a global judgement. Moreover, the information necessary in seizing these aspects most often comes in different scale units and results from evaluations which are subjective or more or less exact. Most models that are developed for loan decision, limit themselves to quantitative financial variables; they also tend to evacuate or ignore the role of the loan officer in the process, and this even if the decision maker's judgement and appreciation of qualitative aspects is recognized as a key factor in the process. This paper shows how a multicriterion methodology based on outranking relationships can assist the loan officer in his or her attempt to surmount the difficulties involved in loan request evaluation. The method proposed is tested over a sample that contains about three hundreds of loan requests addressed to a large Canadian financial institution

1. INTRODUCTION

Financial institutions understandably devote a good deal of attention to corporate lending decisions; corporate loans represent a primary source of both income and risk (Wyman, 1991). On one hand, the financial viability of these institutions is largely dependent on their loans portfolios; on the other, businesses that borrow from banks are generally more vulnerable than those that raise money directly on capital markets. Consequently, lending decisions are among the most significant of those made by commercial banks (Gentry and Shaw, 1989). Moreover, deregulation of financial institutions during the late 1980's has led to greater competition in the industry, thereby increasing the importance of effective corporate lending processes. It is

therefore not surprising that corporate loan screening remains a major concern in the banking literature, both academic and professional (Rodgers and Johnson, 1988; Ensinger, 1991; Wyman, 1991;...). However, the decision making processes that are being proposed in the financial literature are, in many respects, unsatisfactory. For example, they provide a very limited role for loans officers, yet loans officers' judgements and evaluations are recognized as key factors in the decision making process (Braden-Harder, Bhaskar, and Murthy, 1989).

Evaluation of lending risk is based upon many factors, notablly assessment of a firm's financial, managerial, and macroeconomic condition, and each of these assessments rests upon a host of qualitative and quantitative data which are often disparate and conflicting. Loans officers must isolate the information necessary to appraise the various aspects of risk.

Beyond the difficulties related the information selection and integration in formulating a final decision, loans officers are faced with problems of information quality. Not only must they determine which information is relevant and reconcile disparate and conflicting data (Brown and Duren, 1986), they must also assess the reliability of the data. As Zimmer (1981) points out, loans officers tend to weigh lightly or even discount data which they consider insufficiently reliable.

This paper aims to propose a comprehensive approach that will assist loans officers in arriving at their final decisions; this requires an appropriate description of the corporate loan evaluation process.

2. ANALYSIS OF THE DECISION MAKING PROCESS IN CORPORATE LENDING

A number of authors have attempted to describe the decision making process in corporate lending. The main factors identified by Zimmer (1981) are availability of funds, institutional policy, government policy, evaluation of risk and repayment capability, and evaluation of the impact of a loan on the existing loans portfolio. Rodgers and Housel (1987) and Rodgers and Johnson (1988) divide the decision making process into three main steps. Firstly, a loans officer expresses his view, based on a preliminary ratios analysis, of the financial health of the applicant firm and the possible impact of the loan in question on the institution's loans portfolio. The loans officer then proceeds to a deeper analysis of the financial data, and basing his judgement on the results obtained and his personal experience, renders a decision relative to each of the diverse evaluation criteria affecting the loan application.

Finally, the loans officer combines these partial judgements or evaluations to arrive at a final decision.

Unlike other descriptions, such as Zimmer's, this description has the virtue of admitting the influence of loans officers' cognitive processes; nevertheless, it remain incomplete. On on hand, how the loan officer is incorporated into the process is not made explicit, and on the other, it formalizes only one stage of the process, the evaluation of risk and repayment capability, an evaluation that is limited to quantitative financial data.

A complete description must stress the importance of taking loans officers' judgements into account throughout the process : in information selection and preliminary analysis, in evaluating the reliability of data, in evaluating risk, and in making recommendations and arriving at a final decision. Furthermore, this model acknowledge that evaluating risk and repayment capability is a multidimensional process : this evaluation should rest not on financial considerations alone, but also on consideration of macroeconomic factors, strength of management, and value of collateral. This description has been validated in connection with a large Canadian financial institution (hereafter called the Bank), which offers various financial services, principally loans and venture capital.

The main difficulties confronting loans officers are: 1) information selection. The academic literature proposes analysis of a long list of quantitative date, usually consisting of ratios calculated from financial statements, but ratios relevant to evaluating the financial health of an applicant vary from case to case. Assessing the quality of a firm's management, usually ignored in financial distress models, is undeniably important in evaluating a loan application, but this cannot be done using strictly quantitative measurements. Macroeconomic and industry related factors also have a significant influence on a firm's performance, but analysis of banking practices reveals that these factors are inadequately accounted for in assessing risk. Once again, loand officers need help in formulating opinions on the impact of these variables on the overall stability on an applicant firm. The value of the collateral offered for a loan and or managers's level of confidence in a firm's prospects are also important indicators in evaluating risk; 2) an adequate treatment of all this information. There is little to indicate what relative importance is to be accorded to the different variables that could be used to delimit each of these dimensions of a firm. Neither is it clear how evaluations relative to each of these variables can then be combined to produce an overall evaluation of a particular dimension (first level of aggregation) or how these

evaluations can in turn be combined (second level of aggregation) to arrive at a final decision on the desirability of approving a loan

This shows that beyond the essential exercise of identifying the relevant criteria for evaluating corporate loan applications, and assigning each its relative weight, an effective decision making methodology must permit an adequate and integrated teatment of qualitative and quantitative data, which are more or less inaccurate, disparate, and conflicting (Brown and Duran, 1986).

3. PROPOSED PROCESS

To resolve the problems identified above, a multicriteria evaluation process that is similar to that used by the Bank is proposed. In the loans officer procedure an analysis grid containing a large number of variables grouped under eight main headings is used : 1.) review of the applicant firm, 2.) review of management, 3.) market development, 4.) business plan, 5.) forecasts, 6.) working capital, 7.) balance sheet, and 8.) collateral. It should be noted that these headings serve only as a basis for grouping data, not as a basis for evaluation. An implicit aggregation of the variables is then performed to arrive at a rating under each of the following headings : management, profitability, and collateral. These three ratings are then used in combination, to rank a loan application into one of three predetermined categories, low, average and high risk. Interest rates and other lending conditions of a loan are established in conformity with the assigned rank. Despite its relative consistency, the Bank's evaluation procedure is open to question at several levels.

As in the Bank's existing procedure, the proposed evaluation process seeks to base loans officers' recommendations on all relevant data However, to be truly useful to loans officers, the data must be grouped according to the criteria by which officers will make their decisions. Establishing these criteria, a bit like the eight categories in the Bank's evaluation procedure, is crucial to the proposed process. The established set of criteria must satisfy a certain number of requirements and possess two fundamental qualitied : legibility (it must contain a sufficiently low number of criteria) and workability (it must be acceptable as a working instrument). It is of primary importance that loans officers have a clear understanding of each criterion.

Once they have evaluated a loan application under each criterion, loans officers must combine their evaluations to assess the level of risk associated with it; that is to say, they must classify it into one of several predetermined categories (sorting problematic as in Roy, 1985). This step is difficult without appropriate support, the

more so because the criteria are of unequal importance, are often conflicting, and use different scales of measurement. Given the particular characteristics of loan applications, ELECTRE-TRI seems appropriate. Using this method, the criteria are not totally compensatory and have neither absolute discriminating power nor the same relative importance. This method also allows for the explicit integration of loans officers' judgements.

In this study, we use, unlike the Bank which uses only three, five predetermined risk categories (almost nil, low, average, high, and very high); every loan request must be classified into one of these categories. Each category is defined by an upper and lower boundary. It is hypothesized that each boundary be representative of a single bench mark loan application (a fictional one), and that the lower boundary for a given category represent the upper boundary for the one immediately below it. These boundaries are presented as profiles (vectors) consisting of one evaluation of each of the g_j, ,j = 1,2, ..., n criteria.

Each loan application is classified into one of the predetermined categories of risk by systematically comparing its evaluations vector to those that characterize the boundaries between categories. These comparisons are performed using the ELECTRE-III algorithm.

4. ESTABLISHING EVALUATION CRITERIA TO DETERMINE LENDING RISK

As mentioned above, lending risk rests upon many factors : macroeconomic, industrial, financial, managerial, and collateral. Many variables relative to sizing each of these factors have been identified in the literature. The number of variables being so high, it is necessary to group them in terms of a smaller number of criteria. In fact, if it is done correctly, criteria grouping eases loans officers' mastery of the aggregation procedure, without loss of information.

Variables related to the macroeconomic and industry dimension have been regrouped in four categories. Each category constitutes a significant axis to which is associated a criteria for lending risk evaluation: trends in the Statistics Canada Composite Index and of the provincial GIP, industry market risk (beta) and industry expected return. The first two criteria are calculated as the mean variation in % of the corresponding composite index. Market Risk (Beta) is the covariance between the monthly returns of industry and those of the market portfolio. Expected Return on

Investment is obtained from Estep's T model (1987), which relies exclusively on accounting data to calculate an industry's expected return.

A number of variables are used in evaluating a firm's financial dimension. They have been pooled into four categories : profitability, indebtedness, productivity of assets, and working capital. The evaluation of a firm's performance based on financial ratios is accomplished by comparing the ratios for the firms to those for the industry as a whole and the evolution of these ratios over time. Each criterion related to financial considerations thus integrates these two aspects.

Many scenarios or situations can arise relative to the evolution of a given ratio. These scenarios were submitted to an expert committee made up of senior loans officers, which ranked each scenario (from the worst to the best scenarios) on a scale of 0 to 20. The higher the value assigned to a scenario, the stronger the firm. Some scenarios were judged to be equivalent by the panel and so were assigned the same value. The values assigned to each of the variables (ratio) are totalled to obtain an overall value for the corresponding criterion. The higher the resulting values, the more positive the assessment.

This approach to establishing criteria is interesting because it integrates the loans officers' expertise in a manner similar to that of an expert system.

The profitability criterion incorporates the gross margin ratio and the operating costs to sales ratio. Three ratios are used in calculating debt levels : interest coverage ratio, fixed financial payment coverage ratio, and debt ratios. In order to arrive at a value which increases with the assessment, the capitalization ratio is here used in place of the debt ratio. The assets productivity level reveals how efficiently a firm is managing its assets. Based on the literature, the ratios most relevant to an evaluation here are the assets turnover ratio, the stock turnover ratio, and the net profit to total assets ratio. The level of a firm's working capital reveals its ability to cope with short term financial obligations. Two ratios are used under this criterion to evaluate the applicant firm : the working capital ratio and the liquid assets to total assets ratio.

Managers's qualifications is built from nanagers' academic qualifications and experience. Combinations of the two variables were presented to the panel of experts who assigned a value between 0 and 20. The same rating procedure that was used for the financial criteria was applied when officer's evaluations differed. A qualitative scale is used to evaluate Human Resources Planning criterion. Four planning levels, depending on the degree of preparedness of a firm's managers, are considered : unprepared (0) to fully operational (3).

Managers' Level of Commitment derives from the amount invested in the firm by managers relatively to their total wealth. Value of Collateral is simply the loan collateral divided by the total amount of a loan and is thus stated in percentage terms.

The preceeding set of twelve criteria (see Table 1) meet, in our opinion, the basic qualities required from such a set.

5. SORTING PROCEDURE PARAMETERS DETERMINATION

In order, to use the proposed sorting procedure (ELECTRE-TRI) to classify loan requests in their respective risk category, one needs to determine many parameter values : criteria relative importance coefficients, discrimination thresholds and boundary values that define each ordinal risk category.

Criteria relative importance coefficients have been determined using an A H P procedure. Members of the experts committee were asked to perform paired comparisons between criteria related to the same dimension in order to show the importance of one element in relation to another.

Experts were first engaged in paired comparisons between the five dimensions involved in the loan requests risk evaluation and then, in comparisons between criteria related to a same dimension. A criterion relative importance is equal to the relative importance of the dimension to which it belongs, multiplied by its relative importance within this dimension.

The relative importance coefficients, both at the dimension and criteria levels, were determined by running a logarithmic least square regression over the comparisons matrix obtained in the preceding step.

At the dimensions level, one can observe (Table 1) that the financial, managerial and collateral dimensions, are respectively the most important, this relative importance order being the same for all three experts. It is also in line with the order that is implicit in the circular defining the Bank's loan risk classification instructions. But, contrary to our results, the "collateral" dimension is seen as more important than the "managerial" dimension.

The relative weight granted to the "macroeconomic and industry" dimension is relatively weak; one can ask if this result might not come from the fact that the Bank's risk classification procedure does not explicitly consider this dimension. At the criteria level, profitability, managers' qualifications, working capital and value of collateral are, in order, the most important criteria.

Table 1 : Criteria set

DIMENSIONS	CRITERIA	RELATIVE IMPORTANCE COEFFICIENTS
Macroeconomic	Trend in composite Index (g1)	0.02
and	Trend in Provincial GIP (g2)	0.02
industry	Market Risk Beta (g3)	0.04
(0.11)	Expected Return on Investment (g4)	0.02
	Profitability (g5)	0.22
Financial	Debt Level (g6)	0.04
(0.46)	Assets Productivity (g7)	0.08
	Working Capital (g8)	0.13
	Managers' Qualification (g9)	0.21
Managerial	Human Resources Planning (g10)	0.04
(0.32)	Managers' Level of Commitment (g11)	0.07
Collateral	Value of Collateral	0.11
(0.11)		

Table 2 : Discrimination threshold values

Cri. Thres.	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}
q	.03	1.5	.05	.01	1.5	2.3	3.1	1.8	1	.05	.05	0
p	.10	2	.10	.02	2.3	3.7	5.2	3.1	2	.1	.12	1
v	2	10	1	.13	10	20	30	15	10	1	.5	1

Table 3 : Boundary values

Crit. Boun.	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}
b_1	-.24	.5	1.22	-.04	14	17	18	12	6	.05	.65	0
b_2	-.03	3.7	1.02	.01	21	26	26	17	9	.12	.94	0
b_3	.25	5.5	.82	.55	27	37	33	24	12	.27	1.22	1
b_4	.36	7.3	.65	.12	32	43	41	31	16	.40	1.5	1

The ELECTRE-TRI multicriteria segmentation method is based on pseudo-criteria concept. Utilization of pseudo-criteria is justified because : a loan request evaluation over a given criterion often involves a certain indefiniteness (difficulty to perform the evaluation), a certain indeterminateness (criteria definition often involve a certain conceptual fuzziness, and/or, an information loss) and a certain variability (value being subject to variation through time). Consequently, for the loan officer,

existence of a positive difference between evaluations, over a given criterion, for two requests, does not imply that one request is strictly better than the other.

Indifference thresholds result from the loan officer's subjective evaluation. Roy's et al. (1986) suggested procedure to determine these thresholds consists of starting with a very small d_i ($d_i = g_i(x) - g_i(y)$) value and to gradually increase it to the point where the decision maker considers the upper bound of d_i compatible with an indifference situation between x and y requests.

In this application, q_i values have been obtained from a questionnaire that was submitted to the experts committee. This questionnaire applies Roy's et al. (1986) procedure. It brings the expert to find the maximum difference in evaluation, for each criterion, that results in classifying two requests in the same risk category, given the fact that they get equivalent evaluations over all other criteria. In order to facilitate their judgements, the meaning and role of these thresholds were presented in connection with information that summarizes the evaluations obtained over each criterion by each sub-sample requests. This information consists of minimum and maximum evaluations over each criterion.

A procedure similar to the one used for indifference thresholds has been followed to determine the strict preference thresholds (p_i). However, in this case one tries to determine the evaluations minimum difference over a criterion that result in a different sorting of the two loan requests x and y, this even if their evaluations over all other criteria are equivalent. It consists of determining the difference in evaluation over a criterion that allows, without hesitation, to affirm that one loan request is strictly more risky than another. A high value of d_i is presented so request y appears desirable; then, this d_i value is gradually decreased up to the point where the expert (the loan officer) begins to hesitate to declare that one request is strictly better than the other.

The Bank's risk classification system does not allow any inference about the possibility that certain criteria present a veto threshold when sorting loan requests to preestablished risk categories. This does not mean that such thresholds do not exist for certain criteria. In order to determine these thresholds (v_i), start-up value are proposed to the experts; these start-up values are equal to the scale range. They are then gradually reduced up to a point where the criterion does not possess any veto. In this procedure, the expert must keep in mind that a veto threshold cannot be lower than the preceding strict preference threshold; he must also be aware that such theresholds must be used with precaution because they deny, based on only one criterion, the outranking of one action by another.

It must be necessary to pointed out that, even if our procedure gives exact values for the various theresholds (see Table 2), the determination of these values rests on each loan officer in his normal activities : the experts only suggest bench mark values for these thresholds. Then, the officer is free to interpret the reliability level of information and choose the thresholds in accordance.

Five risk categories are used in this study : almost nil, low risk, average, high and very high risk. Each category is defined by an upper and lower boundary (b_1, b_2, b_3, b_4). It is hypothesized that each boundary be representative of a single bench mark loan request (a fictional one), and that the lower boundary for a given category represents the upper boundary for the one immediately below it. These boundaries are presented as profiles (vectors) consisting of one evaluation for each criterion : $b_k = (g_{ik}, g_{2k}, \ldots\ldots\ldots g_{12k})$.

In order to give meaning to the dominance structure between categories, the upper boundary of each risk category must be significantly better than its lower boundary profile. This dominance condition is necessary in order to make sure that a request is not indifferent to more than one profile or fictional request.

Boundary profiles have been implicitly inferred from a sub-sample of 150 requests which have already been categorized by the Bank's loan officer. All loan requests in the sub-sample were classified in the five risk categories. Then, an evaluation of each request in the same risk category was performed, in relation to the twelve criteria used in the study, in order to gain insight into the boundary profiles. The evaluations' lower in higher value, over each criteria, were determined for each risk category; these two values have served to define each category. Sometimes, the maximum value thus obtained for one category, differs from the minimum value for the higher risk category; in such a case, the average, or in between value, was proposed to the expert committee, as joint boundary value.

The risk category k upper boundary profile suggested to the expert committee has been : $b^*_k = (b_{1k}, b_{2k}, \ldots\ldots b_{nk}) = b_k + 1$ = lower boundary for category k + 1. Effectively, profiles that characterized each category where determined by an expert committee; this has been done by means of a questionnaire specifically designed for this task. The purpose of this procedure is to improve uniformity in the request evaluation process.

6. RESULTS OF THE APPLICATION

This section presents the numeric results yielded by the application of the proposed process to hundreds of loan applications from the Bank's files. However, it is important to provide some information about the sample that was used in this study.

The start-up sample consisted of all loan requests received by the Bank, from 1988 through 1992; these applications came from branch offices spread all over Canada. This sample was then limited to loans of 500,000 $ or more, not only to reduce the number of files but also because that category has the biggest impact on risk and profitability of the loan portfolio.

The loan request files contain all necessary information to evaluate the criteria used in this study : 3 years of financial statements from each firm and information reports prepared by the bank about management quality, value of collateral, main activities, risk assessments established by loan officers...Amongst a total number of five hundred sixty three (563) requests for the period in the 500,000 $ or more category, a sample of two hundred fifty (250) requests containing all necessary information was selected. In this sample, 10% of the requests are about loans that were not granted, a percentage in line with the real percentage of loan requests that are rejected by the main office.

The sample was divided, on a random basis, into two sub-samples of 150 and 100 requests respectively. The first sub-sample was used to determine the sorting parameters, such as the discrimination thresholds and the boundary values between risk categories. The loan requests sorting procedure was then tested over the second sub-sample. Table 4 present the preliminary results obtained by applying the ELECTRE-TRI procedure to a sub-sample of 30 requests with the values specified in tables 1, 2 and 3 for the various parameters needed in this procedure.

The risk category to which each of the 30 requests is sorted, with a cutting edge, $\tau = 0{,}67$ the direction of the outranking relationship and each profile that defines the risk categories. The letters I and R respectively mean indifference and "incomparability". Category 1 specifies a high risk and category 5 an almost nil risk. As we can see, depending if we adopt a pessimistic attitude or an optimistic attitude, a given request can be sorted into a more or less risky category. It is the case, for example, for requests D1, D2, D3, which are sorted in categories 4, 3, 3 respectively with a pessimistic attitude and in categories 5, 4, 4 (less risky categories) with an optimistic attitude. One can observe that a difference in sorting happens each time

that there is "incomparability" (R) between the requests involved and one of the profiles acting as boundaries.

Results obtained with the sorting procedure (t = 0.67) are compared to the risk category given by loan officers, this for each of the 30 requests.

On the whole, the proposed sorting procedure gives results that are similar to those observec amongst loan officers. When differences appeart one cannot say taht loan officers tend to adopt a more pessimistic or optimistic sorting.

7. CONCLUSION

The Bank's loans officers have been directly involved in the formulation of the proposed process and attest to its legitimacy. However, it must be pinpointed that the determination of parameters needed in the multicriteria aggregation procedure and the determination of profiles necessary to determine the risk categories have not been an easy task for the experts committee. This difficulty comes from the fact that they do not think of the problem in these terms. The role and exact meaning of each threshold is by no way obvious to someone who is not familar with this sorting procedure. At this stage, it appears that this process will be adopted by the Bank and implemented at its various branch offices, notably for loan officer training.

Table 4 : Sorting

PESSIMISTIC SORTING (τ = 0.67) **OPTIMISTIC SORTING** (τ= 0.67)

Files	Pessimistic					Optimistic					Off.
	b_1	b_2	b_3	b_4	Cat.	b_1	b_2	b_3	b_4	Cat.	Cat.
D1	>	>	>	R	4	>	>	>	R	5	5
D2	>	>	<	<	3	>	>	R	<	4	3
D3	>	>	R	<	3	>	>	R	<	4	3
D4	>	>	<	<	3	>	>	<	<	3	3
D5	>	>	>	>	4	>	>	>	<	4	2
D6	>	>	<	<	3	>	>	<	<	3	4
D7	>	>	>	I	5	>	>	>	I	5	3
D8	>	>	<	<	3	>	>	<	<	3	5
D9	>	>	<	<	3	>	>	<	<	3	4
D10	>	>	<	<	3	>	>	<	<	3	4
D11	>	>	<	<	3	>	>	<	<	3	3
D12	>	>	R	R	3	>	>	R	R	5	3
D13	>	>	>	<	4	>	>	>	<	4	3
D14	>	>	R	<	3	>	>	R	<	4	3
D15	>	>	>	R	4	>	>	>	R	5	4
D16	>	>	>	I	5	>	>	>	I	5	4
D17	>	>	R	<	2	>	R	R	<	4	4
D18	>	>	R	<	3	>	>	R	<	4	5

D19	>	>	R	R	3	>	>	R	R	5	5
D20	>	>	R	<	3	>	>	R	<	4	3
D21	>	>	R	<	3	>	>	R	<	4	3
D22	>	>	R	<	3	>	>	R	<	4	5
D23	>	>	R	<	3	>	>	R	<	4	4
D24	>	>	R	<	3	>	>	R	<	4	5
D25	>	>	<	<	3	>	>	<	<	3	5
D26	>	>	>	R	4	>	>	>	R	5	2
D27	>	>	>	<	4	>	>	>	<	4	3
D28	>	>	>	<	4	>	>	>	<	4	3
D29	>	>	R	<	3	>	>	R	<	4	1
D30	R	R	R	R	1	>	R	R	R	5	5

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DSS: THEY *DO* WORK!

Malin Brännback

Institute for Advanced Management Systems Research

Åbo Akademi University, DataCity A, SF-20520 Åbo, Finland

Email: mbrannba@abo.fi, Fax: +358-21-2654809

Abstract: We have heard that DSS are of little use in supporting decision-making in strategic management so many times that many of us believe it is true. We have a different story.

We have studied the strategic management process of a Finnish drug company, here called C-3, and found that in order for this company to create a sustainable competitive advantage (SCA) it will need superior market position, superior competitive position and superior profitability than its competitors. The competitive position is determined by how well C-3 manages its critical success factors (CSFs). Any event taking place on the drug market that will affect C-3 will affect their competitors and that some action taken by a competitor will affect our SCA - any of the three, or all, elements just mentioned - of C-3. This can be modelled in a DSS. At first we need a ranking of the CSFs and the competitors

In this paper we will describe how we ranked the CSFs for C-3 and then C-3 and their competitors with respect to these CSFs. In both cases we used Expert Choice, a DSS based on the Analytic Hierarchy Process. In using Expert Choice we will acquire a (i) systematic ranking of competitors and (ii) a better ranking of competitors as a result of (i).

1 Introduction

Strategic management is a field under constant controversy [10] where the degree of formalism in the process seemes to be one of the key issues [19, 5] and the information and knowledge being used by decision makers (senior managers) being the other important issue [8, 10].

The use of normative analysis to predict and explain behaviour is often motivated by two assumptions: (i) managers (sic!) are thought to be effective in pursuing their goals, (ii) competition favors rational individuals (managers are considered to be that), thus optimal decisions increase the chances of survival in a competitive environment [23].

Decison support systems (DSSs) have by many authors been critisised for having little or no significant impact on decision-making [2, 3, 4, 6, 9, 18]. DSSs in the context of managerial decision-making (strategic management) is a rarety [9, 22, 17], in fact many people believe that DSSs cannot provide support in strategic management.

With the developments in the field of information systems technology we are beginning to see some changes, i.e. we have the tools that enable us to develop flexible and effective management decision support. In this paper we will describe how we have used a DSS in the knowledge acquisition process in developing a context specific DSS for strategic management described in a previous paper [7].

Decision-making in strategic management is indeed multi criteria decision-making. Nevertheless, managers rarely seem to reason in terms of MCDM, i.e. '..considering m dimensions...', '..with n potential alternatives..', and '..k evaluation criteria..'. Managers tend to think and work for most of their lives with *language* and *ideas* not with *numbers* and *mathematical* symbols [15]. But most of their decision problems *are* as we know, solved in a multicriteria environment. Most managers are confronted with conflicting objectives that they try to satisfy simultaneously. They have to make trade-offs and produce a best compromise solution. This is normally done intuitively supported by their own experience and some data. In a business environment of increasing competition strategic decisions have to be based on something more than that. They need computer-based support that makes sense to them.

Here we will describe how we used Expert Choice, a DSS based on the Analytical Hierarchy Process, in order to find a ranking for the critical success factors (CSF), which is necessary for creating a SCA.

Our context is a Finnish drug company, here called C-3, where we have studied the strategic management process [7]. We have defined strategic management as the process of *creating a sustainable competitive advanatage* (SCA). In order for *this* company to create an SCA it will need some distinctive assets and skills, *critical success factors* (CSFs) that it has to manage better than its competitors. These CSFs will give the means with which C-3 can build a superior competitive position, which in turn will ensure a superior market position, and superior profitability.

In the next section we will give a short description of our case company. We will then describe how we used Expert Choice and acquired a ranking of the CSFs and the competitors on the market.

2 The Context: Antidepressants and Case Company C-3

The psychotropic drug market in Finland is going through rapid changes as the market is growing and competition is becoming stronger. *Psychotropic drugs* (PD) can be divided into several subcategories, we have in this study focused on antidepressants. There are various categories of antidepressants (AD) on the Finnish drug market, the most commonly used are called tricyclic ADs. One new category was released in 1989; the *SSRI* (selective serotonin reuptake inhibitors). The AD market has grown as a result of the SSRIs entering the market. The reasons for a rapid breakthrough of the SSRIs come from the fact that they have real advantages over the old tricyclics in the treatment of depressions. They are effective, safer, and better tolerated. When total treatment costs are taken into account the newer ADs are also more cost effective [20]. The SSRIs have all

gained substantial market shares in very short time after their release (there are five competing SSRIs). The older ADs have all subsequently lost market share.

Our case company, C-3, is a small drug company marketing some twenty different drugs in Finland. They have specialised in psychotropic drugs; antidepressants and neuroleptics, and they have two or three drugs that are not psychotropics. C-3 is a subsidiary with headquarters abroad. This means that C-3 in Finland has no responsibility for production or basic R&D (some post-release, phase IV, R&D is carried out in Finland). The Finnish subsidiary is thus only responsible for marketing and sales. The entire personnel counts for ten people.

C-3 released an SSRI in 1990. The release was the result of a successful R&D process that had taken many years. When the release took place C-3 had approximately 3% of the AD market share value. In 1990 there where 5 major competitors, today three new competitors have entered. Within three years C-3 has acquired almost 50% of the market, and is now *the drug of choice among* physicians.

C-3 has managed to grow despite new competitors entering the market and an attempt to start price competition - a very rare event in pharmaceuticals - the former market leader (prior to C-3's release) released a "me-too" SSRI and priced it 20% below C-3's and other competitors' drugs.

Why would the managing director of C-3 need to "systemise his thoughts" or "augment his memory", i.e. use any kind of support system for strategic decision-making? With a market share of 50% the business appears to run splendidly and the managing director seems to make the right decisions.

The managing director does not believe the success will last forever. What if the R&D at headquarters cannot discover and develop new drugs at a *steady* rate? What is a *steady* rate? What are competitors doing in their laboratories? And even if he has some knowledge of competitors, will a future release be a success? Will it be a big success? How big is big and will big be enough? We know that Glaxo's success-product released in the 1980's, the anti-ulcer drug Zantac, overwhelmed even its most optimistic proponents. According to Corstjens [11] there are many examples of how drug companies have underestimated market potentials.

Hence, the rapid market growth and the growing competition have made it clear for the managing director that he needs to base his strategic decisions on something more than some data, experience and gut feelings. In other words the managing director is looking for a tool that will organise complexity, support systematic analysis and include subjective considerations. Such a tool would be a decision support system.

A company is able to create a sustainable competitive advantage by possessing superiour assets and skills, which they have to manage better than their key competitors. These assets and skills we know as *critical success factors* (CSFs). The critical success factors will give the company the means with which they compete and build their competitive position, which in turn will influence the company's market position, and also the profitability of the company. These CSFs will be different in different industries, and they will certainly vary among competitors even in the same industry and companies operating in the same

market segment. Hence, the outline of an SCA that we present here has to be seen as *company specific.*

The market position element will be expressed in terms of the company's market share value. In the profitability element, we have the financial result of C-3, both actual and estimated. These elements do not present a problem since they can be regarded as modules holding market facts. However, these two elements are dependent on the company's competitive position [7], which is determined by the CSFs.

We managed to determine the CSFs quite easily by interviewing the managing director. The CSFs were *new products, marketing knowledge, product quality, R&D, marketing resources, good reputation,* and *wide product range.* Our problems occured when we wanted to find out which of the CSF was the most important and how the competitors were performing with reference to these CSFs. Why was this important?

3 Decision Support With Expert Choice

Because of the author's background in the pharmaceutical field the knowledge about the field was plentiful. However, we were trying to *understand* how the key elements were connected to each other and thus affecting C-3's performance. This was essential for developing a DSS for strategic management of C-3.

Initially we ranked the CSFs and the competitors using a competitor strength grid (CSG) [1]. This was done manually and the grid has no numerical scale, only verbal, from weak to excellent [7]. The respondent lists the CSFs whereafter each competitor is plotted in the grid based on whether the CSF is considered a strength or a weakness for the particular company. We would like to call this an *intuitive ranking.* The CSG is comparable with repertory grids for problem construction, described by Eden and Jones [14]. Repertory grids are simply a matrix that relates a repertoire of elements to a set of constructs.

The research project started in 1990 and the first CSG was done. However the market was extremely dynamic and the situation kept changing, with the competitive scope changing. A second CSG was done at the end of 1992 and a third in the beginning of 1993. They were all different. Partly the difference can be explained by the dynamic market, but the CSG-method does obviously not force the user to consider each element carefully enough. We realised that we needed another tool to rank the CSFs and the competitors.

We decided to use Expert Choice (EC) [13], a computer software based on the Analytic Hierarchy Process (AHP) [21]. The decision was rather an ad hoc one, based on the fact that the manager could be regarded as a computer non-user and we needed a tool that was easy to use and to understand. We knew EC had these characteristics from having used EC in teaching.

Using an AHP involves the user making pairwise comparative judgements inputting their ratio judgemental scores to EC to derive global priorities, which measure the relative importance of each element in terms of achieving the top hierarchical goal. AHP allows the user to combine both qualitative and

quantitative criteria into a single judgement by using a ratio scale of relative importance [16, 12]. This is very useful as it is easy to see the difficulty in attempting to quantify some of the CSFs. AHP is particularly useful in treating the problem hierarchically as a MCDM model by linking the importance of CSFs (objectives) to the performance of competitors. In recognising the interdependence of elements between hierarchical levels reduces the likelihood of making too optimistic or holisitc decisions. Defining the problem hierarchically is likely to lead to more effective decisions [12] (see Fig.1).

In order to be able to compare the intuitive ranking of the different competitors and the ranking using EC we transformed the intuitive ranking with EC. The CSFs were only ranked with EC because initially the CSFs were thought to be equally important.

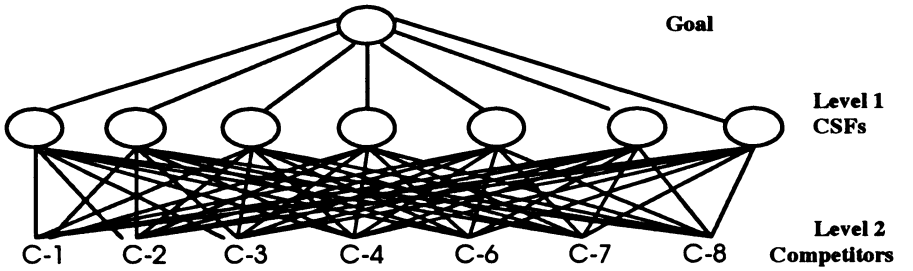

Figure1: The problem as a hierarchy of problem elements.

3.1 The result of the ranking

The CSFs were ranked as follows: new products (.292), marketing knowledge (.209), product quality (.196), R&D (.108), marketing resources (.076), good reputation (.074), and wide product range (.044). The inconsistency ratio was 13% which was satisfying.

When looking at the ranking one might wonder whether the rather low weights of 'marketing resources', 'good reputation', and 'wide product range' indicate a possible overlap. Could they be "included" in the other CSFs? We discussed this with the managing director and our comments were recognised as valid. However, the managing director wanted to consider them separately.

Having ranked the CSFs and having checked that the ranking was consistent with the managing director's intuitive perceptions of C-3's reality, we then continued to rank C-3 and each competitor with respect to each CSF.

As we can see the values obtained through ranking competitors using EC seem to be varying to a greater extent than those values obtained through intuitive ranking. This is better shown in Figs. 2, 3, 4, 5, 6, 7, and 8, where the EC ranking and the intuitive ranking is shown as profiles. Nevertheless, most of the profiles show that there is at least a definite but small degree of relationship between the intuitive and EC ranking procedure. Only the rankings of *high product quality*

(prqual), Fig. 2, and *marketing resources* (mkres), Fig 7, show a negligible relationship (the verbal description of the magnitude of the correlation coefficients is adapted from Williams, [24, p. 132]). Using Pearsons product-moment correlation we find that *r* for prqual and mkres are .05 and (-).1 respectively.

The profiles for *marketing knowledge* (mkknow), Fig. 8, and *wide product range* (wiprora), Fig. 5, show a very dependable relationship with *r* values of .95 and .96 respectively ($p < .001$). The profiles for *R&D*, Fig. 4, show a marked relationship with *r* equalling .78 ($p < .025$) and *good reputation* (goodrep), Fig. 3, has an *r* value .54, which is a substantial relationship ($p < .25$). In the profiles of *new products*, Fig. 6, we see a definite but small relationship - $r = (-).37$ ($p < .25$).

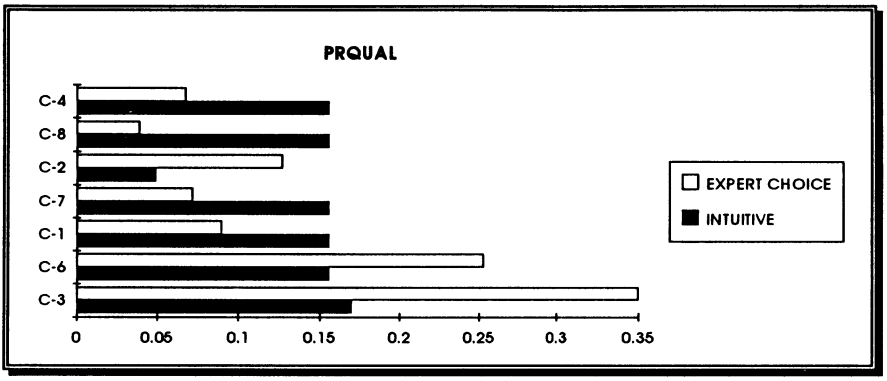

Figure 2: Product quality (prqual)

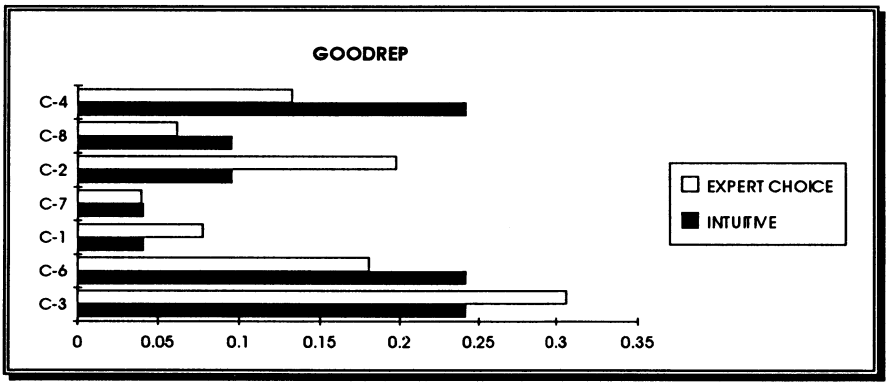

Figure 3: Good reputation (Goodrep)

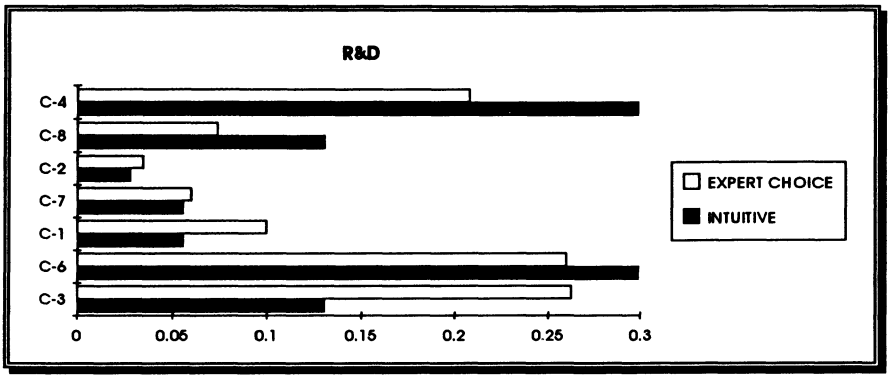

Figure 4: Research and Development (R&D)

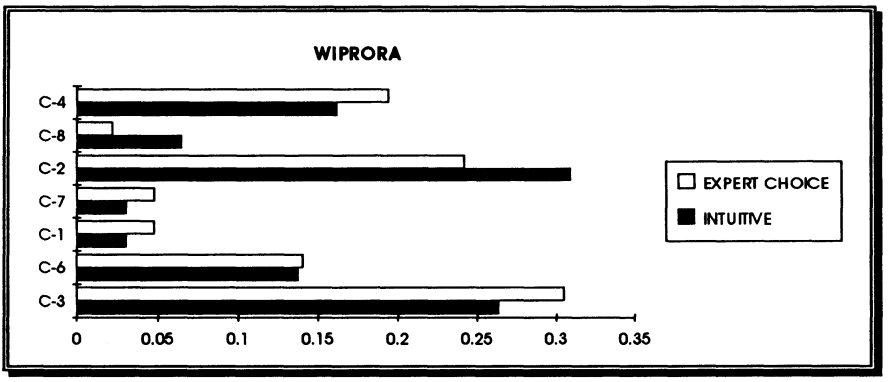

Figure 5: Wide product range (Wiprora)

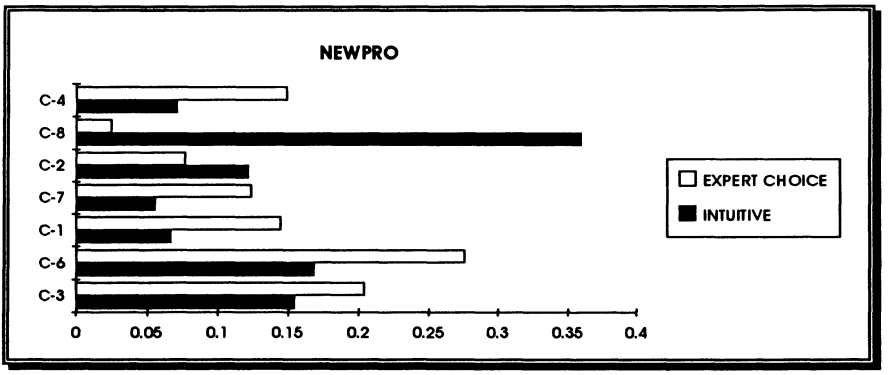

Figure 6: New products (newpro)

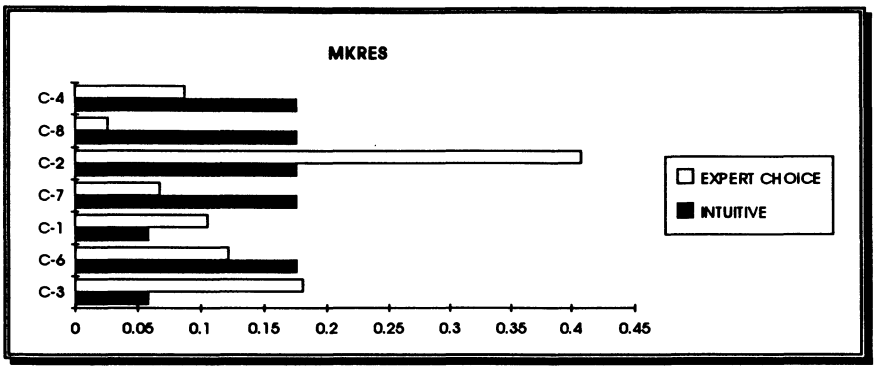

Figure 7: Marketing resources (Mkres)

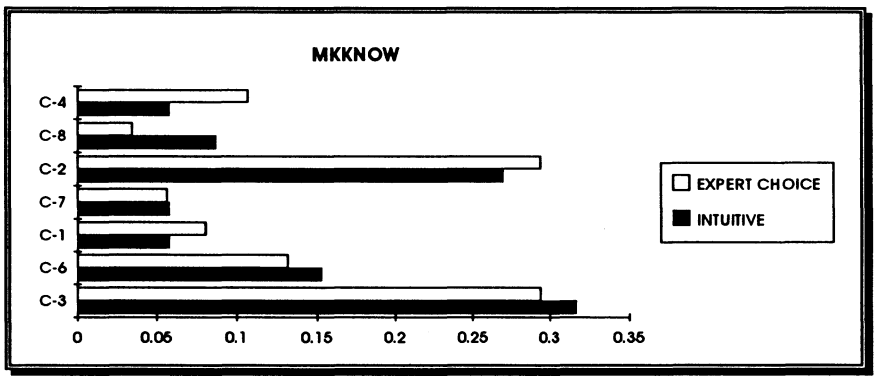

Figure 8: Marketing knowledge (Mkknow)

4 Discussion

What have we accomplished with this comparison? First, we have been able find a more visible difference in performance among the competitors. Take a look at, for instance, *product quality*, in Fig. 3 or *marketing resources* in Fig. 7, we can see that the intuitive ranking of competitors argued that most competitors were equally competitive, however, the ranking done with EC tells us a different story. The managing director has also verified that the EC-ranking corresponds to his true perception of the competitive situation.

Secondly, we believe that we have been able to capture a more precise picture of the competitors' assets and skills this way. We have also acquired a better understanding of the strengths and the weaknesses of C-3. Knowing a company's assets and skills is not enough to understand a company's competitive strengths. It is only after you compare the company's performance with the competitors that the knowledge becomes meaningful.

Finally, we have found that when confronted with a tool like Expert Choice, the managing director has been pressed to consider the important issues and their relationships in a systematic fashion, quite unlike what he has been used to. We can also see that the conditions of invariance are violated [23]. As we know, the principle of invariance, states that different representations of the same choice problem should yield the same preference. In other words we should have had the same ranking. From the profiles of all CSFs shown in Figs. 2-8 we find considerable variation, although they may indicate a similar ranking. However, we also find completely diverting ranking results (e.g. Fig. 2; prqual for competitors C-2, C-7, and C-1; Fig. 6; newpro for competitor C-8; and Fig. 7; mkres for competitors C-2 and C-3).

5 Conclusions

We have allowed the manager to use both his own judgement and intuitive feel in ranking his company relative to his competitors. We have then allowed the managing director to redo the ranking in a more organised framework using EC. Expert Choice has put the managing director on the spot and forced him to consider each element and its interdependencies. Thus we have accomplished an improvement in two ways: (i) we have acquired a *systematic* ranking of competitors and (ii) a *better* ranking of competitors as a result of (i). This process has certainly updated and strengthened our understanding of the drug company and its market. We have used the results from this study and built a knowledge-based support system for strategic market management of C-3 [7].

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OpTiX-II: A Software Environment for MCDM Based on Distributed and Parallel Computing

Manfred GRAUER and Harald BODEN

Information and Decision Systems Institute, Faculty of Economics, University of Siegen, Hölderlinstr. 3, D-57068 Siegen, Germany; e-mail: grauer@fb5.uni-siegen.de

1. Introduction

The aim of this contribution is to provide an introduction to the OpTiX-II Software Environment. This software system supports nonsequential solution approaches for so-called multidisciplinary decision problems based on nonlinear optimization tasks (e.g. [1]). The underlying mathematical programming problem can be formulated as follows: find an n-dimensional vector $x = (x_1, ..., x_n)^T \in R^n$ that minimizes $f(x):R^n \rightarrow R$ subject to the constraints $g_i(x) = 0, g_j(x) \leq 0$ and the bounds $x_l \leq x \leq x_u$, where $g_k(x):R^n \rightarrow R$ (i = 1, ..., m_e; j = m_e+1 , ..., m; k = 1, ..., m).

The solution of decision problems based on nonlinear optimization arising from this multidisciplinary field are usually characterized by the fact, that either the objective function or the constraints are set up with simulation software systems, based on finite element analysis. The repeated calculation of the simulation model leads to very time consuming solution procedures. To overcome this problem, the use of multiprocessor architecture offers a possibility to reduce the solution time. But then a software environment for formulating, starting and controlling the concurrent tasks of a decision problem and for managing the communication effort during the solution process is required. In order to accord with these necessities, the OpTiX-II Software Environment has been designed to support the formulation of these decision problems and their solution on computer networks by applying coarse-grained parallel and distributed approaches. This allows the use of local area networks as cost efficient alternatives to supercomputers in this problem area.

The following section will give a brief overview on the nonsequential solution approaches in nonlinear optimization. They are, at least to some extent, supported by OpTiX-II. In section 3 the conceptual design targets of the software environment for distributed and parallel solution are presented. Section 4 gives a review on the implementation and use of OpTiX-II. Results obtained by solving test problems are given in section 5.

2. Nonsequential solution approaches in nonlinear optimization

The nonsequential solution approaches can be divided into two classes. The first class makes use of parallelism that is inherent to a given algorithm. It can be represented by classical optimization algorithms using data parallelism, parallel extensions of hybrid optimization methods, and heuristic solution approaches applied in parallel. The second class utilizes parallelism inherent in the structure of the optimization problems. It can be is represented by the use of parallelism inherent to decompositon methods.

2.1 Parallelism inherent to the algorithm

Parallelism inherent to a given algorithm or a combination of algorithms is procured mainly to speed-up the solution process and, especially in the last few years, to extend the class of solvable problems with respect to the number of available processors in a computing system

(scale-up).

The first group of algorithms includes classical nonlinear mathematical programming methods such as Quasi-Newton-Methods. The design concept of the algorithms as a whole is serial with, for example, direction search followed by line search. Thereby, the speed-up of the internal operations is based on the use of data parallelism (e.g. for linear algebra computations). Such algorithms are mostly modified for the use on supercomputer systems with massive parallelism or vector processing units. The algorithms can be included in the solution process by the OpTiX-II Software Environment if they are accessible on dedicated supercomputers within a local area network. A review on the research in this field of parallel nonlinear optimization algorithms can be found in [2] and [3].

A different approach in designing a parallel algorithm has been introduced in [4], where a scalable multidirectional search method is suggested. In this case, "scalable" means that a parallel algorithm adapts itself to the dimension of the optimization problem and to the number of processors available. Thus, the two parameters determine different parallel algorithms with different characteristics. This concept is demonstrated in [5] for the class of evolutionary algorithms.

Hybrid optimization methods as described in [6] represent a further approach which can be extended to parallel computations. They consist of a combination of a globally convergent algorithm with a locally superlinearly convergent algorithm and a test for switching between both methods. For convex optimization problems, the resulting method is globally and superlinearly convergent. This leads in itself to a type of coarse-grained or control parallelism, if both methods are applied in parallel. The necessary exchange of information after a certain number of iterations is asynchronous. A similar heuristic method, which makes use of the same type of parallelism, is to concurrently start a random search method, a method for nondifferential problems, and a local convergent method using derivative information. This helps to locate the global solution and to reduce computing time for the multidisciplinary optimization problems. The controlled exchange of information between these algorithms, operating in parallel, is thereby the basis for a more robust and, in some cases, even faster solution of the optimization problem (see [7]).

2.2 Parallelism inherent to the problem structure

The application of decomposition methods for the solution of large scale decision problems leads to the most basic realization of coarse-grained parallelism by implementing the two iterative tasks: multiple subproblem solutions (concurrent and asynchronous) and coordination of subproblems´ results. From this viewpoint, however, there is a broad range of alternatives in terms of the complexities referring to subproblems and coordination procedures. Even within this divide-and-conquer parallelism for the global solution strategy, it is possible to apply several forms of parallelism inherent to the solution methods, such as described in the previous section.

The OpTiX-II Software Environment is designed to support a distributed solution strategy by allocating dedicated algorithms and processors to subproblems. To our knowledge, such a software environment including algorithms for parallel and/or distributed solution of decision problems has not yet been presented.

3. Conceptual design of OpTiX-II

In this section we introduce the objectives for the conceptual design of the OpTiX-II Software Environment for the parallel and/or distributed solution of decision problems based on multidisciplinary optimization. The design objectives can be summarized as follows:

(I) The software environment should be an integrated system supporting the formulation of the decision problem as an optimization task, the design of a parallel/distributed solution strategy and the control of the solution process. It should support several hardware architectures being available for the solution. Thereby the efficiency of alternative nonsequential solution strategies for different problem classes in dependency of several types of multiprocessor architectures (see Fig. 1, Workstation cluster, Transputer system, Multiprocessor system) can be studied.

(II) The problem formulation language should support the analytical description of a decision problem in form of an optimization task as well as the inclusion of external software written in the programming languages C or Fortran. This requirement evolves from the necessity to include existing algorithms or commercial software as part of a problem description. In the case of a problem formulation being given in analytical form, the software environment should provide first and second order symbolic derivatives.

(III) The problem formulation should be independent of the hardware that is subsequently being used for the problem solution. The transformation of the problem formulation and the inclusion of external routines written in C and Fortran should be provided automatically for all supported hardware architectures.

(IV) The user must be able to formulate an overall (parallel) optimization strategy, utilizing the nonsequential solution approaches described in section 2. Therefore it must be made possible to allocate the resulting individual optimization problems to computers within the available testbed and to choose algorithms with default parameters for the solution of the individual optimization problems from a proposed list of methods. This list should be extendable for further solution methods. The parameters of each algorithm must be adjustable to the problem under research.

(V) The user should not be confronted with synchronisation and communication issues that arise from the use of heterogeneous computer networks. These problems should be solved on the basis of standards for Workstations and Transputers and should be hidden from the user.

(VI) A graphical user interface which is based on standards set in the workstation

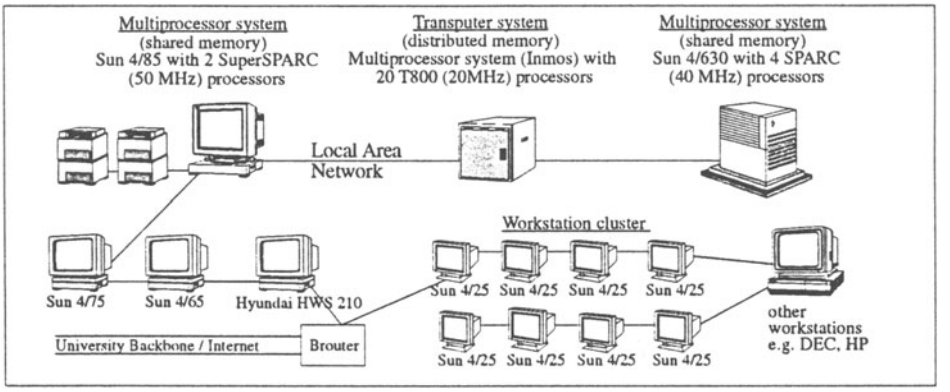

Figure 1: Multiprocessor testbed for studying different nonsequential solution approaches for decision processes based on mathematical optimization.

market should be provided. Thus the user will be confronted only with a minimum of new syntax and semantics for the solution of the optimization problems.

In the following we will describe the logical design of the OpTiX-II Software Environment from a user's view and its implementation by demonstrating the solution of a decision problem from the field of mechanical engineering.

4. Logical design and implementation of OpTiX-II

The objectives (I) to (IV) for the conceptual design are realized in OpTiX-II on the basis of dividing the entire solution process into three phases (see Fig. 2). During the problem formulation phase, the user defines the analytical formulation (including the use of external codes) of the current decision problem (see section 4.1). In the second phase this formulation is then translated into a machine code representation, which is suitable for parallel processing in heterogeneous networks (see section 4.2). The third phase (see section 4.3) is designated for the control of the optimization problem solution. Within latter phase the user has to define an optimization strategy by choosing the optimization algorithms and by allocating computing resources from the network to the subsystems (individual optimization tasks).

Fig. 2 shows the flow of data and control within the OpTiX-II Software Environment. The realization of the design objective (V) and implementation issues are discussed in section 4.4. In the following the features and the application of OpTiX-II will be demonstrated in designing a gear reducer (see Appendix A). This nonlinear optimization problem consists in minimizing a weight function under the given constraints arising from structural mechanics. It is a very common test example in mechanical engineering and may be solved by the application of decomposition methods.

4.1 Problem formulation phase

In this phase the optimization tasks of the decision problems are formulated, the generation of optimization servers for different platforms are controlled, and the execution environment is started. The problem description is entered into a graphically controlled problem editor using the OpTiX-II problem description language (see Fig. 3). The problem description consists of a declaration section, in which variables, constants and starting points are declared, and a problem definition section, in which the user defines one or more subsystems, each defining an individual optimization problem or a computational task. These subsystem tasks will be distributed to the resources of the computer infrastructure during the problem solution phase.

The statements made within the problem definition section correspond with the mathematical notation for nonlinear optimization tasks. This is demonstrated by formulation of the gear reducer design problem in the text-editor window in Figure 3. For more complex decision problems, which cannot be described by simple algebraic expressions, OpTiX-II allows the inclusion of external functions written in the C or FORTRAN programming languages. Calls to these functions may become a subexpression of the objective function or a constraint. Thereby it is also possible to define subsystem tasks that include commercially available simulation packages for the solution of complex mathematical models.

4.2 Problem translation phase

In the translation phase the problem description is transformed into a machine code suitable for the execution in a heterogeneous network of (parallel) computers. The transformation can be addressed by the "Compile" and "Build executables" menus in the Edit and Compile Environ-

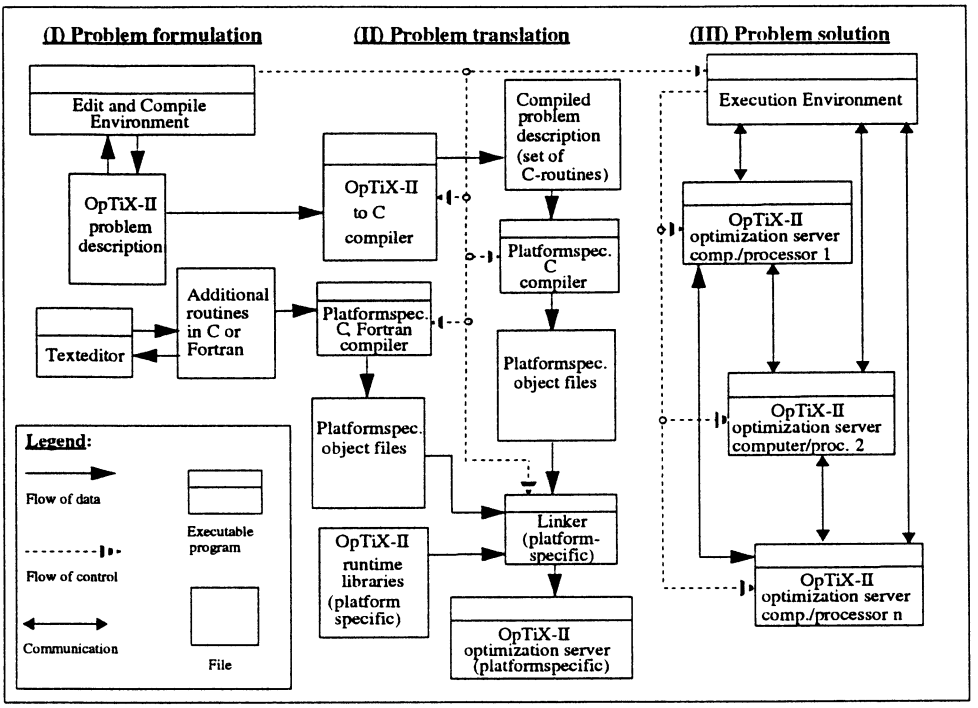

Fig. 2: Flow of data and control in the OpTiX-II software environment (logical design)

ment (see Fig. 3). A compiler translates the problem description into a set of statements in the programming language C. Thereby, the compiler produces symbolic expressions for first and second order derivatives of the objective function and the constraints. In the case of external function calls, numeric gradient approximations are inserted into the generated code. The set of C functions is then compiled and linked into optimization servers (see Fig. 2) for different computing systems (e.g. Transputer systems under the Helios Operating System, Unix-based workstations with SPARC or MIPS processors). The OpTiX-II Software Environment, in its current stage of development, supports the following hardware/software architectures: (I) Unix workstations from various manufacturers, (II) Unix-based Multiple Instruction stream Multiple Data stream (MIMD) type computers with shared memory (e.g. Sun SPARCstation 10, SPARCserver 600 and SPARCcenter 1000/2000 series) and (III) Helios based Transputer systems (MIMD-type computer with distributed memory).

4.3 Problem solution phase

The OpTiX-II Execution Environment allows the formulation of parallel and/or distributed solution strategies as well as the recording of the problem solution processes and the display of the results. The user interacts with the control module of the Execution Environment (see Fig. 4). The nonsequential solution strategy is formulated by writing a strategy script, which consists of a sequence of solution steps. In each step the user may solve several individual optimization subproblems in parallel. Thus, the user must choose algorithms for the solution of the individual optimizations (subsystems) and allocate computing resources within the

network to the individual optimization tasks (subsystems in Fig. 4).

The nonsequential solution approaches discussed in section 2 can be realized in this manner. The gear reducer can, for example, be decomposed into two subproblems (shaft_and_bearing_1, shaft_and_bearing_2). So three individual optimization tasks have to be processed iteratively: the two subproblems in parallel and subsequently the coordination problem. For each individual optimization task, a host (computing facility) and an algorithm must be chosen. The subwindow on the bottom left (Fig. 4) gives the history of the solution and the subwindow on the bottom right displays the current best solution.

4.4 Implementation issues

The main parts of the OpTiX-II Software Environment have been developed using the programming language C. The graphical user interface has been designed with the Sun Microsystems GUIDE program. This tool was selected, because it supports different user interfaces (e.g. OPENLOOK and MOTIF). Software developed for Unix-based computers con-

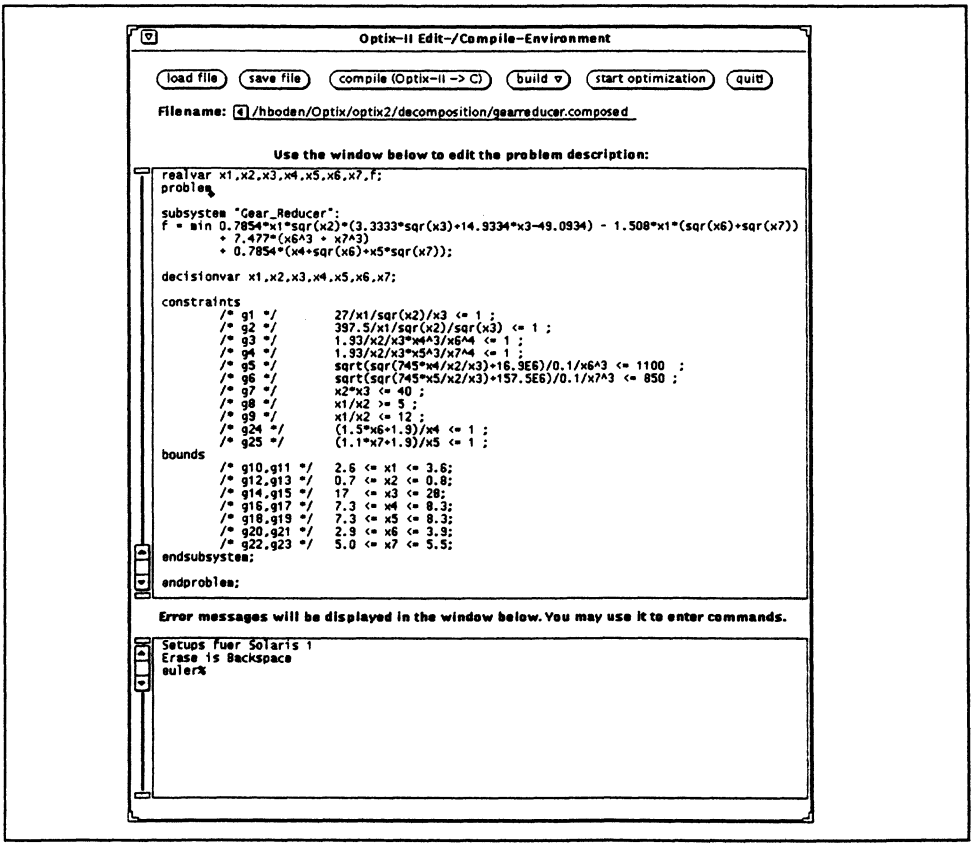
Optix-II Edit-/Compile-Environment
load file
save file
compile (Optix-II -> C)
build
start optimization
quit
Filename: /hboden/Optix/optix2/decomposition/gearreducer.composed
Use the window below to edit the problem description:
realvar x1,x2,x3,x4,x5,x6,x7,f;
problem
subsystem "Gear_Reducer":
f = min 0.7854*x1*sqr(x2)*(3.3333*sqr(x3)+14.9334*x3-49.0934) - 1.508*x1*(sqr(x6)+sqr(x7))
+ 7.477*(x6^3 + x7^3)
+ 0.7854*(x4+sqr(x6)+x5*sqr(x7));
decisionvar x1,x2,x3,x4,x5,x6,x7;
constraints
/* g1 */ 27/x1/sqr(x2)/x3 <= 1 ;
/* g2 */ 397.5/x1/sqr(x2)/sqr(x3) <= 1 ;
/* g3 */ 1.93/x2/x3^4^3/x6^4 <= 1 ;
/* g4 */ 1.93/x2/x3^x5^3/x7^4 <= 1 ;
/* g5 */ sqrt(sqr(745*x4/x2/x3)+16.9E6)/0.1/x6^3 <= 1100 ;
/* g6 */ sqrt(sqr(745*x5/x2/x3)+157.5E6)/0.1/x7^3 <= 850 ;
/* g7 */ x2*x3 <= 40 ;
/* g8 */ x1/x2 >= 5 ;
/* g9 */ x1/x2 <= 12 ;
/* g24 */ (1.5*x6+1.9)/x4 <= 1 ;
/* g25 */ (1.1*x7+1.9)/x5 <= 1 ;
bounds
/* g10,g11 */ 2.6 <= x1 <= 3.6;
/* g12,g13 */ 0.7 <= x2 <= 0.8;
/* g14,g15 */ 17 <= x3 <= 28;
/* g16,g17 */ 7.3 <= x4 <= 8.3;
/* g18,g19 */ 7.3 <= x5 <= 8.3;
/* g20,g21 */ 2.9 <= x6 <= 3.9;
/* g22,g23 */ 5.0 <= x7 <= 5.5;
endsubsystem;
endproblem;
Error messages will be displayed in the window below. You may use it to enter commands.
Setups fuer Solaris 1
Erase is Backspace
euler%

Fig. 3: The OpTiX-II Edit and Compile Environment (screen hardcopy) with the problem formulation of the gear reducer design problem (see Appendix A).

forms to the X/Open portability guide and the POSIX standards. The transputer-cluster operates under the Helios operating system. It offers POSIX compatible libraries and supports the Network File System (NFS). Communication within the workstation network is based on Sun Microsystems's Remote Procedure Call concept and makes use of TCP/IP based services (e.g. remote copy, remote login, NFS). The decision for the tools mentioned above was made at the beginning of the project in 1990. Meanwhile the Remote Procedure Call (RPC) concept has become an industry-standard in workstation computing.

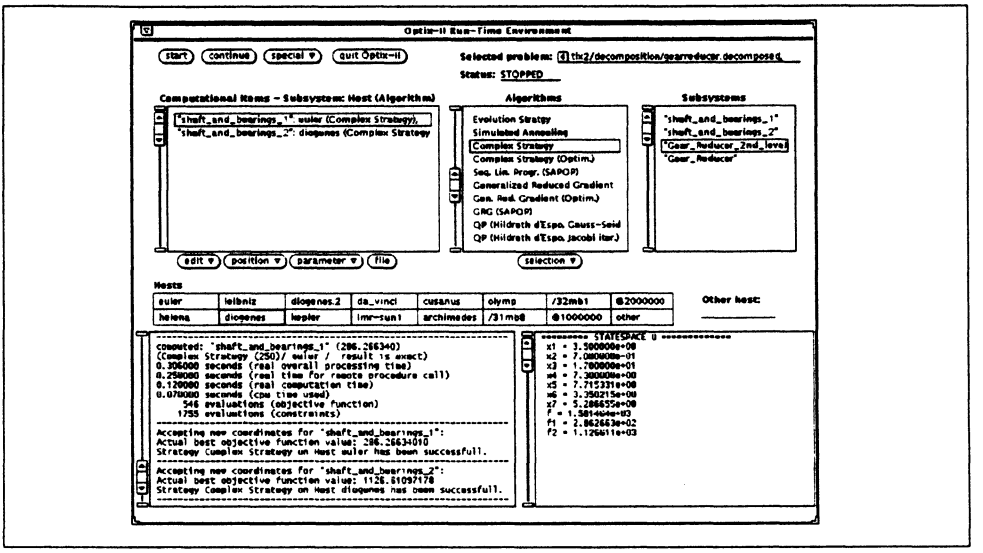

Figure 4: The OpTiX-II Execution Environment (screen hardcopy), with a partial view on a strategy script for the solution of the decomposed gear reducer design problem.

5. Numerical test results

In the following some test results are provided to demonstrate the ability of OpTiX-II to serve as a testbed for parallel and distributed solution strategies and algorithms in decision problems based on nonlinear optimization on different multiprocessor systems. The optimization problems, with which OpTiX-II was tested, are the 100-dimensional Rosenbrock Problem (see Appendix B, RO-100) and the generalized Rastrigin Problem (see Appendix C, RA-30). Both problems were solved for comparative purposes with a sequential and two nonsequential approaches. Of all nonsequential solution approaches, a parallel extension to a hybrid method and, since both problems are separable, a decomposition approach (distributed computing) were applied. The three solution approaches are applied on three different architectures of multiprocessor systems (see Table below). The results for the multimodal function RA-30 are the average of 10 runs. Of course, the implications of these results (in the Table) are very restricted, but it is nevertheless not difficult to recognize, that current transputer systems are not a competitive hardware for parallel optimization. On the other hand, workstation clusters and shared memory systems give promising results.

Type of Multiprocessor-system \ Solution strategies and testproblems	Test-problem	Sequential approach*	Nonsequential approaches	
			Parallel hybrid**	Distributed computing***
Multiprocessor system (shared memory, see Sun 4/630 in Fig. 1)	RO-100	46.00	29.52	13.61
	RA-30	341.03	90.18	7.10
Workstation Cluster (see Sun 4/25 in Fig. 1)	RO-100	137.30	78.87	45.08
	RA-30	574.00	160.00	10.97
Transputersystem (see Fig. 1)	RO-100	829.00	531.16	157.63
	RA-30	3374.63	2552.76	68.87

The following algorithms and solutions strategies are used:
* RO-100: Quasi-Newton-Method; RA-30: Adaptive-Random-Search,
** RO-100: Quasi-Newton-Method, Steepest-Descent;
RA-30: Steepest-Descent, Nelder-Mead, Adaptive-Random-Search,
communication after 100 iterations.
*** RO-100: 3 subsystems and coordination by Quasi-Newton,
RA-30: 3 subsystems by Adaptive-Random-Search.

Table 1: Computation times (in seconds) for the solution of the Rastrigin (RA-30) and the Rosenbrock (RO-100) test functions (see Appendix B and C).

In addition to these results, OpTiX-II is used to realize a parallel solution strategy for a structural optimization problem, thereby optimizing a complex mechanical structure (automotive wheel) in parallel on a network of workstations [8] and for the solution of decision problems from production planning and water management.

6. Summary and conclusion

The OpTiX-II Software Environment which supports the steps from the formulation of a decision problem based on nonlinear optimization to the solution on networks of parallel computers was presented. It enables the user to define the problem and a nonsequential solution strategy without necessarily taking into account the architecture of the hardware that will subsequently be used to solve this problem. Numerical tests have shown that workstation clusters and multiprocessor systems with shared memory give very promising results for the use of parallel and distributed solution strategies in this field of decision support.

7. References

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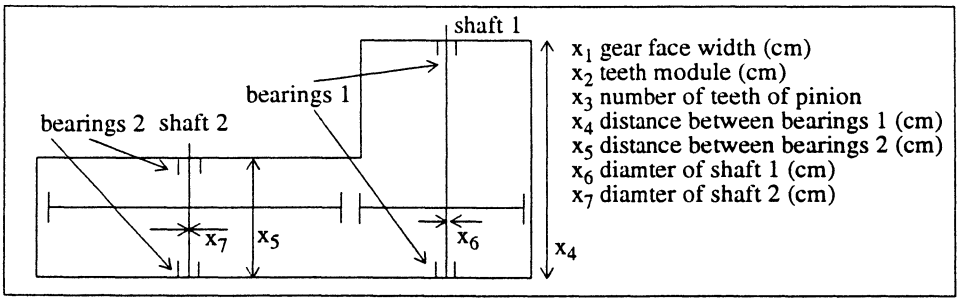
Appendices

Appendix A:

The decision problem consists in the design of a gear reducer with minimal weight under a set of given constraints (see [9]).



Appendix B:

The 100-dimensional Rosenbrock Banana Problem is used as follows:

$$min\ f(x) = \sum_{i=1}^{99} \left[100 \times (x_{i+1} - x_i^2)^2 + (1 - x_i)^2 \right]. \qquad \text{(RO-100)}$$

The starting point is $x_{start} = (1.2, -1.2, \ldots, 1.2, -1.2)^T$, and the bounds are $x_l = (-3.0, \ldots, -3.0)^T$ and $x_u = (3.0, \ldots, 3.0)^T$. The optimal solution is $x_{opt} = (1.0, \ldots, 1.0)^T$ with $f(x_{opt}) = 0$.

Appendix C:

The 30-dimensional generalized Rastrigin Problem is used as follows:

$$min\ f(x) = nA + \sum_{i=1}^{n} (x_i^2 - A\cos(\omega x_i)) \text{ with n=30, A=50 and } \omega = 2\pi. \qquad \text{(RA-30)}$$

The starting point is randomly generated from within the bounds $x_l = (-5.0, \ldots, -5.0)^T$ and $x_u = (5.0, \ldots, 5.0)^T$. The optimal solution is $x_{opt} = (0.0, \ldots, 0.0)^T$ with $f(x_{opt}) = 0$.

Testing a Bicriterion Location-Allocation Model with Real-World Network Traffic: The Case of Edmonton, Canada

M. John Hodgson[1], Kenneth E. Rosing[2], and A. Leontien G. Storrier[3]

[1,3] Department of Earth and Atmospheric Sciences, The University of Alberta, Edmonton, Canada, T6G 2H4

[2] Department of Applied Economics, Erasmus University, Rotterdam, Postbox 1738, NL-3000 DR, Rotterdam, NL

Abstract: The *p*-median model in a network treats demand for service as weights expressed at nodes. The flow-capturing location-allocation model treats demands expressed by traffic (OD) flows. We consider a bicriterion model, a hybrid of these two, which trades off node- and flow-based demand, and apply it to real-world data. These are journey to work data in a system of 177 traffic zones in Edmonton, Canada. The system is relatively complex, comprising 703 nodes, 2198 links and 23,350 OD pairs.

We consider how the solutions for each of the two criteria meet the objectives of the other, i.e.: how well a *p*-median objective would accommodate flow-based demand and how well a flow-capturing objective would accommodate node-based demand. On this comparison, the *p*-median model is the more tolerant model, though neither solution is particularly well-suited to the needs of the other type of demand.

We conclude that it is important to understand how demand for service is expressed when modelling location decisions, and that it is important to use a multiobjective approach to model location decisions when demand is expressed in multiple ways.

1. INTRODUCTION

The *p*-median model [5], a traditional location-allocation (LA) model on a network, treats demand for service as weights expressed at nodes. Flow-capturing LA models (FCLM) [2] treat demands expressed on paths, by traffic flows between origins and destinations. Most retail businesses cater to both types of demand; they serve persons who travel specifically to the outlet from their homes and they serve passing traffic. Hodgson and Rosing [3] presented a bicriterion LA model based on the trade-off between serving demands centred in nodes and demands arising from flows. In this paper, we consider the polar extremes of this model with traffic in the road network of Edmonton, Canada.

2. THE BICRITERION LOCATION-ALLOCATION MODEL

The model trades off the objectives of the flow-capturing location-allocation model and the *p*-median model. We applied weights totalling 1.0 to the separate objectives to produce the combined objective:

MINIMIZE: $z = wz^p - (1-w)z^f$

where w and $1-w$ are the relative weights assigned to the p-median objective, z^p ,and the flow capturing objective, z^f, respectively. At $w = 0.00$, the model is a pure flow capturing problem, at $w = 1.00$,it is a pure p-median problem. Intermediate values of w specify different mixes of the two objectives. The bicriterion model is defined as:

MINIMIZE:

$$Z = w\sum_{i=1}^{n} P_i \sum_{j=1}^{n} d_{ij} X_{ij} - (1-w)\sum_{q=1}^{n_q} f_q Y_q$$

SUBJECT TO:

$$\sum_{j}^{n} X_{ij} = 1 \qquad \forall\, i$$

$$X_{jj} - X_{ij} \geq 1 \qquad \forall\, i, j$$

$$\sum_{j=1}^{n} X_{jj} = p$$

$$\sum_{j \in \Lambda_{jj}} X_{jj} \geq Y_q \qquad (q = 1, \ldots, n_q)$$

$$X_{ij} = (0 \vee 1)$$

Where:

n is the number of nodes in the network.

P_i is the population at node i.

d_{ij} is the distance between nodes i and j.

X_{ij} is the proportion of node j's demand covered at i, by the p-median convention,

X_{jj} = 1 if there is a facility at j, = 0 otherwise.

f_q is the flow from O to D (OD_q).

Y_q = 1 if f_q is captured, 0 if not.

p is the number of facilities to be located.

Λ_{jj} is the set of nodes capable of capturing a particular OD pair q's flow, ie: the set of all nodes on the shortest path between that origin and destination.

X_{ij} is the proportion of the demand at i that is served at j.

The model is a straight-forward hybrid of the flow capturing and the traditional p-median models with minor variable redefinition for compatibility. We used LAMPS [4] to solve the problem as a continuous linear program and to then resolve non-binary solutions with branch and bound.

We had hypothesised that weights favouring the flow capturing model would have a relatively benign effect upon quality of service for residence-based demand. We based this hypothesis upon the common knowledge that the p-median objective tends to be shallow in the vicinity of the optimum and that the model thus tolerates facility location shifts reasonably well. We believed that deviations from p-median optimal locations to flow gathering nodes on the network would be relatively short and that solution quality would change only slightly. We had also hypothesised that the flow capturing model would be more sensitive to impairment by weights favouring the p-median model. We based this hypothesis on our intuition that because traffic tends to become bundled into a few major arteries and nodes, facilities being shifted from these through a p-median emphasis would suffer a considerable drop in the amount of flow captured. We tested the multiobjective model on a small, 25-node network with flows generated by a simple gravity-type trip distribution model.

In contrast to our hypotheses, the p-median model appeared to be more susceptible to damage than the flow capturing model; flow capturing generally resulted in poor p-median solutions. We considered this departure from our hypotheses to be an anomaly resulting from the artificial structure of the test network and from the concentration of flows generated by the gravity model between well-connected nearby nodes in the network. We were therefore enthusiastic to test the model in situations such as an intra-urban networks in which flows tend to be concentrated on a few major arteries and much of the population is located in backwater neighbourhoods. This paper is an initial step toward testing the multiobjective model in such a real world setting.

3. THE REAL WORLD NETWORK

Edmonton, Canada, is a prairie city with an approximate area of 700 km^2 and a 1991 population of 616,714 [6]. The roadway system of 2927 km, comprises 103 km of freeways and expressways, 729 km of arterials, 451 km of collectors and 1644 km of local roads. Many roadways and two bridges are one-way, which makes the flow structure asymmetrical. The *City of Edmonton Forecasting and Assessment Branch* (FAB) provided network topology, population, and flow data for 1989. The basic data aggregation unit is the traffic zone; the full street network is aggregated to a system of zonal centroids and freeway and arterial road network intersections. Zone centroids are connected to the transportation network by one or more feeder links and intrazonal traffic, travel times and distances are estimated for each traffic zone's centroid.

As flow data, we used FAB's 1989 estimate of the morning peak hour, all purpose, OD trip volume table. The system of 177 zone centroids produces a total

of 23,350 non-zero flow pairs (OD pairs with zero flow are not considered by the FCLM). The FCLM will capture flows on paths however they are defined; we assumed that all travel is by the shortest path for the purpose of generating objectively a reasonable data set for this experiment. We pared the network to remove all nodes and links which do not fall on the shortest paths between nonzero OD pairs. A total of 703 nodes and 2198 links remained in the network.

4. ANALYSIS

In this application, the bicriterion program is very demanding computationally; mobilising it and performing sensitivity analysis is a top priority. Here, we report on the two poles represented by the two solution criteria of the multiobjective LA program. That is, we evaluate the p-median solution on the FCLM objective, and evaluate the FCLM solutions on the p-median objective. We view this as the numerical evaluation of the consequences of using the inappropriate model if the other were entirely appropriate, i.e.: the consequence of "modelling the incorrect attendance behaviour." We solve the FCLM with LAMPS, which computational efficiency limits to $p \leq 15$. Solving a p-median problem of this size optimally is now barely possible using cutting edge software and hardware. For this experiment, we solved the p-median model using the Tietz and Bart [7] heuristic as provided by [1]. For technical reasons, our version of this program allows $p \geq 5$. Our comparisons are thus for $p = 5,...,15$.

Because the effects of using the inappropriate model are adverse, we use the concept of *damage* as articulated by Hodgson and Rosing[4] for the comparison measure. To evaluate the damage done by using Model A where Model B is appropriate, we assume that the optimal (or best heuristic) solution for B is correct, calculate the Model B objective function for the locations derived by Model A, and compare the two objective functions with a function which indicates the degree to which the inappropriate solution is worse than the optimal solution. For the p-median and FCLM, this involves the following terms:

Λ_p = the p-median facility locations

Λ_f = the FCLM facility locations

Z_{pp} = The p-median objective function value for Λ_p

Z_{ff} = The FCLM objective function value for Λ_f

Z_{pf} = The p-median objective function value for Λ_f

Z_{fp} = The FCLM objective function value for Λ_p

Using the FCLM when the p-median model is appropriate results in excess aggregate weighted distances. We use the expression:

$$\frac{Z_{pf}}{Z_{pp}} - 1.0$$

to describe this type of damage. The damage resulting from using the *p*-median model when the FCLM is appropriate, is the degree to which the flow captured falls short of the optimum. To make the damage index from this maximisation directly comparable with the other (from a minimisation), we use the expression:

$$\frac{Z_{ff}}{Z_{fp}} - 1.0$$

which describes the degree to which the flow captured by the correct model exceeds that captured by the *p*-median.

5. RESULTS

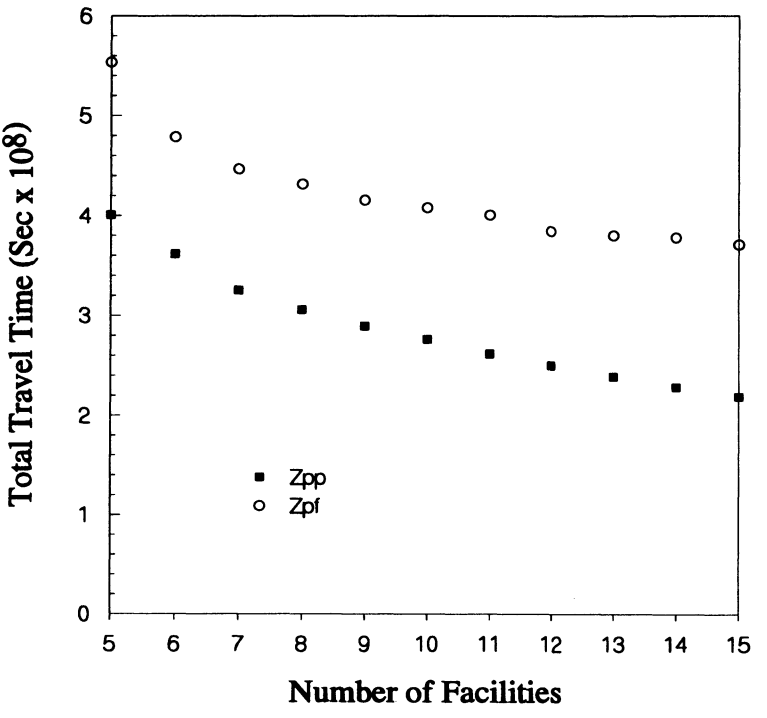
Numerical results are documented in Table 1. The *p*-median results (Z_{pp})

Table 1: Damage Caused by Modelling Incorrect Attendance Behaviour

p	Z_{pp} (Sec x 10^8)	Z_{pf} (Sec x 10^8)	Damage to p-Median	Z_{ff} (Persons x 10^4)	Z_{fp} (Persons x 10^4)	Damage to FCLM
5	4.01	5.54	0.38	2.95	1.39	1.13
6	3.62	4.79	0.32	3.33	1.40	1.37
7	3.25	4.47	0.37	3.64	1.69	1.16
8	3.06	4.31	0.41	3.92	1.49	1.63
9	2.89	4.15	0.44	4.19	1.82	1.30
10	2.76	4.07	0.48	4.42	1.97	1.24
11	2.61	4.00	0.52	4.63	2.17	1.13
12	2.50	3.84	0.54	4.84	2.13	1.27
13	2.39	3.80	0.59	5.02	2.29	1.19
14	2.28	3.78	0.65	5.18	2.39	1.17
15	2.19	3.71	0.69	5.32	2.52	1.11

exhibit the expected diminishing declining travel time returns to *p* (Fig. 1); the FCLM (Z_{pf}) solutions show monotonically diminishing returns to *p*, naturally higher than the optimal *p*-median solution. The FCLM (Z_{ff}) results exhibit the expected diminishing increasing flow returns to *p* (Fig. 2); the *p*-median

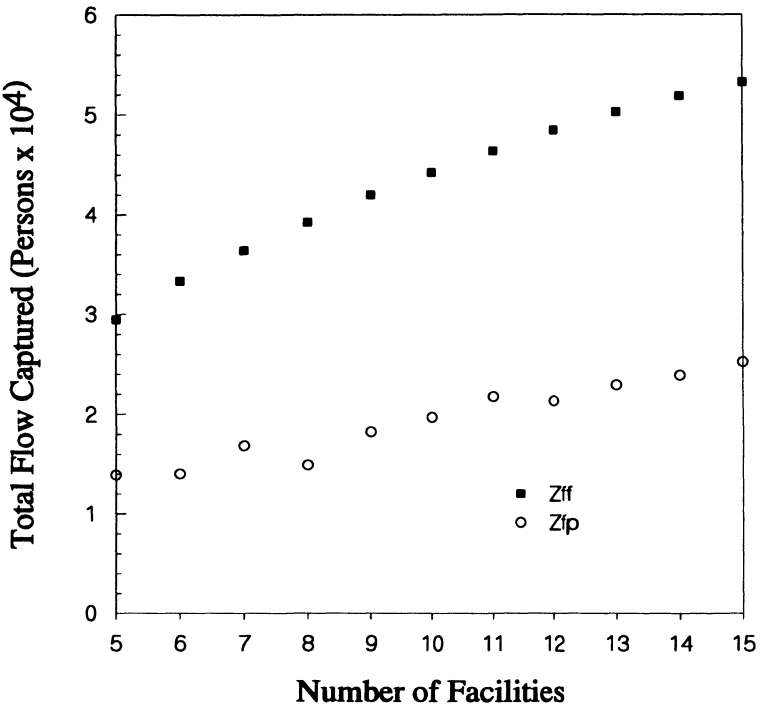
Figure 1: Performance of p-median and FCLM on p-median objective



solutions (Z_{fp}) naturally capture less flow than the FCLM ones. Although the amount of flow captured by the p-median solutions generally increases with p, the relationship is not monotonic. Non-monotonicity in Z_{fp} obviously occurs where a particular p-median solution is poorly located for capturing flows, where placing facilities to minimise aggregate travel times happens to miss good network connections.

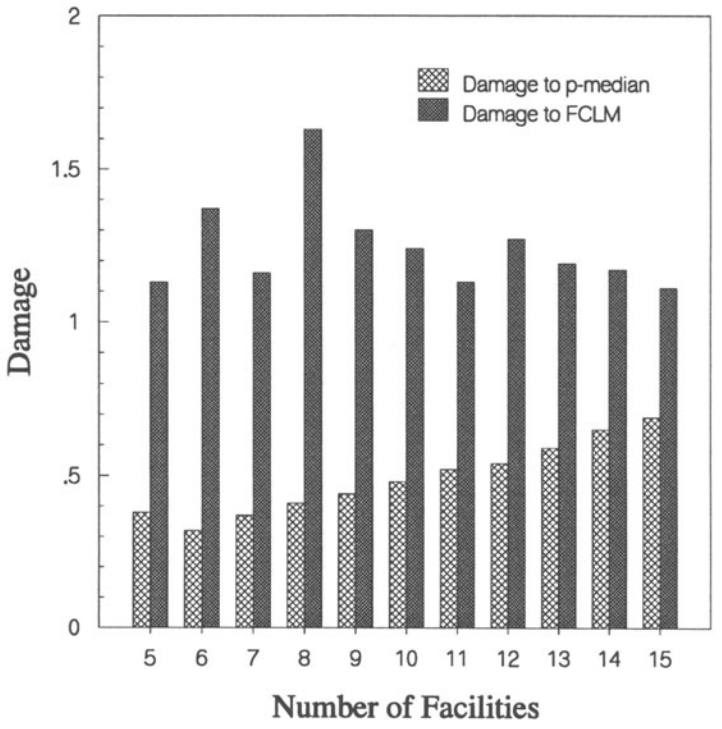
The damage caused by assuming flow capturing travel when the p-median model is appropriate is, as Hodgson and Rosing surmised, less than the damage caused by assuming p-median travel when the FCLM is appropriate (Fig. 3). Damage to the FCLM is in the 1.1 to 1.6 range, indicating that for all tested values of p, the flow capturing solution captures more than twice as much flow as the p-median. Damage to the FCLM appears to be independent of the value of p. Damage to the p-median model generally increases with p, and is in the 0.3 to 0.6 range, indicating that the flow capturing solution never endures more than 60 percent of the excess travel time of the optimal p-median.

Figure 2: Performance of FCLM and *p*-median on FCLM objective



Maps of the optimal and damaged p = 15 solutions illustrate the sources of damage quite clearly. The optimal FCLM solution locates facilities at strategic flow bundling points on the transportation network. Because morning peak traffic is modelled, it tends to focus on central locations, resulting in bundling of traffic at relatively central locations, most on intersections other than traffic zone demand points (Fig. 4). These optimal FCLM locations are too close together near the city centre; the least-time demand allocation desire lines (Fig. 5) clearly indicate the suboptimal mix of highly clustered facilities with small hinterlands and an outer ring of still quite central facilities with large, eccentric suburban hinterlands. The *p*-median solution also somewhat oriented toward the downtown, with a looser cluster of facilities reacting to the higher population densities in the centre, but it exhibits a greater spread of facilities throughout the city and a better balanced set of service areas (Fig. 6). None of these locations capture large flows and many capture very little flow at all (Fig. 7). Being located to minimise average distance, they are located on major flow paths only by chance, if at all.

Figure 3: Damage done by assuming incorrect travel pattern

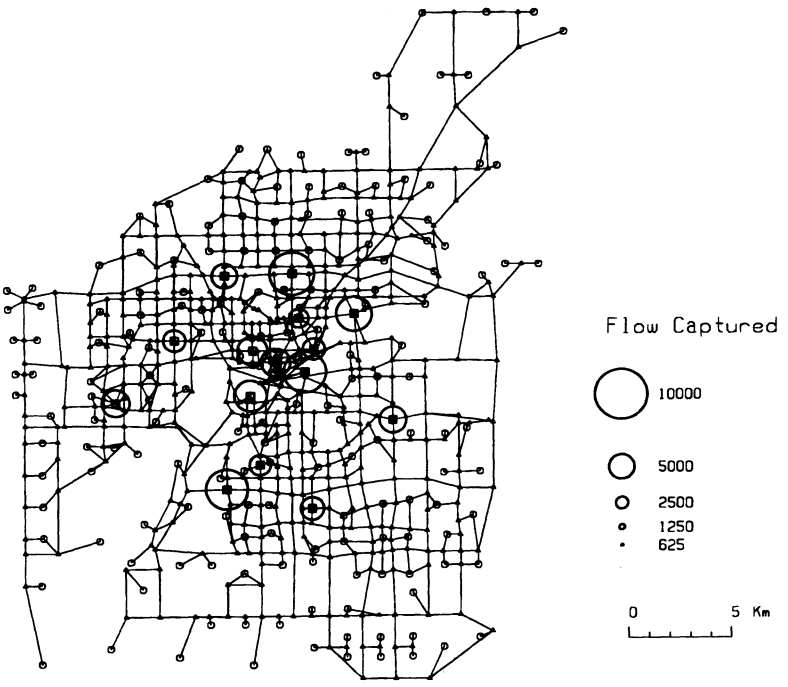


6. CONCLUSION

We have compared FCLM and *p*-median results in an authentic urban network. Our bi-polar analysis highlights the importance of knowing whether a facility's custom is derived from passing traffic of from local residence based sources. In reality, it is unlikely that either one or the other extreme is correct; there will almost certainly, within urban areas at least, be a mix of behaviour for any type of business. Gasoline stations will be more highly flow oriented, small supermarkets will be more oriented to residences, but there will be a mix. Our findings indicate that, in Edmonton, with morning traffic, residence based demand will tolerate flow capturing optimality better than would flow based demand tolerate residence oriented optimality. The solution to the mixed location problem is not so simple, it *demands* a multiobjective approach.

Two research problems leap out from this demand. To activate the multiobjective approach in a realistic setting, valid values of W in (9) are required for whatever type of retail activity is addressed. This in turn requires commensurability of the *p*-median and FCLM objectives, which might most directly be produced by assessing monetary values for each kind of custom. At this time, we prefer still to leave this type of work to marketing specialists. To

Figure 4: Flows captured by FCLM, $p = 15$



satisfy our curiosity and to proselytise the importance and applicability of such empirical work, we regard performing the sensitivity analysis of the multiobjective approach to be our highest research priority. We invite the comments and collaboration of researchers who could help us solve bicriterion integer problems of this size.

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Figure 5: Least time allocations to FCLM facilities

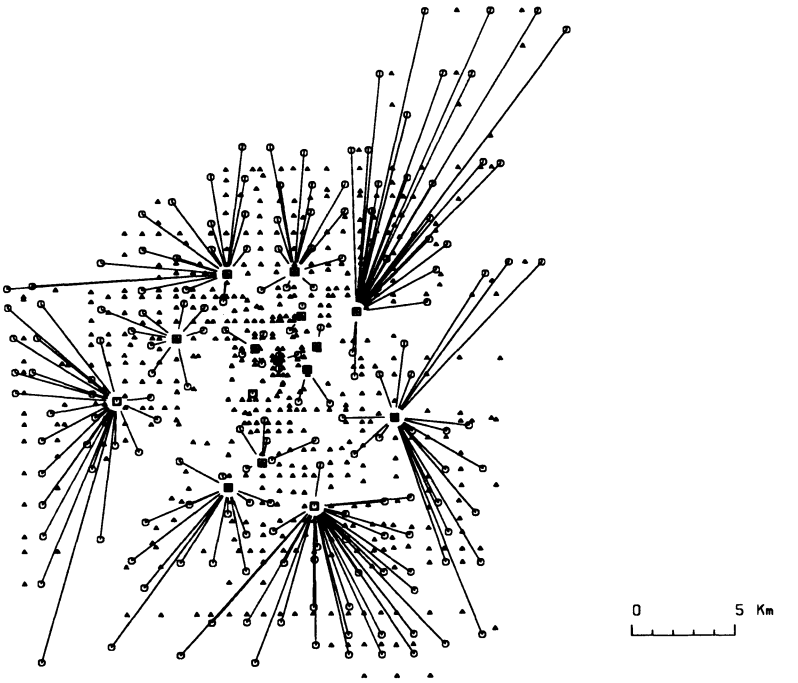


Figure 6: Least time allocations to *p*-median facilities

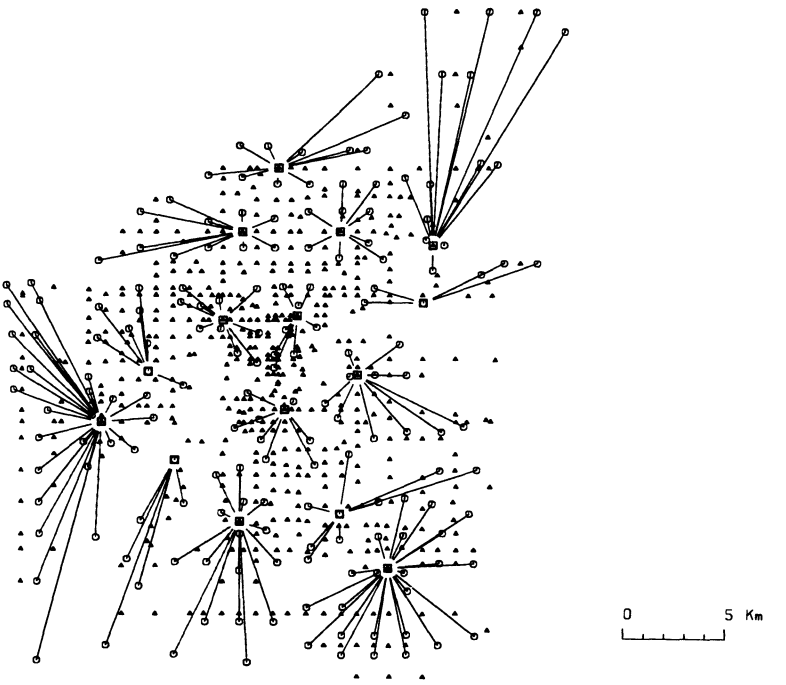
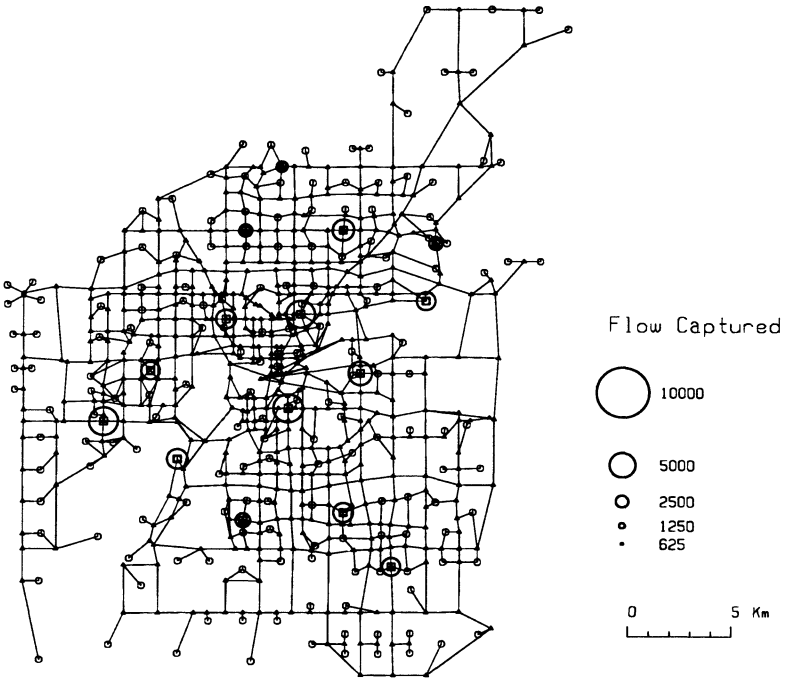


Figure 7: Flows captured by *p*-median model



ON THE USE OF MULTICRITERIA DECISION AID METHODS TO PORTFOLIO SELECTION

Ch. Hurson[2] and C. Zopounidis[1]
[1]D.S.S. Laboratory, Technical University of Crete, University Campus, 73100 Chania, GREECE. Fax: (30) 821 648 24 Tel: (30) 821 64 186
[2]G.R.E.Q.A.M, Université d'Aix-Marseille II, 2 rue de la charité, 13002 Marseille, FRANCE. Fax: (33) 91 90 02 27 Tel: (33) 91 14 07 27

Abstract: This paper proposes the use of different multicriteria decision aid methods for management of stocks' portfolios. The ELECTRE TRI (ELimination Et Choix Traduisant la REalité) method and the MINORA (Multicriteria INteractive Ordinal Regression Analysis) system are used to sort and rank respectively a sample of stocks. Then, the ADELAIS (Aide à la DEcision pour systèmes Linéaires multicritères par AIde à la Structuration des préférences) multiobjective programming system is employed for the constitution of a stocks' portfolio according to the results obtained by ELECTRE TRI and MINORA. The data used were obtained from the Athens Stock Exchange for the period 1990-91.

1. Introduction

Portfolio selection concerns the problem of finding the most attractive stocks and the determination of their proportions in a portfolio, which is essentially a matter of arbitration between the risk and the return. In 1952 Markowitz proposing the mean-variance model gives the start of a theory that had known since a great development. According to this approach, an investor follows two conflicting objectives that are the maximisation of the expected return and the minimisation of the risk measured by the variance of the return. On the same basis several other models were developed.

Recently, the development of multicriteria analysis gave rise to new way of modelling portfolio management. Multicriteria analysis provide the methodological basis to resolve the multicriteria nature of the portfolio selection problem and builds realistic models by taking into account, besides the two basic criteria of return and risk, a number of important criteria such as marketability, price earnings ratio, and others. In this paper we propose a methodological

framework in which the ELECTRE TRI method and the MINORA system are used to select stocks and, then, the ADELAIS system is used to constitute a portfolio of the attractive stocks. This methodological framework is applied to the Athens Stock Exchange (ASE) for the period 1990-1991. A more detailed presentation of this work that the one proposed in this paper can be found in Hurson (1995). Section 2 presents the proposed methodological framework and section 3 its application to the ASE. Finally, we summarise the used methodological framework, underline its advantages and give some concluding remarks.

2. The methodological framework

The basic components of the proposed methodological framework are presented in Figure 1.

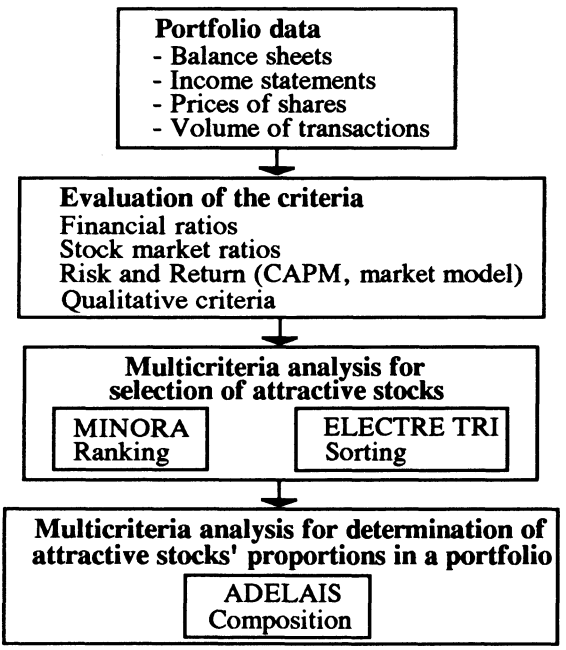


Figure 1: Basic components of the methodological framework

2.1. Evaluation of the criteria

On the basis of the portfolio data one can calculate financial and stock market ratios (return on equity, current ratio, earnings per share, price earnings ratio,...), the pair risk/return from the theoretical portfolio models as Capital Assets Pricing Model. The portfolio manager needs also to have more general information in a qualitative form (quality of management, image,...) so that his evaluation would be as objective and complete as possible. Financial and stock market ratios have become an accepted evaluative technique of financial analysts and portfolio managers. They provide a quantitative view of every element that concerns the internal operation of a firm as well as its relations with the outer world. The risk and the return issue of theoretical portfolio models (CAPM and market model) are the basic criteria in portfolio selection. The risk is measured by the beta coefficient, which measures the systematic risk (part of the risk which cannot be eliminated by diversification). Qualitative criteria are modelled according to the preferences of the portfolio manager with the aid of an ordinal scale.

2.2. Multicriteria analysis for selection of attractive stocks: The MINORA system and the ELECTRE TRI method

In this step the two multicriteria decision aid methods MINORA and ELECTRE TRI are used to help the portfolio manager in the selection of a set of attractive stocks taking into consideration his preferences and experiences. Here, only a short description of the two methods is given.

MINORA is an interactive method (cf. Siskos et al., 1993; Spyridakos and Yannacopoulos, 1995) used to rank the stocks from the best to the worst. On the basis of a preordering asked to the portfolio manager on a small number of well-known stocks (reference set), MINORA estimates a certain number of separable additive utility functions. The interaction takes the form of an analysis of the inconsistencies between the ranking established by the portfolio manager and the ranking assessed by Minora. During the interaction the portfolio manager can modify his ranking, the scale of one or more criteria, the marginal utilities or the problem formulation. When an acceptable compromise is found; the result is extrapolated to the full set of stocks. MINORA presents some important advantages for portfolio selection: (1) It leads to a ranking, and consequently the portfolio managers have the habit to rank stocks and/or

portfolios in order to advise their clients. (2) The interaction aids the portfolio manager to construct his own model in a non normative way; totally involving at the same time the portfolio manager in the resolution process, whose task is made easier by a graph system. MINORA has proved itself in numerous management problems and more particularly in portfolio management (cf. Hurson and Zopounidis 1993; Zopounidis 1993; Zopounidis et al., 1993).

ELECTRE TRI (Yu, 1992) is used to proceed to a classification of the stocks in categories delimited by some reference alternatives or reference profiles. In ELECTRE TRI there are two different kinds of assignment: the pessimistic and the optimistic one. For the assignment ELECTRE TRI compares the alternatives with the profiles using the classical concept of valued outranking relation. Then, this valued outranking relation is used to define the preference, the indifference and the incomparability. Pessimistic and optimistic assignments are different only if some incomparabilities appear between the stock and some reference profiles. The advantages of the ELECTRE TRI method related to the portfolio selection are the following: (1) ELECTRE TRI by sorting the stocks in categories, fits well with the purpose of the portfolio selection. (2) The ELECTRE family accepts the incomparability and the intransitivity, in such a way that this kind of methods will point out the stocks which have particularities in their evaluations, using techniques easy to understand for the portfolio managers. The methods belonging to the ELECTRE family are very popular methods, successfully used for a great number of studies and in portfolio management by Martel et al. (1988) and Szala (1990).

2.3. Multicriteria analysis for determination of attractive stocks' proportions in a portfolio : The ADELAIS system

Once a set of attractive stocks is selected using MINORA and ELECTRE TRI, the ADELAIS (Siskos and Despotis, 1989) system helps to determine the proportions of capital invested in theses stocks. ADELAIS is an interactive computer system developed to support the search for a satisfactory solution to Multiple Objective Linear Programming problems. The system provides a "two levels" interaction: interactive assessment of the portfolio manager's utility function (as in MINORA) and interactive modification of the satisfaction levels. The advantages of the ADELAIS system for the portfolio manager, are the following: (1) It allows to determine the proportions of the attractive stocks in the portfolio that is not possible with ranking or sorting

methods. (2) The interactive analysis of the inconsistencies as in MINORA system, which helps the portfolio manager to well understand the portfolio selection problem. (3) The interactive revision of the satisfaction levels orientates in a way easy to understand the research of the best compromise which is well adapted to the portfolio manager's preferences. The ADELAIS system has also been applied in some studies and, especially, in portfolio management by Zopounidis et al. (1993, 1995).

3. Application of the three multicriteria decision aid methods to the ASE

3.1. Stocks and evaluation criteria

The sample considered in the study consists of 40 stocks coming from the financial and commercial sectors of the ASE. The portfolio diversification by sector is a way to eliminate the unsystematic risk (risk due to a stock). The study period includes the years 1990 and 1991 and the closing prices were recorded in a monthly basis. We have used the data average of the period of two years for the calculation of criteria. The used criteria are considered the most significant both in theory and practice and they are the following:

- g_1: Return (Max)
- g_2: Marketability (percenage of shares traded) (Max)
- g_3: Beta (Min)
- g_4: Price Earnings Ratio (Price of the share / earnings per share). Since there are negative values of earnings (losses), we choose to maximize the inverse of this criterion (Max)
- g_5: Growth of Dividend Per Share (Max)
- g_6: Acid Test (current assets - inventories / current liabilities) (Max)
- g_7: Return On Equity = net income / stockholder's equity (Max)

All the stocks of the sample have been evaluated on the five stock market criteria $g_1,\ldots,g_5$, while, in order to obtain most significant results, the criterion g_6 is used for the stocks of the commercial sector (CS_1 to CS_{20}) and the criterion g_7 for the stocks of the financial sector (FS_1 to FS_{20}). Tables 1 present the multicriteria evaluation for the stocks of the commercial sector. Qualitative criteria are not included because they are dependent to the portfolio manager's personal information which was not available in this application.

Table 1: Multicriteria evaluation of stocks for the commercial sector

Criteria	CS_1	CS_2	CS_3	CS_4	CS_5	CS_6	CS_7	CS_8	CS_9	CS_{10}
g_1	0.82	0.41	0.57	0.24	0.00	0.93	0.01	0.86	2.16	1.24
g_2	0.45	0.63	0.20	0.02	0.46	0.02	0.69	0.86	0.60	0.12
g_3	0,26	0,03	0,10	0,08	0,62	0,14	0,77	0,86	0,12	0,62
g_4	-4.7	2.28	6.08	2.41	5.04	2.82	7.55	4.28	2.11	11.65
g_5	-100	-20	-33.3	-53.5	-76.5	6.38	-40	3.71	56.3	12.50
g_6	0.45	2.04	1.08	0.62	3.02	0.72	3.23	0.57	0.51	1.17
Criteria	CS_{11}	CS_{12}	CS_{13}	CS_{14}	CS_{15}	CS_{16}	CS_{17}	CS_{18}	CS_{19}	CS_{20}
g_1	0.80	1.23	0.24	0.26	1.10	1.79	1.02	1.32	1.36	0.57
g_2	0.58	0.37	0.28	0.65	0.76	0.55	1.06	1.12	0.04	0.17
g_3	0.62	0.64	0.73	0.58	0.54	0.73	0.82	0.94	1.02	0.23
g_4	13.70	8.97	-1.75	4.88	0.29	5.88	5.50	12.06	1.79	-11.5
g_5	34.6	45.9	0.00	7.14	0	-100	6.38	-61	110	0
g_6	1.54	0.96	0.72	0.90	0.73	2.69	0.73	2.69	2.31	0.52

3.2. The ranking of stocks using the MINORA system

The reference set and his ranking for the commercial sector is the following: (1) CS_{16}, (2) CS_{18}, (3)CS_{12}, CS_{19}, (4)CS_9,CS_3, (5) CS_7, CS_8, (6) CS_1, (7) CS_{20}, for this purpose the portfolio manager can use a graphic help.

The MINORA system provides two basic results: the criteria's graphics (i.e. marginal utilities, Figure 2) and the ordinal regression curve (ranking versus global utility, Figure 3). An important graphic help is at the disposal of the portfolio manager in order to interpret theses results. Table 2 shows the result of the extrapolation phase for both the two sectors.

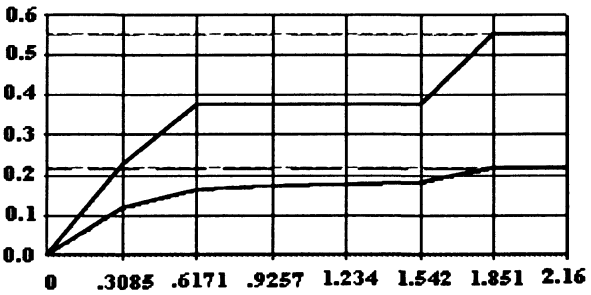


Figure 2: Marginal utility curves of the criterion return (commercial sector)

	D.R.	A.Act.	G.U.	M.R
1	1	CS_{16}	.761	1
2	2	CS_{18}	.745	2
3	3	CS_{12}	.656	3
4	3	CS_{19}	.656	3
5	5	CS_{9}	.632	5
6	5	CS_{3}	.632	5
7	7	CS_{7}	.596	7
8	7	CS_{8}	.596	7
9	9	CS_{1}	.554	9
10	10	CS_{20}	.362	10

Kendall's $\tau = 1$

$F = 0$

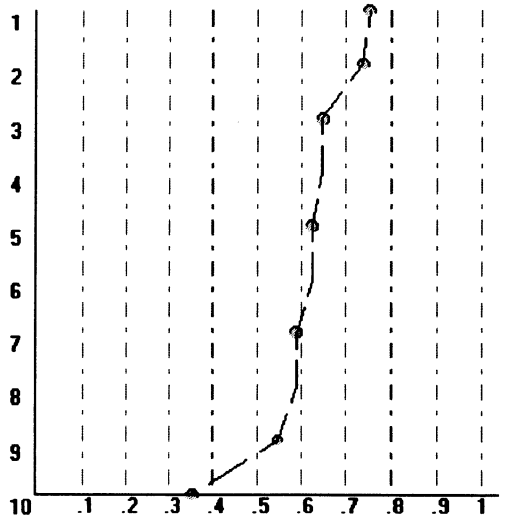


Figure 3: Portfolio manager's regression curve(commercial sector)

Table 2: Global utility for the sample of 40 Greek stocks (extrapolation phase)

Actions	G U	D.R.	M.R.	Actions	G.U.	D.R.	M.R.
CS_{16}	0.761	1	1	FS_{5}	0.679	*	1
CS_{18}	0.745	2	2	FS_{10}	0.665	1	2
CS_{11}	0.684	*	3	FS_{20}	0.570	2	3
CS_{12}	0.656	3	4	FS_{9}	0.570	2	3
CS_{19}	0.656	3	4	FS_{6}	0.555	*	5
CS_{2}	0.643	*	6	FS_{2}	0.554	*	6
CS_{17}	0.635	*	7	FS_{18}	0.549	4	7
CS_{3}	0.632	5	8	FS_{3}	0.549	4	7
CS_{9}	0.632	5	8	FS_{17}	0.534	*	9
CS_{10}	0.622	*	10	FS_{19}	0.532	*	10
CS_{7}	0.596	7	11	FS_{4}	0.511	6	11
CS_{8}	0.596	7	11	FS_{15}	0.504	*	12
CS_{15}	0.582	*	13	FS_{14}	0.499	7	13
CS_{5}	0.559	*	14	FS_{7}	0.490	8	14
CS_{1}	0.554	9	15	FS_{8}	0.490	8	14
CS_{14}	0.554	*	16	FS_{16}	0.484	*	16
CS_{13}	0.495	*	17	FS_{12}	0.476	*	17
CS_{6}	0.471	*	18	FS_{1}	0.433	*	18
CS_{4}	0.388	*	19	FS_{11}	0.395	10	19
CS_{20}	0.362	10	20	FS_{13}	0.210	*	20

(*) Stocks not included in the reference set

In the Figure 3 are shown: the preordering of the portfolio manager (DR), the stock names (ACT), the global utility of every stock (G.U.), the ordering from the global MINORA (MR) and the ordinal regression curve (stocks' ranking versus global utility). In Figure 2 there are three utility curves for the return criterion (min, middle and max). The middle one corresponds to the above presented model of additive utility and, also, gives the relative weight for the criterion. The two others show the entire range of the possible marginal utility functions and gives an idea of the sensitivity of the criteria. A sensitivity analysis of the results of MINORA was performed and gave good results.

3.3. The sorting of stocks using the ELECTRE TRI method

The objective is to sort the stocks in the following three categories: attractive stocks (C_3), uncertain stocks to be studied further (C_2), non attractive stocks (C_1). The reference profiles and the thresholds related to the concordance and discordance indices are fixed by the portfolio manager according to his experience and preferences. Table 3 presents this preferential information for commercial sector.

Table3: Reference profiles and thresholds for commercial sector

Parameters	g_1	g_2	g_3	g_4	g_5	g_6
High reference profile	1	0.35	0.6	6	10	1.1
Low reference profile	0.5	0.7	0.25	2.5	0	0.7
Indifference threshold	0.05	0.05	0	0.1	8.72	0.05
Preference threshold	0.25	0.2	0.2	0.5	10	0.25
Veto threshold	2	1	1	10	180	2.75

Tables 4 presents the sorting results of the ELECTRE TRI method for commercial sector. The stocks which belong to the best category (C_3) in both optimistic and pessimistic sortings, are proposed without hesitation to the portfolio manager for selection. The stocks which belong to the worst category (C_1) in both optimistic and pessimistic sortings are not proposed to the portfolio manager. When the stocks belong to the uncertain category (C_2) for both optimistic and pessimistic sortings, this means that these have moderate values on all criteria and, consequently, they must be studied further. In the cases where some stocks belong to different categories in both optimistic and pessimistic sortings, this means that they are incomparable with one or two reference profiles. This is due to the fact that these stocks have good values for

some criteria and, simultaneously, bad values for other criteria. Thus the notion of incomparability underlignes the particularities of these stocks which must be examined further and brings an important information to the portfolio manager.

Table 4: Sorting of stocks for the commercial sector

	C_1	C_2	C_3
Optimistic	CS_4, CS_{13}, CS_{20}, FS_1, FS_7, FS_{11}, FS_{12}, FS_{13}.	CS_1, CS_2, CS_3, CS_5, CS_6, CS_8, CS_{14}, CS_{15}, CS_{17}, FS_8, FS_9, FS_{14}, FS_{19}.	CS_7, CS_9, CS_{10}, CS_{11}, CS_{12}, CS_{16}, CS_{18}, CS_{19}, FS_2, FS_3, FS_4, FS_5, FS_6, FS_{10}, FS_{15}, FS_{16}, FS_{17}, FS_{18}, FS_{20}.
Pessimistic	CS_1, CS_4, CS_{13}, CS_{19}, CS_{20}, FS_1, FS_7, FS_9, FS_{11}, FS_{12} FS_{13}, FS_{14}.	CS_2, CS_3, CS_5, CS_6, CS_7, CS_8, CS_9, CS_{12}, CS_{14}, CS_{15}, CS_{17}, FS_3, FS_8, FS_{10}, FS_{16}, FS_{19}.	CS_{10}, CS_{11}, CS_{16}, CS_{18}, FS_2, FS_4, FS_5, FS_6, FS_{15} FS_{17}, FS_{18}, FS_{20}.

Regarding the ranking results of the MINORA system with the sorting results of the ELECTRE TRI method, one can remark, generally, that there is an agreement, that is, the stocks which are well ranked (i.e. top of the ranking) by MINORA are in the best category C_3 by ELECTRE TRI, and vice versa. This agreement between these two methods asserts the interest of the study and allows the portfolio manager to be confident with their results. A sensitivity analysis of the results of ELECTRE TRI was performed and gave good results.

3.4. The determination of the attractive stocks' proportions in a portfolio using the ADELAIS system

According to the results obtained above, the portfolio manager can choose a subset of the best stocks from each sector and then, using the ADELAIS system, determine the proportions invested in each selected stock. The choosen set is the following: FS_{10}, FS_5, FS_{20}, FS_6, FS_2 from the financial sector and CS_{16}, CS_{18}, CS_{12}, CS_{11}, CS_9 from the commercial sector.

The decision variables, X_i, are the percentages of capital invested in each stock. For the needs of the study we note: $X_1=FS_{10}$, $X_2=FS_5$, $X_3=FS_{20}$, $X_4=FS_6$, $X_5=FS_2$, $X_6=CS_{16}$, $X_7=CS_{18}$, $X_8=CS_{12}$, $X_9=CS_{11}$, $X_{10}=CS_9$.

The objective functions are:

- Maximize the return (g_1): Max $R_1X_1+\ldots+R_{10}X_{10}$.
- Maximize the marketability (g_2): Max $M_1X_1+\ldots+M_{10}X_{10}$.
- Minimize the beta (g_3): Min $B_1X_1+\ldots+B_{10}X_{10}$.
- Maximize the price earnings ratio (g_4): Max $PER_1X_1+\ldots+PER_{10}X_{10}$.
- Maximize the growth of dividends per share (g_5): Max $GD_1X_1+\ldots+GD_{10}X_{10}$.

Where R_i, M_i, B_i, PER_i, and GD_i are the values of the corresponding objective for the stock i.

The constraints are:

- $X_1+\ldots+X_{10}=1$, all the available capital must be invested.
- $0.05<X_1<0.2,\ldots,0.05<X_{10}<0.2$ upper and lower limits of the amount to be invested in each stock (% of capital).

At the beginning (preliminary phase) ADELAIS proceeded to the estimation of an initial efficient portfolio of stocks (compromise) and present it as in Table 5. A pay-off table for the objectives was also given to the portfolio manager.

Table 5: Initial efficient solution

Criteria / Initial solution	g_1	g_2	g_3	g_4	g_5
Upper bound	1.61	0.64	0.672	13.98	49.92
Compromise	1.44	0.58	0.745	11.9	47.5
Satisfaction Level	12.05	0.34	0.933	9.08	-6.33
Lower bound	12.05	0.34	0.933	9.08	-6.33
R_c*	58.2%	81.2%	60.5%	57.5%	95.7%

*R_c is the rate of closeness to the ideal (upper bound)

With respect to the initial solution, the portfolio manager being satisfied by the attained values of the marketability and the growth of dividends per share asked for an improvement of the other objectives. On the basis of this information the system generated a set of portfolios, and asked the portfolio manager to rank them. On the basis of these data, ADELAIS assesses an additive utility model and use it to determine a new reference solution. Finally, after three iterations ADELAIS determines the best compromise solution which could not be improved. Table 6 presents the values of the objectives and the rate of closeness to the ideal point, the corresponding portfolio appears in Figure 4.

Table 6: Best compromise solution

Solution	g_1	g_2	g_3	g_4	g_5
Attained values	1.47	0.52	0.73	11.9	47.22
R_c	65%	61.9%	65.8%	57.5%	95.2%

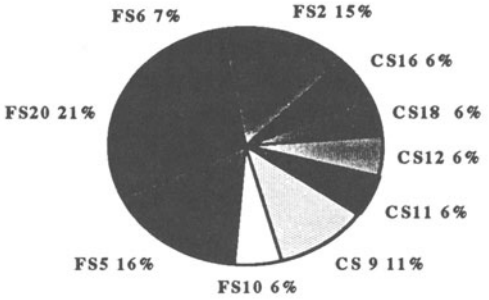


Figure 4: Portion of capital invested in the portfolio

4. Concluding remarks

In this paper a methodological framework for portfolio selection is proposed and applied on the ASE. This framework is a new supportive tool which uses multicriteria analysis to cover all the process of portfolio selection. The use of multicriteria decision aid methods allows to take into consideration all the relevant criteria, whatever their origins, for portfolio selection. At first, the conjoined use of MINORA system and ELECTRE TRI method have shown some advantages as the complementarity and the similarity of the obtained results that state the portfolio manager's confidence in the constitution of his portfolio; also, they satisfy one of the portfolio managers' preoccupation which is the ranking and the sorting of stocks in portfolio selection. Secondly, ELECTRE TRI with the notion of incomparability brings an important information to the portfolio manager, especially when the evaluation of stocks appears difficult. Thirdly, ADELAIS and MINORA systems provide a considerable aid to the portfolio manager to understand his preferences and his own evaluation criteria, in order to construct his own model of portfolio selection in an interactive way. This portfolio selection model is without any normative consideration proposed by the classical portfolio theory.

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Interfirm Performance Evaluation under Uncertainty: A Multi-dimensional Framework

Jaap Spronk[1] and **Erik M. Vermeulen**[2]

[1] Department of Finance, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR, Rotterdam, The Netherlands.

[2] Department of Finance, Erasmus University Rotterdam, and EIM Small Business Research and Consultancy, P.O.Box 7001, 2701 AA, Zoetermeer, The Netherlands.

Abstract: In this paper a multidimensional framework for performance evaluation is proposed, which is elaborated and illustrated through a real-life case. A particular feature of the approach is that it takes account of the influence of risks, which are beyond the control of the decision maker. Here risk is seen as a multidimensional measure; it is expressed by means of sensitivities to unexpected changes of a multitude of risk factors. Furthermore, the approach corrects for differences in operational characteristics between the firms that are being compared. Some operational characteristics are fixed and thus uncontrollable for the decision maker, whereas other operational characteristics may be changed and controlled by the decision maker. The approach also considers the fact that it might have been better to change some of the controllable firm characteristics.

1. Introduction

In this paper a multidimensional framework for performance evaluation is proposed, which is elaborated and illustrated through a real-life case of interfirm comparison. A particular feature of the approach is that it takes account (and corrects) for the influence of external sources of risk which are beyond the control of the decision maker. For this purpose the multi-factor method is used. This method explains unexpected performance by means of sensitivities to unexpected changes of risk factors (cf. Hallerbach (1994), Goedhart (1994), Vermeulen (1994), Van Aalst (1995) and LoCascio and Spronk(1992)). By modelling risk as a multi-dimensional measure, any decision problem involving risk becomes automatically a multi-criteria decision problem.

Secondly, the approach takes account and corrects for differences in operational characteristics between the firms that are being compared. Finally, the method takes account of the fact that the decision maker may

be able to change some of the operational firm characteristics. In other words, it also checks whether the decision maker has made the "right" decision in changing the characteristics. The framework for performance evaluation is outlined in the next section.

2. Performance evaluation: outline of a framework

An important question is how the future (ex post) performance of an activity can be improved. In other words, how to estimate the effects on performance when making changes in an activity? For instance, how can we get an idea of what will happen if we would provide a soccer player with a better training or if we would change his position within the playing field? When considering a change in activity, one may compare the unchanged activity on the one hand and the 'manipulated' activity on the other as mutually exclusive decision alternatives, the better of which has to be selected.

Generally, when the decision which activity to choose is being made, it is still uncertain what ex post performance will result from each of the activities. Depending on the situation, more or less information on the future (ex post) performance is available. Between the extremes of complete ignorance and full information about future results there is among others the case that a well defined probability distribution over the future performance can be defined. Very often the information available is not sufficient to construct such a probability distribution. An example is the effect of the planned marketing efforts in a given year. The marketing manager may have expectations on the future sales volume and even have some idea about the variability of sales, but that is not yet a well-defined probability distribution. In our framework, we adopt the view (cf. Vermeulen et al. (1993)) that part of the uncertainty is caused by a number of external forces which cannot be influenced by the decision maker. Examples are the price of energy, the exchange rate, the wage rate and the business cycle.

Furthermore, it is assumed that the sensitivity of the performance of the activity at hand for changes in the external factors are fairly stable and can be measured (or reliably estimated). It can be argued that such a sensitivity is a firm characteristic which is a measure of the risk the firm runs with respect to the source of risk considered. For example, a transportation firm with a low energy price sensitivity in comparison with another transportation firm runs less 'energy price risk'.[1]

The framework for performance evaluation of a firm presented in this paper takes account of the fact that part of the result is due to external factors and thus beyond the control of the firm.

Another element of the framework is that it tries to take account of differences in operational characteristics between firms. Consider for

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[1] Instruments that the firm can use to change its sensitivities are discussed in Vermeulen et al. (1996).

instance the comparison between a **hotel without many facilities and a *****hotel with seven restaurants, conference and golf facilities: a straightforward comparison of financial ratios and risk profiles may easily lead to false conclusions! Following Van der Wijst (1990), our interfirm performance framework corrects for differences in operational characteristics between firms.

Finally, in the framework we explicitly take account of the fact that management can manipulate some of the operational characteristics. As an example, take a baker that can change its product range. In evaluating ex post performance, we should consider the possibility that given the information the baker had at the beginning of the period to be evaluated, it would have been better to manipulate the operational characteristics and, for instance, to change the product range. As regards the sensitivities this implies that management may consider a reduction in the sensitivity to those risk factors that are expected to become extremely unstable.

In the framework used to evaluate ex post performance we use a one period model in which I_0 denotes the set of information available at t_0, I_1 stands for the information available at t_1 and D_0 represents decisions made at t_0. The objective of the decision maker is to maximize ex post performance at t_1. At t_0 the information set "I_0" is available about the expected course of the risk factors and their stochastic behavior, e.g. their variance. Based on this information, decisions "D_0" have to be made whether or not to change the firm characteristics, such as the firm's debt level and its product range. At t_1 new information "I_1" is available concerning realized performance and the realized values of the uncontrollable risk factors. This information is needed to evaluate ex post performance.

A distinction is made between fixed and changeable firm characteristics. Firm characteristics that are fixed cannot be changed in the one period model presented here in contrast to the changeable firm characteristics[2]. The firm characteristics determine both expected performance, E(P), and unexpected performance, $\tilde{P}$ - E(P). Furthermore, expected performance depends on the expected factor values, $E(\tilde{f})$. Unexpected performance is modelled by means of sensitivities, "b", to unexpected changes of risk factors, "$\Delta \tilde{f}$". The sensitivity caused by the fixed firm characteristics is denoted as "b_f", and the sensitivity caused by the changeable firm characteristics as "b_c". The total sensitivity is denoted as "b_t". As stated, the vector of sensitivities is called the risk profile and can be seen as a multi-dimensional risk measure. When risk is modelled

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[2] Some of the changeable firm characteristics will be contingent. The decision maker wants to and can change these characteristics at t_1 given the realized value of the risk factors. As an example take an abandonment option that enables management to sell, for instance, a factory at a predetermined price if the factory's performance appears to be weak.

in this multi-dimensional way, it is possible to take account of the influence of all kinds of risk factors.

A change in firm characteristics may affect both expected and unexpected performance. For instance, an investment in a more efficient machine - i.e. a machine that uses less input in order to obtain the same level of output - will lower the firm's expected costs and thus increase expected cash flows. It also influences unexpected performance, since a more efficient machine is most probably less sensitive to unexpected changes of risk factors - such as the prices of the input factors - as well.

3. Existing approaches

In this section we shall describe some existing approaches that can be used to determine ex post performance.

Simple performance evaluation

One of the easiest ways to evaluate performance consists of comparing expected performance by realized performance. Management is hold responsible for the difference:

$$P_r = P^* - E(P) \qquad (1)$$

where P^* is realized performance, and E(P) stands for expected performance.

Here, expected performance "E(P)" is based on the average performance of similar firms over a number of years. To give an example, take return on investment as a performance measure. In that case a bakery's performance would be measured by comparing its return on investment with the average return on investment over a sample of bakeries over a number of years.

This way to evaluate performance has two shortcomings. Firstly, performance is evaluated without taking account of the firm's characteristics. In other words, all bakeries are expected to perform similarly and independent of their size and product range and so on.

Secondly, management is implicitly held responsible for performance changes that are beyond its control. For example, management is also hold responsible for bad results that can be attributed to a downturn in the business cycle.

The first drawback is solved by means of an interfirm comparison technique developed by Van der Wijst (1990). A solution to the second drawback is provided by Barnea et al. (1975).

Interfirm comparison, Van der Wijst (1990)

Van der Wijst (1990) suggested to take account of the firm's characteristics when evaluating performance. The corresponding measure for over/ underperformance is

$$P_r = P^* - E(P \mid fc) \qquad (2)$$

with E(P | fc) the expected performance based on the firm's characteristics. Thus, performance is compared with the expected level of performance given the firm's characteristics. Referring to the bakery case, the bakery's return on investment would be compared with that of a comparable bakery with similar characteristics.[3]

Conditional performance review and the multi-factor method

Barnea et al. (1975) argue that management should not be held responsible for changes in performance caused by unexpected changes of risk factors that are beyond control of management. The corresponding measure for over/underperformance is:

$$P_r = P^* - (E(P) + \sum_{i=1}^{k} b_{ti}\, \Delta f^*_i) \qquad (3)$$

where b_{ti} stands for the sensitivity to unexpected changes of risk factor i, and Δf^*_i is the realized unexpected change of risk factor i.

4. Interfirm conditional performance review

In comparison with simple performance measurement, the strength of Van der Wijst's method is that it also takes account of the firm's characteristics, and the strength of Barnea's method is that it considers the effects of uncontrollable risk factors.

Our new approach, which is called ICPR: interfirm conditional performance review, combines the best of the methods developed by Van der Wijst and Barnea et al.: it considers both the firm's characteristics and the realized values of external risk factors when evaluating ex post performance.

Moreover, we extend the approach by making a distinction between fixed and changeable firm characteristics. It is argued that given the information at time t_0, good management changes the changeable characteristics such that either expected performance is increased or the risk of unexpected performance is decreased.

The corresponding measure for over/underperformance is as follows:

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[3] Of course, it is hard to find a large sample of bakeries with similar characteristics. Van der Wijst solved this problem by means of regression analysis. First the relation between performance and the firm's characteristics is determined. Next, this relation is used in combination with the specific characteristics of the firm under investigation in order to obtain its own taylor-made expected performance.

$$P_r = P^* - (E(P \mid fc_f, fc_c) + \sum_{i=1}^{k} E(b_{ti} \mid fc_f, fc_c) * \Delta f^*_i) \qquad (4)$$

where fc_f are the fixed firm characteristics, fc_c are the changeable firm characteristics, and $E(P \mid fc_f, fc_c)$ is the expected performance given the fixed firm characteristics and the fact that some firm characteristics could have been changed, $E(b_{ti} \mid fc_f, fc_c)$ is the expected sensitivity given the fixed firm characteristics and the fact that some firm characteristics could have been changed, and as before Δf^*_i stands for the realized unexpected change of risk factor i.

This measure takes account of all relations included in the framework described in Section 2. Both expected and unexpected performance are considered and related to firm characteristics. The firm characteristics are split into fixed and changeable characteristics.

5. Putting the model into practice

It is seen that the interfirm conditional performance review presented in sectiont4 makes use of the relation between firm characteristics on the one hand and expected performance and sensitivities to unexpected changes of risk factors on the other. Regression analysis can be used to quantify these relations. In this section this will be illustrated by the relation between firm characteristics and sensitivities.

Using data concerning 67 bakery shops the influence of the product range on the business cycle sensitivity has been estimated by means of regression analysis. The parameter estimates and sales levels are shown in Table 1. For the purpose of illustration, we restrict ourselves to one risk factor. Extension to more than one risk factor is straightforward.

Table 1: Product range, business cycle sensitivity and sales levels

	Business cycle sensitivity	Sales
Own make bread	0.0060	657 000
Own make pastry	-0.0006	408 000
Resale articles	-0.0071	362 000

It is seen that own make bread has a positive influence on the business cycle sensitivity, whereas the influence of own make pastry and resale articles is negative. Using these parameter estimates the business cycle sensitivity of abakery can be computed, if the sales levels of its various products are known.

The change in the business cycle from time t to time t+1 is -14. The bakery's expected cash flow was Dfl. 600 000.- and its realized cash flow only Dfl. 540 000.- (Dfl. 1.- is about $ 0.60). Its total sensitivity to the business cycle is Dfl. 5049.-[4]. Implying that a decrease of one point in the business cycle leads to a decrease in cash flow of Dfl. 5049.-. With this figures we can evaluate performance while using the three performance measures discussed before[5]. In the remainder of this paper the amounts will be expressed in thousands of Dutch guilders.

Simple performance evaluation

According to this measure management has performed badly: $R_r = 540 - 600 = -60$

Conditional performance review

In this case we take account of the fact that the firm's total sensitivity is $ 5049.- and that the business cycle decreased by 14. The corresponding (over)performance is: $R_r = 540 - (600 + 5.049*-14) = 10.7$. This time, the conclusion is that management performed 10.7 above expectation and has done considerably well.

Interfirm conditional performance review

The question remains whether the sensitivity was justified by the firm's characteristics. The expected sensitivity given the firm's characteristics can be computed from Tables 1. Multiplication renders a sensitivity to the business cycle of ((0.0060*657) + (-0.00060*408) + (-0.0071*362) =) 3.441, and (under)performance is: $R_r = 540 - (600 + 3.441*-14) = -11.9$. When both the unexpected change in risk factor, and the deviation of the sensitivity from its expected sensitivity is taken into account, at first glance performance of management would be evaluated as being poor. However, to evaluate management properly, a further analysis is needed.

The part of the sensitivity that cannot be explained by the firm characteristics equals 5.049-3.441 = 1.608. Let us assume that the contribution of 3.441 to the sensitivity is caused by fixed firm characteristics and that the contribution of 1.608 is the result of changeable firm characteristics caused by decisions of management at t0. It is seen that in this case increasing the sensitivity by changing the changeable firm characteristics appeared to be disadvantageous for the firm, since the business cycle dropped unexpectedly.

However, without more information it cannot be said whether the decision to increase the sensitivity was wrong. After all, if management had good reason to believe that the risk factor was going to be less risky at t1 then increasing the sensitivity may be a good decision.

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[4] Because of lack of space, the way to compute this number is not presented here. The interested reader is referred to Vermeulen (1994), page 130 *ff.*

[5] In this illustration the method of Van der Wijst is left out of consideration.

6. Graphical representation

In Figure 1 the three performance measures are illustrated. The vertical axis denotes the firm's performance. Point A refers to the level of expected performance, which is 600. Point C denotes realized performance which is 540. Using simple performance evaluation, realized performance (Point C) is compared to expected performance (Point A) and the difference is -60.

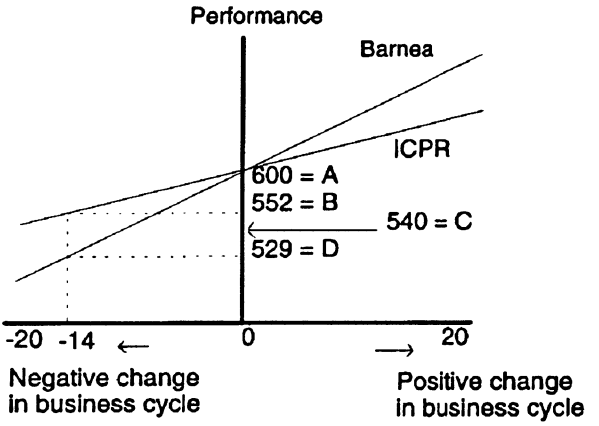

Fig. 1: Conditional performance measurement

Barnea's conditional performance review takes account of both the sensitivity to unexpected factor changes and of these changes themselves. The horizontal axis of Figure 1 refers to the realized unexpected changes of the business cycle. The line indicated "Barnea" relates realized unexpected changes in the business cycle to the level of performance that corresponds with this change. In accordance with the firm's actual sensitivity the slope of the line is 5.049. Using Barnea's line it can be seen which level of performance would be expected given the realized change of the business cycle.

Given the realized unexpected decrease of the business cycle which is -14, the expected level of performance equals 529, which is indicated by Point D.

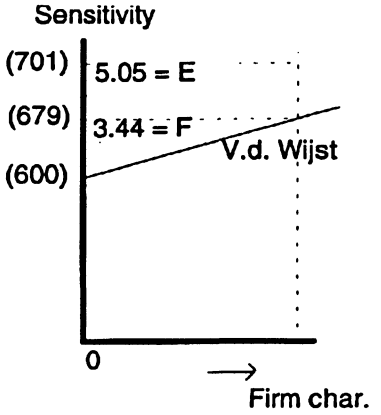

Fig. 2: Interfirm comparison

Comparing realized performance, Point C, to this expected level, management is evaluated as performing well. However, we advocate that performance evaluation should firstly be based on the sensitivity that would be expected given the fixed firm characteristics. Figure 2 illustrates this point. The horizontal axis refers to the fixed firm characteristics and the vertical right axis to the corresponding sensitivity. The sloping line in the figure (called V.d. Wijst) indicates what the expected level of the sensitivity is, given the fixed characteristics of the firm. The vertical difference between the line and the realized sensitivity denotes the level of the oversensitivity.[6]

The firm under consideration has a sensitivity of Dfl. 5.049/one point change in the business cycle, Point E[7]. However, it is seen that given its fixed firm characteristics a sensitivity of only Dfl. 3.441/one point change in the business cycle would be expected, Point F. Management is responsible for the difference in expected and realized sensitivity and of possible consequences thereof.

The consequences are that instead of using the Barnea-line to correct for unexpected changes in the business cycle, we have to use the ICPR-line in Figure 1. In that case we compare realized performance Point C with expected performance given the fixed firm characteristics Point B and see that the decision to change the firm characteristics, which led to a higher business cycle sensitivity, appeared to be disadvantageous. As stated before, without further information it is not possible to say whether this decision was good or bad.

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[6] In Van der Wijst's analysis the vertical axis denotes performance and the horizontal axis the firm characteristics.

[7] A sensitivity of 5.049/one point change in the business cycle, accords with 101/20 points change in the business cycle. As a result, the corresponding level on the right vertical axis is: expected performance + 20*sensitivity = 600 + 101 = 701.

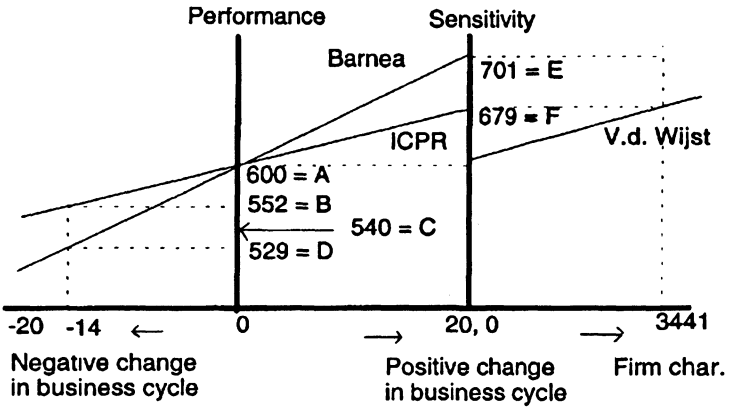

Fig. 3: Interfirm conditional performance review

Finally, Figure 3 presents the three performance measures in one graph. The left part of the graph refers to the multi-factor part that corrects required performance for performance changes caused by unexpected changes in risk factors beyond control of management. The right part of the graph presents the interfirm comparison part that takes account of the firm's characteristics when evaluating performance.

7. Conclusion

In this paper we presented and illustrated a multi-dimensional framework for interfirm performance evaluation under uncertainty. Uncertainty is modelled by means of the so-called multi-factor method, which relates unexpected performance changes through sensitivities to unexpected changes of risk factors. In this way, a multi-dimensional view on risks is developed, which makes it possible to take account of uncontrollable risk factors. The proposed framework leads to a tailor-made evaluation since it takes account of the differences between the characteristics of different firms including their respective risk sensitivities characteristics.

Apart from the simple illustration in this paper different elements of the framework have been tested and implemented in real situations. These include applications to retail industries, airlines and banks.

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Scenario Analysis and Multicriteria Decision Making

Theodor J. Stewart
Department of Statistical Sciences
University of Cape Town
Rondebosch 7700
South Africa

Abstract The use of scenarios as internally consistent representations of possible futures is being widely used for strategic planning, in preference to attempting to forecast the future in detail. This has two implications for the application of MCDM to strategic planning problems in large corporations or in the public sector. Firstly, "policy scenarios" may be used as a representation of possible policy directions, but which are neither complete nor exhaustive. Here the challenges to MCDM relate to the selection of good scenarios, and to the provision of meaningful methods of comparisons between these scenarios. Secondly, future uncertainties will not be expressed as probability distributions, but rather as a small number of "state scenarios" (again neither exhaustive nor complete). We discuss reasons why such scenarios should be treated as part of the objectives' hierarchy, rather than to try to collapse across them by taking expectations, for example. This, too, has implications for choice of MCDM procedures and for subsequent sensitivity studies.

1 Introduction

The concept of *scenario planning* (see, for example, Ascher and Overholt [1]) is being widely adopted as an important tool in strategic planning and analysis. Under conditions of uncertainty and imprecision, the idea of "forecasting" future trends becomes meaningless, and in response to this the aim becomes to identify policies or courses of action which are robust in the sense of providing "satisfactory" performance under all scenarios. The scenarios themselves are meant to represent internally consistent possible future trends, and are chosen so as to provide a sufficiently wide coverage, in some sense, of future uncertainties.

If MCDM is to contribute to strategic planning, then the tools of MCDM need to be appropriate for use in this scenario planning mode. There are a number of issues before this can become a practical reality, however. The

situations which give rise to the need for scenario planning tend not only to be characterized by imprecise visions of future exogenous states, but by equally imprecise visions of potential policies and how they will ultimately be implemented. For example, in river basin planning in the Eastern Transvaal region of South Arica, one policy may be to reduce the intensity of exotic commercial forestry to some stated degree. But it is almost impossible to state at the outset precisely where the reductions might occur, how the areas will be cleared, what changes in species mix might occur, how well compliance will be monitored, etc. Decisions will have to made on the basis of some realistic, and internally consistent assumptions regarding these details, perhaps with key uncertainties represented in terms of "sub-alternatives", but with many of these details simply left unspecified on the basis that the they represent a second-order effect when expressing preferences between broad policy directions. In this sense we are dealing with *policy scenarios* in much the same way as we deal with exogenous *state scenarios*, such as political and socio-economic developments.

The set of policy scenarios can never be fully identified, and in fact only a very small number of policy scenarios are likely to be taken into consideration at any one time. This is a characteristic of state scenarios also. From the perspective of decision analysis, therefore, the aim is not simply to assist the decision makers in constructing preferences across a given list of alternatives, or to support the process of selecting one item from such a list. Decision analysis must, if it is to be useful, contribute to the process of constructing and identifying policy options (scenarios) in parallel with developing a preference structure. This might be achieved, we argue, by establishing initially a preference structure on a small set of policy scenarios, using this to establish some partial ranking and the elimination of some potential policy directions. This information may then aid in refining the remaining scenarios and/or in seeking or creating new compromise scenarios. This approach has been elaborated in references [4] and [5].

Formally, let $\mathbf{X}$ represent the set of all policy options which exist in principle and Z a (possibly multi-dimensional) random variable representing the unknown state of the world. For any element $x \in \mathbf{X}$, and any realization of z of Z, let $g_k(x, z)$ be the consequences for the decision maker in terms of *attribute* k (for $k = 1, 2, \ldots, p$). For this paper we shall assume that choice of x has no influence on the exogenous state random variable Z, and that attributes map one-to-one on to the criteria relevant to the decision maker, in such a way that maximization of each attribute is desirable, all else being equal. The available policies and possible states are represented by the finite sets of policy and state scenarios, say $\Xi = \{\xi_1, \xi_2, \ldots, \xi_m\}$ and $\Psi = \{\psi_1, \psi_2, \ldots, \psi_n\}$ respectively. A particular policy scenario ξ_i cannot in general be associated with a specific element of $\mathbf{X}$, but would be representative of a small region of $\mathbf{X}$. Similarly, a specific state scenario ψ_j will not simply be a realization of Z, but will represent outcomes within a subset of the sample space of Z. Nevertheless, it would still be usual to model consequences in terms of at-

tribute k as a function $\gamma_k(\xi, \psi)$ of the state and policy scenarios; but the form of model required may be strongly dependent upon the degree of refinement of definition of the scenarios, and may be quite different in structure to the hypothesized $g_k(.)$ functions. The imprecision of these relationship between the scenarios and the underlying unseen reality raises questions about the validity of many of the underlying assumptions or axioms of MCDM, decision analysis, etc. It is these questions, and their implications for the principles and practice of MCDM that form the topic of this paper.

2 Modelling uncertainty by state scenarios

State scenarios represent the range of uncertainties in the exogenous world which may impact upon the consequences of the action(s) taken. The characterization of such scenarios is the basis of conventional scenario planning. Each scenario ψ_j is an internally consistent, but generally incompletely defined picture of how the future, in so far as it affects the outcomes of policy decisions, will evolve. For any given policy (scenario) ξ_i, the consequences for each state scenario ψ_j are assumed to be given by $\gamma_k(\xi_i, \psi_j)$ for each attribute. But this does not allow us directly to compare the policies in terms of even the single attribute k, as no means of aggregation across state scenarios is yet specified. An initial reaction may be to generate some type of expectation of the form $\sum_{j=1}^{n} p_j \gamma_k(\xi_i, \psi_j)$, where p_j is a "probability" associated with state scenario ψ_j. But since the set of state scenarios is not a complete sample space, it is not clear how these probabilities are to be defined. The intuitive reaction may be to use some form of "relative likelihoods", standardized to sum to unity. These relative likelihoods presumably refer to regions of the sample space of Z which are represented by the scenario ψ_j, but this is problematical in that the regions are not well defined.

The problems of establishing probabilities to be associated with each scenario are evident even in the simplest possible case, *viz.* when Z is a univariate random variable with a known probability distribution, and each "scenario" is simply a specific possible value for Z. Keefer and Bodily [2] show that if Z has probability density $f(z)$, and $g(z)$ is a monotone function of z, then the expectation of $g(Z)$ can be very well approximated by the sum of three terms: $0.185g(z_{0.05}) + 0.63g(z_{0.5}) + 0.185g(z_{0.95})$, where z_α represents the α-th quantile of the distribution of Z. The weights are the same, irrespective of the shape of the distribution, and thus relative probability densities give little or no clue as to what the weights should be. In any case, if Z has a normal distribution for example, the densities at the 5th and 95th percentiles, relative to that at the median is approximately 0.067, which is far from 0.185/0.63=0.286. There appear thus to be substantial dangers in assessing scenario "probabilities", and using these to aggregate performances of different policy alternatives across scenarios.

For the multiattribute utility theory approach, the Keefer and Bodily re-

sults suggest that it may be possible to obtain a satsfactory approximation to the expected utility of a specific element $x \in \mathbf{X}$ by the following:

$$\mathrm{E}[U(g_1(x,Z),\ldots,g_p(x,Z))] \approx \sum_{j=1}^{n} p_j U(g_1(x,\psi_j),\ldots,g_p(x,\psi_j))$$

If we can further approximate the utility function $U(.)$ in additive form (which, as we have argued in [3], is often a well-justifiable approximation), then the above simplifies further to:

$$\mathrm{E}[U(g_1(x,Z),\ldots,g_p(x,Z))] \approx \sum_{j=1}^{n}\sum_{k=1}^{p} p_j w_k u_k(g_k(x,\psi_j))$$

where the $u_k(.)$ functions are the marginal utility functions, standardized to a convenient range. This constitutes an additive value function over the $n \times p$ attributes, representing performance in terms of each of the p original attributes under the assumption of each exogenous state scenario. If we re-standardized the $u_k(.)$ functions separately for each state scenario (which might facilitate elicitation in some contexts), then the weight for (attribute k)$\times$(scenario j) would be $p_j w_k$ multiplied by the relative range for attribute k under scenario j, expressed in utility terms, across all alternative policies. Since, as previously noted, even the "probabilities" p_j do not have a natural interpretation, these weights may be difficult to assess. Nevertheless, we speculate that decision makers might yet have an intuitive feel as to what weight needs to be attributed to performance in terms of a particular attribute given a particular scenario. In eliciting such weights, however, the analyst must avoid phrasing questions in terms of how likely the scenario may be; questions should rather direct attention to the importance of seeking good performance under the assumption of the particular scenario.

In MCDM approaches which are not utility-based, it seems that we need even more to view performance under each scenario, for each attribute of interest, as separate criteria, giving an extended MCDM problem with $n \times p$ criteria (or, indeed, giving an MCDM problem even when there is only one true criterion). Certainly, the replacement of the uncertainty in any attribute by a single aggregate measure (such as an expectation) would be most inappropriate in many situations, as this would tend in general to misrepresent risk preferences. The appropriateness of different MCDM methods for aiding the solution of such super-multicriteria problems requires further consideration and research. Before entering on any discussion, however, let us first look at the analytical implications of the use of policy scenarios.

3 Effects of assessing policy scenarios

The selection of policy scenarios needs to be viewed differently to the identification of "alternatives". Policy scenarios need to characterise a rich range

of available policy options, but are not an exhaustive list of such options. In [4] we have suggested systematic procedures for selecting an appropriately representative set of policy scenarios. This is not the central theme of the present paper, but it is worth noting the two-stage nature of the proposed procedures, as follows:

1. Use of statistical experimental design ideas to select out a "background set" of scenarios on which detailed information can be collected, in such a way that statistical models of responses can be used to interpolate between the selected scenarios with maximum precision;

2. Use of a combined weighted Archimedean and Tchebycheff norm to identify those policy scenarios in the background set which are most frequently closest to ideal, when weights are randomly generated (subject to a restriction that all scenarios need to be sufficiently different).

Apart from the fact that the policy scenarios do not represent an exhaustive set of alternatives, each policy scenario is also not a complete definition of every detail of policy implementation. The question thus arises: even in the absence of any uncertainty regarding outcomes, can we meaningfully talk of a preference structure on the set of policy elements? If someone states that "scenario A is preferred to scenario B", to what extent is this preference dependent upon assumptions regarding the undefined features of the scenarios?

Our assumption when defining the concept of policy scenarios was that the omitted, or arbitrarily specified details, were "second-order" in not materially influencing preferences between policy alternatives. It has long been a central tenet of the "European" school of MCDM that the comparison of two alternatives should allow not only preference or indifference statements, but also incomparability (*cf.* Vincke [6]). This incomparability seems to have been viewed primarily as a consequence either of incompletely formed preferences, or of alternatives which differ substantially on so many criteria at a time that information overload occurs. But in scenario planning, incomparability between two scenarios can occur at a more fundamental level, if the differences between them depend on unresolved details. Incomparability of this nature can arise for either of two reasons, *viz.* (i) the supposed "second-order" details are in fact of substantial importance; or (ii) the incomparability is an indication of near-indifference. If incomparability issues arise while attempting to assess policy scenarios from the point of view of a particular criterion, it is essential that the reason be established. In the first case, it would be necessary to determine what additional policy issues need to be incorporated in the scenario statements, before meaningful comparisons can be made. The latter reason is no cause for concern, however; it simply states that at the present stage of evaluation (see comments below on the iterative nature of the process), these two policy scenarios cannot be distinguished in terms of this criterion.

Apart from the problem of incomparability, there seems to be no good reason why transitivity of strict preference should not remain a reasonable

normative principle. Thus we would still expect preferences between policy scenarios to constitute a semi-order. The concept of preferential independence between criteria (which is necessary for some forms of preference elicitation, and for some forms of preference models) may create problems, not because of the use of policy scenarios *per se*, but because the criteria in strategic planning problems are so often linked to the interests of different parties. The resulting politics may make the desirability of certain trade-offs between the interests of two parties dependent upon how well another party is perceived to be doing. The extent of this problem, and whether it can be ameliorated by re-definition of criteria is difficult to judge in general terms. It is worth commenting, however, that in work we have carried out on assessing regional water resource development policies [4], different groups did seem to be perfectly content to arrange policy scenarios along a value scale, without questioning effects on other interest groups, suggesting that additive difference independence (stronger than preferential independence) was in effect accepted as a desirable normative principle.

The application of decision analysis to policy scenarios in this sense, rather than to completely defined alternatives, must be iterative in nature. The results of one round of analysis will be (depending on the MCDM method used) a partial or complete rank ordering of the policy scenarios *currently under consideration*. But the analysis will not end there. The one or more policy scenarios selected will generally form the basis for development of new scenarios by combining the better features of the selected scenarios and/or the refinement of the selected scenarios by resolving previously unspecified details in a number of alternative ways. The whole process of evaluation according to the different interests and criteria will then be repeated, until such time as sufficient consensus emerges. An important implication of this iterative process is that a strict preference order is not in any way necessary, particularly during early iterations, as long as there can be some classification of policy scenarios into more or less desirable options.

4 Implications for the practice of MCDM

Let us consider first the implications of the scenario planning approach for utility based MCDM procedures. In principle, the aim is to find the option $x \in \mathbf{X}$ which maximizes the expected utility, and we have argued earlier that there is little loss in practice in using an additive approximation of the form: $\sum_{j=1}^{n} \sum_{k=1}^{p} p_j w_k u_k(g_k(x, \psi_j))$. It should be emphasized that one of the key results coming from [3] is that the most sensitive part of this approximation is neither departure from additive independence of attributes nor the assessment of weights, but the correct representation of the form of the marginal functions $u_k(x, \psi)$. In particular, reduction to a p-criterion problem defined on expectations of attributes, *i.e.* a value function of the form $\sum_{k=1}^{p} w_k \mathrm{E}[g_k(x, Z)]$, with or without approximation of the expectation by $\sum_{j=1}^{n} p_j g_k(x, \psi_j)$, leads

to extremely poor results.

When also introducing the concept of policy scenarios, we need further to replace the true policy option x by a policy scenario ξ. If the impacts $\gamma_k(\xi, \psi)$ can be assessed with sufficient precision, we might simply attempt to order the policy scenarios on the basis of a simple additive score of the form $\sum_{j=1}^{n} \sum_{k=1}^{p} \lambda_{jk} v_{jk}(\xi)$, where λ_{jk} represents the relative joint importance of criterion k and scenario j (i.e. the importance of the range of consequences for criterion k under the assumption of scenario j, relative to that for other criteria and/or scenarios), and $v_{jk}(\xi)$ the marginal "value" of policy scenario ξ (relative to all other policy scenarios, expressed on a scale satisfying additive difference independence) in terms of criterion k under the assumption of state scenario j. The assumption of an additive model raises a number of questions:

1. Is it better to assess importance weights separately for criteria and for state scenarios, and then to obtain λ_{jk} by multiplication; or to assess the joint weights directly? (Recall again the need to phrase elicitation questions in importance rather than in likelihood terms.)

2. Should separate value scales be used for each criterion-state scenario combination, or should one scale be used for each criterion, giving evaluations of all outcomes (over all state and policy scenarios) on this one scale?

3. Even though the original criteria satisfy preferential independence, is the implied assumption of preferential independence between all criterion-state scenario pairs still valid?

Given the general robustness of additive value function models, we conjecture that the answers to the above questions should be based primarily on what is easiest for the decision maker, and (for the second question) on what option leads to the most consistent interval scale of strength of preference.

As indicated earlier, however, the main impact of the use of policy scenarios is the introduction of some increased degree of incomparability between policy scenarios, which can be viewed in terms of uncertainty or imprecision in the mind of the decision maker(s) as to the precise implications of scenario ξ for each attribute; in other words the consequences $\gamma_k(\xi, \psi)$ might need to be associated with some measure of imprecision. In practical terms, it is difficult to conceive of anything much more sophisticated than a simple interval estimate, i.e. simple lower and upper bounds $\gamma_k^L(\xi, \psi)$ and $\gamma_k^U(\xi, \psi)$. The implications of this for utility-based approaches would be that the v_{jk} scores for each policy scenario might be expressed as an interval rather than as a point, and the question arises as to how then to aggregate these intervals. It would, of course, be quite simple to assess the corresponding maximum and minimum values of $\sum_{j=1}^{n} \sum_{k=1}^{p} \lambda_{jk} v_{jk}(\xi)$ for each policy scenario ξ. This might give an unrealistically wide band of uncertainty in the scores of each policy scenario, however. A Bayesian view of such uncertainty in the mind of the decision

maker would be to represent it in terms of a probability distribution (possibly uniform or triangular) over the stated intervals. If these uncertainties were approximately independent, then the resultant distribution of the weighted sum is in principle easily obtained (probably by Monte Carlo methods). The end result would be a need to order distributions, but we recall that we are not seeking an absolute "optimum".

There is a further aspect of scenario-based, strategic level MCDM which needs comment. At the first level in the objectives hierarchy, at least, each objective tends to be linked to some specific interest group. Furthermore the assessments by each group of necessity must at least partially include subjective evaluations; no single decision maker assesses the policiy scenarios against all criteria. It is essential therefore that the assessments in terms of each interest be capable of clear and unambiguous communication between groups. This has important implications for the selection of MCDM technique. Perhaps the simplest unambiguous assessment is a rank ordering (or partial ordering). Since an ordinal scale should not be interpreted in a cardinal sense, any methods based on trade-offs would be inappropriate, which rules out both utility and goal programming methods. The non-compensatory property of outranking methods (ELECTRE) do seem potentially more appropriate, but even there the concept of discordance needs to be carefully thought out: is there any basis for defining discordance purely in terms of rank differences? The extension of a pure rank order to a value scale introduces other difficulties, in ensuring that the origin and scaling used for each interest are understandable and comparable. One way is to fix origin and scale by assigning values (e.g. 0 and 100) to the worst and best ranking policies, in terms of the criteria under consideration, after which the other options can be slotted in with due consideration to the relative "gaps" between alternatives. This seems to more-or-less force an interval scale, in which case simple additive aggregations are both mathematically justifiable and easy to understand. It is for this reason that we have tended to focus on the additive value function approach, and have certainly found in work on water resource planning in South Africa that such scales do seem to be clearly understood between groups. It is not evident what advantages there are in adopting any other approaches in this context.

The above concepts have relevance for non-utility based approaches as well. The idea of viewing performance under the assumption of each state scenario as one level in an objectives hierarchy, is equally applicable to other MCDM methods (e.g. outranking methods, goal programming and reference point methds). The same questions arise as before as to how to assess weights on these "super-criteria": separately for scenarios and criteria, or jointly? In addition, it might make a difference whether the hierarchy of objectives splits first on scenarios at the highest level, and then on criteria within each scenario, or *vice versa*. This opens avenues for further research.

One important principle does seem to have applicability to non-utility based methods. We have commented on the fact that the taking of ex-

pectations of inappropriately scaled functions of the valuations on each criterion (e.g. expectations of raw evaluations, rather than of von Neumann-Morgenstern utility functions of these) can result in highly misleading results. Although we have not examined the implications for other MCDM methods, there does seem to be *prima facie* evidence at least that collapsing across state scenarios by taking expectations should preferably be avoided.

5 Future work

This paper has attempted to open questions for discussion, rather than to offer final solutions to what is an extremely complex class of problems, *viz.* those involving choice at a strategic planning level between "policy scenarios", when uncertainties are represented by "state scenarios", and where neither set of scenarios is completely or exhaustively defined. A key proposal has been that in treating uncertainty in MCDM in the scenario planning context, one should not aggregate directly across scenarios. One should rather treat performance under various state scenarios as part of the objectives hierarchy, and we have suggested important research questions which derive directly from this approach.

A further avenue of research relates to the manner in which scenarios are defined. Proposals regarding the construction of policy scenarios has been made in [4] and [5], but these do need further research. Criteria for selecting suitable state scenarios are still rather undefined, although there are some hints from the work of Keefer and Bodily [2] that an attempt should be made to capture specific quantiles of the underlying distributions which are not too extreme (e.g. 5% or 10% quantiles), at least in a rough qualitative way. It is not immediately evident, however, how this might be put into practice in multivariate situations where the distributions are not well-understood.

Many questions remain, therefore, but MCDM needs to address these and related questions if it is to make any contribution to strategic planning – but, at the same time, strategic planning is precisely the area in which conflicts of objectives are most evident.

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A Multicriteria Approach for Evaluating Efficiency and Appropriateness of Industries: A Case in China

Mario T. Tabucanon[1] and Yu Zhijing[2]

[1]Industrial Systems Program
School of Advanced Technologies
Asian Institute of Technology
G.P.O. Box 2754, Bangkok 10501, Thailand

Abstract: This paper proposes a multiple criteria decision making approach to investigate the relative strengths and weaknesses of industrial zones and to analyze the appropriateness of industries for promotion. The approach is two-tiered. Firstly, it examines the industrial sector using Data Envelopment Analysis (DEA), assessing the efficiency of the decision-making units (special economic zones) with respect to certain important factors of development. For those so-called "inefficient" units, DEA is used to identify comparable "efficient" units and then set performance targets for the former vis-a-vis the latter. The findings obtained from DEA on the appropriateness of criteria are then used for developing an Analytic Hierarchy Process (AHP) model to determine the type(s) of industries suited for the conditions of the region for the improvement of the industrial sector's efficiency. The proposed approach was applied to the case of selected special economic zones in the Southeastern part of China.

1. INTRODUCTION

Industrial development is a complex process that varies from country to country, depending upon their social structures and goals, population patterns, natural resources' endowment and capital formation to mention some. Experience has shown that there is a direct correlation between economic development and the level of industrialization of a country; an increase in the level of per capita income is usually indicative of an economic structural change involving declining share of agriculture and rising proportion of manufacturing output. Against the backdrop of the importance of the industrial sector is the fundamental imperative of prioritizing industries in certain economic zones and evaluating their strengths and weaknesses with respect to certain performance criteria.

2. METHODOLOGY

The approach is two-tiered. Firstly, it utilizes the Data Envelopment Analysis (DEA) to assess the efficiency of the decision-making units (special economic zones) with respect to certain important factors of development. For

those so-called "inefficient" decision-making units (DMUs), as subsequently identified by DEA, performance targets are set on the basis of the performance of the so-called "efficient" units. In other words, DEA pinpoints comparative gaps of development among the decision-making units. The findings obtained from DEA are then used to develop the Analytic Hierarchy Process (AHP) model to determine the type(s) of industries suited for the conditions of the decision-making units. The advantages of the approach is that it not only identifies the strength and weakness of the target unit regarding factors at the industry level, but also guides the "inefficient" unit to improve its performance with priorities. It helps overcome the limitation of DEA or other methods that do not use comprehensive environmental data in the evaluation process.

2.1 The Data Envelopment Analysis (DEA)

DEA is a technique for assessing the relative efficiency of comparable DMUs with a view to improving their performance. This implies a basic assumption that differences in performance among "like" units exist and are measurable. The factors (both inputs and outputs) characterizing the performance of all units in the group are supposed to be identical except for differences in intensity or magnitude. **Bowlin [1987]** suggested that the number of units should be at least twice the number of inputs and outputs. DEA has an inherent strength in its ability to separate all DMUs which defines the performance frontier from those which are under-performing, as well as its ability to relate organizational outcomes to resources utilized, adjusting for environmental factors not under management control but which affect the organization's effectiveness. Conceptually, DEA defines a total output for each DMU as a weighted sum of outputs, i.e.

$$S_k = \sum_{i=1}^{s} u_i y_{ik}, k = 1,2,\ldots,n \qquad (1)$$

where u_i is the weight to be attached to output measure i and y_{ik} is the value of output measre i for DMU k. The units u_i are such that the product $(u_i y_{ik})$ is dimensionless. The total input T_k is defined in a similar way, i.e.

$$T_k = \sum_{j=1}^{t} v_j x_{jk} \qquad k=1,2,\ldots, n \qquad (2)$$

so that it is also dimensionless. The efficiency (e_o) of DMU o is determined by solving the fractional programming problem below:

$$\text{Max} \quad e_o = \frac{\sum_{i=1}^{s} u_i y_{io}}{\sum_{j=1}^{t} v_j x_{jo}} \qquad (3)$$

subject to

$$\frac{\sum_{i=1}^{s} u_i y_{ik}}{\sum_{j=1}^{t} v_j x_{jk}} \leq 1k = 1,\ldots,o,\ldots,n \qquad (4)$$

$$u_i \geq \varepsilon, i = 1,2,\ldots,s \qquad (5)$$

$$v_j \geq \varepsilon, j = 1,2,\ldots,t \qquad (6)$$

where ε is a non-Archimedean infinitesimal. Efficiency, which is restricted to equal or less than 1.0, is taken from the engineering analogy that total output does not exceed total input **[Charnes et al., 1978]**. The above fractional programming model can be replaced with a linear programming (LP) equivalent, either emphasizing input eduction where the outputs are held at a fixed unit value for each DMU, or emphasizing output enhancement by holding the inputs at a fixed unit value. The dual formulations can also be taken to enable determination of real slacks of factors for each DMU vis-a-vis the efficient frontier. Another reason for the use of the dual formulations is computational since the dual, in this case, tend to be smaller in size as the primal formulations.

2.2 The Analytical Hierarchy Process (AHP)

The AHP, developed by **Saaty [1980]**, is a technique for organizing the information and judgments used in making complex decisions. The AHP facilitates the decomposition of decisions into a hierarchy of micro-decisions and uses relative importance weighting. The AHP model works on the three basic principles: The principle of constructing hierarchies, the principle of establishing priorities, and the principle of logical consistency. The approach to structuring the hierarchy depends on the kind of decision to be made in order to capture the basic elements of the problem. Priorities are set on the basis of the relative importance of each element on the next higher level. The exercise is repeated until all combinations of elements have been considered. For each pair of set of pairwise comparison the user is asked which element is more desirable, likely, or important and by how much or by how many times. In order to accomplish this, a 9-point scale of relative importance (1 being of equal importance and 9 being of extreme importance) is used in typical AHP studies (**Saaty,1980**). This step is the heart of AHP where logical thinking and feeling are used to make judgment value. The original pairwise matrix itself need not be consistent. What we would like is a measure of the error due to inconsistency which provides information on how serious are violation of numerical and transitive (ordinal) consistency. The result could seek additional information and the data used in constructing the scale could be reexamined in order to improve consistency.

2.3 Model of Appropriate Industries' Selection

There are two phases involved in the model formulation. Phase I uses DEA for evaluating the efficiency of DMUs considered, and phase II develops an AHP model using the findings of DEA. Phase I consists of three main steps: (1) Definition and selection of DMUs to enter into analysis; (2) Determination of input and output factors which are relevant and suitable for assessing the relative efficiency of the selected DMUs; (3) Application of DEA model and analysis of the outcomes. In phase II, the results obtained from DEA evaluation is used to identify the factors which need to be improved and the factors under which the industrial sector can operate properly. These two categories of factors can be used to form two criteria. One is "taking the existing advantage", and the other is "improvement of weak performances". The weights of these two criteria can be deduced from DEA results. After that, the factors can be chosen as subcriteria which correspond to the criteria described above. The weights of subcriteria can be obtained by using AHP. Then establishing an evaluation function, using the result of AHP, can lead to the evaluation value $V_i(x)$ of each individual industry. Comparing this value with the standard value $V(x)$, one can determine as to which industries the industrial efficiency should be improved.

3 THE CASE STUDY

3.1 Phase I: DEA Evaluation

Because of their similar social and economic environment, sixteen cities in Guangdong province of China are chosen as DMUs. Two years of annual data are used to measure the DMUs activities. The factors chosen as appropriate criteria to measure the efficiency of Zhuhai industrial sector are as follows: The input factors are Foreign Investment (FI), Domestic Investment (DI), and Government Policy (GP). The output factors are Industry Net Output (INO), Industry Profit Growth (PG), Improvement of Employment (IE), Industry Productivity (IP), and Improvement of Foreign Exchange Earning (FER). Except for government policy which was subjectively graded between 1 (good) to 5 (average), all factors had data obtained objectively from China's statistical Yearbook.

The CCR model **[Charnes et al. ,1978]** was used to assess Zhuhai industrial sector's efficiency. From Table 1 we can see that the relative efficiency of Zhuhai industrial sector is 0.9094 and there are non-zero slack variables on Industry Net Output (INO), Industry Profit Growth (PG), Improvement of Employment (IE), and Improvement of Foreign Exchange Earning (FER) for 1990 and INO. IE, and FER for 1991. The comparison of Zhuhai with its selected reference sets of efficient units is given in the Table 2. The reference set includes Shenzhen and Fushan for 1990, and Shenzhen, Fushan and Shantou for 1991.

The value of information shown in Table 2 is that they identify comparable efficient and inefficient units, and pinpoint the areas in which the inefficient units are weak in comparison with the efficient units. Such information is useful both in seeking an improved understanding of the performance of the units concerned, and also in setting performance targets for the inefficient units. For example, comparing Zhuhai with its reference's efficient unit Shantou in 1991, it is found that domestic investment in Zhuhai was comparatively lower, but the industrial performances in Shantou were better than in Zhuhai. The industry net output and foreign exchange earning ratio were much higher in Shantou.

The above application of CCR model has resulted in the determination of the overall technical and scale efficiencies. To separate these efficiencies a BBC model **[Bankers et al., 1984]** has been applied to the above sixteen cities. The results have shown that the Zhuhai industrial sector's operating efficiency was improved to 1 during 1990, 1991. It means that it operates well in this period . But the indicator of return to scale of Zhuhai is small compared to Guangzhou and Shenzhen, and has increasing return character. This means Zhuhai industrial sector has great potential to develop and efforts should be made to attract investors. From the result of DEA, one can find that Zhuhai industrial sector's performance was weak in some factors, such as: INO, IE, and FER. The local government should take these into consideration. The next phase will show one way to improve the efficiency of Zhuhai industrial sector.

3.2 Phase II: Hierarchical Analysis

The result obtained from DEA of 1991 data (Table 2) will be used to construct the hierarchy model. In output contribution part one can classify that factors INO, IE, FER are under the criteria "Improving the Weak Performance". The two factors, PG, IP, which comparatively give good performance, can be classified under "Taking Existing Advantage" criterion. In input contribution part, all slack variables corresponding to input factors are equal to zero, which means that all three mentioned input factors give good performance. The hierarchy model is shown in Fig. 1. At level 1, the weights of C1 and C2 are calculated from the results of a rating survey and are equal 0.4936 and 0.5064 respectively. At level 2, the weight for input contribution equals to 1 because there are no slack variable corresponding to the input factors.

For output contribution, the weights of SC22 and SC23 are calculated as follows: (a) Delete the three outputs factors which have non-zero slack variables from output set and solve the linear programming problem of CCR model based on Zhuhai, (b) Delete the two output factors which have zero slack variable from output set and solve the linear programming problem of CCR model based on Zhuhai, (c) The result of (a) and (b) will give us the efficiency rating of DMUs (Table 3), from which the ranking of Zhuhai industrial sector is deduced: 11 in (a) and 13 in (b). By scaling the ranking in the domain [0,1], the local weight of

SC22 and SC23 are calculated and equal to 0.4583 and 0.5416, respectively. At level 3, the weights of subcriteria can be obtained from the consensus pairwise judgment.

Table 1: Efficiencies of Guangdong Industrial Cities, with Slack Variables.

			Slack Variables							
			Input Factors			Output Factors				
Cities		h*	GP	FI	DI	INO	PG	IE	IP	PER
(DMUs)										
1990										
Guangzhou	C1	1	-	-	-	-	-	-	-	-
Shenzhen	C2	1	-	-	-	-	-	-	-	-
Zhuhai C3	0.91	-	-	-	-	34.3	2.8	0.65	-	-
Shantou C4	1	-	-	-	-	-	-	-	-	-
Shaoguan	C5	1	-	-	-	-	-	-	-	12.8
Meizhou	C6	0.67	-	-	0.29	-	-	9.02	-	-
Huizhou C7	1	-	-	-	-	-	-	-	2653.8	-
Donggutang	C8	1	-	-	-	-	-	-	-	-
Zhongshan	C9	1	-	-	-	-	-	-	-	-
Jiangmen	C10	0.95	-	-	-	25.2	17.5	2.5	-	-
Fushan C11	1	-	-	-	-	-	-	-	-	-
Yangjiang	C12	1	-	-	-	-	-	-	-	-
Zhanjiang	C13	0.74	-	-	0.48	-	2.4	-	-	4.67
Maoming	C14	1	-	-	-	-	-	--	-	-
Zhanqing	C15	0.65	-	-	-	-	3.24	-	-	-
Qingyuang	C16	1	-	-	-	-	-	-	-	-
						-	-	-	-	-
1991						-	-	-	-	-
Guangzhou	C1	1	-	-	-	32.4	-	4.87	-	21.9
Shenzhen	C2	1	-	-	-	-	-	-	-	-
Zhuhai C3	C3	0.91	-	-	-	-	3.67	13.8	10370	-
Shantou C4	C4	1	-	-	-	-	-	-	-	-
Shaoguan	C5	0.94	-	-	-	-	-	-	-	-
Meizhou	C6	1	-	-	-	-	-	-	-	-
Huizhou C7	C7	1	-	-	-	-	-	-	-	-
Dongguuang	C8	1	-	-	-	-	-	-	-	-
Zhongshan	C9	1	-	-	-	-	-	-	-	-
Jiangmen	C10	0.95	-	-	-	-	-	-	-	-
Fushan C11	C11	1	-	-	-	-	-	-	-	-
Yangjiang	C12	1	-	-	-	-	-	-	-	-
Zhanjiang	C13	0.91	-	-	0.74	3.44	-	-	11304	16.8
Maoming	C14	1	-	-	-	-	-	-	-	-
Zhanqing	C15	0.62	-	-	-	4.96	-	-	-	3.4
Qingyuang	C16	1	-	-	-	-	-	-	-	-

Note: Legend ' - ' means zero.

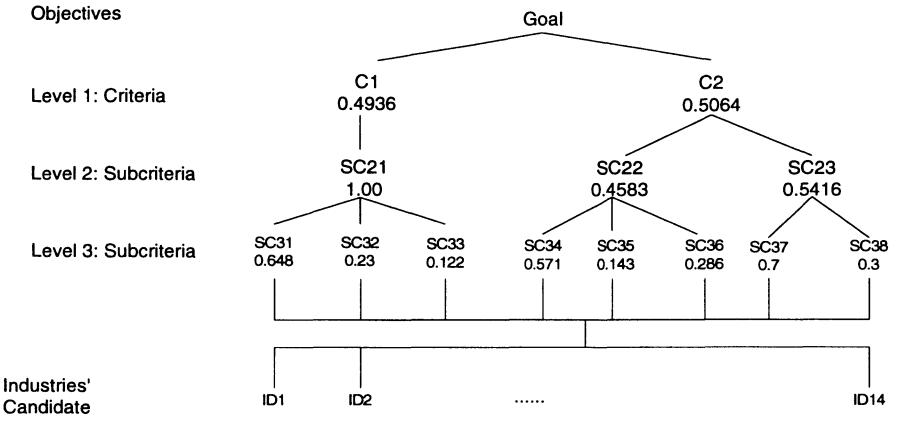
Table 2: Input and Output of Zhuhai Industrial Sector Relative to Selected Efficient Units.

		Inefficient Unit	Reference Set of Efficient Units		
1990		Zhuchai	Shenzhen	Fushan	
Relative Efficiency		0.90941	1	1	
Inputs:	GP	1	1	3	
	FI	2.184	4.44	1.48	
	DI	7.85	18.61	1.5	
Outputs:	INO	20.13	87.48	128.44	
	IPG	2.32	3.06	23.27	
	IER	0.1	1.23	1.48	
	IP	24976	51950	32334	
	FER	12.77	59.65	20.59	
1991		Zhuchai	Shenzhen	Fushan	Shantou
Relative Efficiency		0.99429	1	1	1
Inputs:	GP	1	1	3	1
	FI	1.88	5.28	2.21	1.11
	DI	7.52	18.25	3.01	7.58
Outputs:	INO	37.63	128.41	150.71	75.12
	IPG	9.72	13.09	23.27	13.23
	IER	0.22	8.07	3.05	7.93
	IP	27604	59495	37573	24115
	FER	18.31	59.65	21.16	66.8

Table 3: Efficiency Rating of DMUs

	Efficiency Rating	
DMUs	(a)	(b)
Guangzhou	0.4518	1
Shenzhen	1	1
Zhuhai	0.9943	0.9286
Shantou	1	1
Shaoguan	0.5597	0.9494
Meizhou	1	0.6838
Huizhou	1	
Dongguuang	0.7341	1
Zhongshan	1	1
Jiangmen	1	1
Fushan	1	1
Yangjiang	1	1
Zhanjiang	0.5914	0.8418
Maoming	1	1
Zhanqing	0.6157	0.5357
Qingyuang	1	1

(a): CCR model with output factors having non-zero slack variables.
(b): CCR model with output factors having zero slack variables.



Goal: Improving Industrial Sector

C1: Input Contribution
C2: Output Contribution

SC21: taking Existing Advantage in Inputs
SC22: Improving the Weak Performance in Outputs
SC23: Taking Existing Advantages in Output

SC31: Government Policy
SC32: Foreign Investment
SC33: Domestic Investment
SC34: Industry Net Input
SC35: Improvement of Employment
SC36: Foreign Exchange Earning Ratio
SC37: Industry Profit Growth
SC38: Industry Productivity

ID1: Electronic & Communication Facilities
ID2: Machinery
ID3: Textile
ID4: Clothing
ID5: Light*
ID6: Food
ID7: Beverage & Tobaco
ID8: Chemical & Fiber
ID9: Medical
ID10: Plastic Products
ID11: Metal Refine & Processing
ID12: Building Materials
ID13: Transport equipment
ID14: Electric Power, Coal & Gas Products

*: Include Leather Processing, Lumber Processing, Furniture Manufacture, Rubber Products, Industrial Art Products, Paper & Printing, Toy Producing

Figure 1 - A Hierarchy Model of Appropriate Industries' Selection (Local Weights)

Establishing the Evaluation Function

In this study, minimization in subcriteria regarding input value is expected, so the evaluation fuction is calculated as follows:

$$V_k(x) = \frac{\sum_{i=4}^{8} sc_{3i} I_{ki}(x)}{\sum_{j=1}^{3} sc_{3j} I_{kj}(x)}, k = 1,\ldots,14 \qquad (7)$$

where $I_{ki}(x)$ is a dimensionless performance's measurement of industry k under subcriterion. The result of evaluation function are presented in the Table 4.

Table 4: The Results from Evaluation Function

Evaluation Function	Value	Evaluation Function	Value
$V_1(x)$	2.07342	$V_8(x)$	0.65319
$V_2(x)$	0.75142	$V_9(x)$	2.3610
$V_3(x)$	1.35927	$V_{10}(x)$	0.73690
$V_4(x)$	1.1858	$V_{11}(x)$	0.44800
$V_5(x)$	1.10736	$V_{12}(x)$	0.84522
$V_6(x)$	1.49009	$V_{13}(x)$	0.49985
$V_7(x)$	0.95082	$V_{14}(x)$	0.38780

From the above results some observations can be made: at output contribution part, when comparing their corresponding weights, the criterion of "Taking Existing Advantage" (SC23) is more important than "Improving Weak Performance" (SC22). But here the two criteria are mutually beneficial because the difference in weight is not so big. Under SC21, SC31 (GP) is dominant because in a socialist-type economy, the government policy affects every aspects of economic development. The SC32 (FI) is the main factor to induce establishment and growth of industries as suggested in phase I. This criterion plays an important role in foreign oriented policy of government in special economic zones due to the limited domestic savings. Under SC22, the INO (SC34) is the most important. The net output is a major measurement of industry since it concerns with the gross industry input and its consumption. Its improvement will be beneficial to the industrial development such as creation of employment, improvement of industrial efficiency, etc. Under SC23, SC37 is more important than SC38. Because in developing countries labor cost is comparatively cheaper, improving industry profit (SC37) is more beneficial compared to improving industry productivity since it attracts more investment.

Based on the evaluation value of industries (Table 4) one can see that the *Electronic & Communication Facilities* and *Medical* industries have the highest ranking. They are the industries including many new and high-technology subindustries which have the highest growth potential in the future. They also meet the local government policy to shift from labor-intensive to technical-intensive industries. The *Textile, Clothing, Light* and *Food* industries also have high evaluation value. Although regarded as highly labor-intensive and having no long-term comparative advantage, these industries still have priority in improving their efficiency. The government should take some incentive measure to encourage upgrading efforts from these industries through automation and mechanization to improve operational efficiency.

4 CONCLUDING REMARKS

An integrated model has been built to evaluate the industry efficiency and the evaluation results have been used to select appropriate industries that need efficiency improvement. The application of DEA has the advantage of requiring no excplicit prespecified set of weights to indicate the relative importance of the different measures. However, there are problems in determining which input(s) and output(s) should be selected for inclusion in the model, and difficulty of making judgment about industries employing unusual mixes of inputs or producing unusual mixes of outputs. With a hierarchy model, the industries which are suited for improving a DMUs industrial sector's efficiency are determined. The results (for the case of Zhuhai, China) indicate that industries which are technically-intensive are most suited for the zone's development (*Electronic & Communication Facilities* and *Medical*). And those labor-intensive such as *Textile, Clothing, Light* and *Food* industries, at the present stage, still have the priority to be developed.

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Valuation of Environmental Goods with Expert Panels

Fred Wenstøp[1], Arne J. Carlsen[1], Olvar Bergland[2] and Per Magnus[3]

[1] Norwegian School of Management, p.o.box 580, 1341 Sandvika , Norway. Fred.wenstop@bi.no

[2] Norwegian Agricultural University

[3] National Institute of Public Health

Abstract: The present work introduces the use of decision panels to valuate environmental goods for public policy purposes as a supplement to more traditional contingent evaluation methods. It addresses problems arising from road traffic and seeks to specify the most important environmental- and health impacts in a useful way, both for valuation and policy making purposes. Particularly, we want contribute with reasonable appraisals of how much it is worth for the society to reduce the various impacts by one unit.

Three independent expert panels where created consisting of members from different state authorities. The panels were requested to perform judgments as concerned citizens with the added insights and values which might have arisen from their professional work. They were not supposed to consciously reflect official viewpoints of their organization. They were asked to express what in their opinion *the society* should be willing to pay to avoid the various adverse impacts.

Two different weight elicitation methods were used, one numerical-cardinal and one verbal-ordinal. Two different cases where used as background for the valuation processes, one national and one local. There were three independent valuation sessions with each panel.

There turned out to be considerable uncertainties *within* the panels about proper judgment. The differences *between* the panels were not strong except for criteria concerning domestic reduction of emission of toxic or climate gases. Where direct comparisons can be made, the valuations agrees well with previous values published by the State Pollution Control Authority. The choice of elicitation method had little impact on the results, but the choice of background case did have a significant impact. The valuations per afflicted person turned out systematically higher when fewer people were afflicted. This observed framing effect was also manifest in a similar and parallel study of student panel valuations. The expert panels valuations were generally higher than the student panels.

1. Introduction

There is a growing consensus that non-market effects like environmental- and health impacts need to be assessed on equal terms with monetary ones. This makes it possible to take such impacts into account in a consistent fashion during the project evaluation phase instead of treating them as subsidiary effects or afterthoughts.

It is common practice today, at least in public enterprises, to perform systematic project evaluations where the advantages and disadvantages of different alternatives are compared, taking into account several evaluation criteria at the

same time. There are several evaluation methods available. Some rely on a predominantly qualitative process and some are more strictly quantitative. See Lawrence [11] for a discussion. The qualitative approaches are often *ad hoc* and tailored to the decision problems at hand, whereas the quantitative approaches are usually more general and based on theory to assure consistency (Stewart [14]). When there are several decision criteria to take into account, evaluation of alternatives will imply some kind of weighting of the criteria whether this is done consciously or unconsciously. This task is often accomplished, explicitly or implicitly, by *expert panels* within the responsible governmental agencies (Stauth et al. [13]). In other cases, various interest groups form separate expert panels and an effort is made to find alternatives that are good compromises, a pioneer implementation being reported by Fredrikson et. al. [3]. Such approaches are primarily *decision tools* for given decision problems. Our point of departure is to utilize such tools with the express purpose of valuating environmental and health goods so that the results can applied to any decision situation were such impacts occur.

Health and environmental goods are certainly being valuated for the same purpose but with other methods. Standard welfare economics stipulates that policy should be based on the preferences of the population. In *contingent valuation* surveys (CV), the population's *willingness to pay* for health- and environmental goods is estimated through surveys of stated preference. Questions regarding costs, validity and ethics calls, however, for supplementary approaches. The method is obviously costly and time consuming. Second, while CV purports to reveal existing preferences in the population, there are strong indications that such preferences actually tend to be constructed during the interview. The responses are therefore susceptible to framing and anchoring effects. To increase validity, Gregory et. al. [5] argue, preferences should first be constructed through suitable processes. It is also questionable whether such important issues as health and environment should be left to the discretion of the consumer. See for instance Cowen[2].

2. The expert panels

Three expert panels where established. *The panels were told not to reflect possible official viewpoints of their organization, but rather act as concerned citizens with the added insights and values obtained through their professional work.*

One panel was from the *State Pollution Control Authority* consisting of four persons from high level management, the second was from the *Directorate of Public Roads* consisting of five persons from middle-to-higher level management, and the last was from the *National Institute of Public Health* consisting of three physicians.

3. Criteria selection

The the health- and environment impact criteria to be valued were chosen carefully with regard to their relevance, understandability and operationality as defined by Keeney [9]. The more understandable a criterion is, the more *valid* the resulting weight estimate. The more *operational* a criterion is, the more reliable are the predicted effects of alternative measures. Understandability and operationality are in practice conflicting properties. Health authorities currently set recommended upper limits to concentrations of toxic gases and noise levels in urban areas. "The number of people exposed to above recommended levels" is used as an indicator of a health problem and an administrative policy instrument. In reality, there is very limited knowledge of the real health effects in persons that are exposed to above recommended concentrations. From a decision making point of view, the *understandability* of "one person above recommended limit to NO_2" is therefore low. Consequently, we agreed with our panels to focus on *end-impact* criteria rather than *environmental-load* criteria. It is obviously much easier to value a person-day of chest irritation, than a day of exposure to above recommended concentrations of NO_x. The disadvantage of end-impact criteria, on the other hand, is that they are less *operational.* It is harder to use them for planning purposes, since it requires dose-response relations from concentrations to end impacts.

Effects to be valued

Criteria that express the contribution towards achievement of national commitments

- % reduced level in SO_2 emissions from the 1991 level
- reduced level of NO_x emission, % contribution to the national goal
- reduced level of CO_2 emission, % contribution to the national goal

Adverse health effects

- A person-day less per year of chest irritations or coughing
- A person-day less per year with headache
- A person-day less per year with dizziness
- A person-day less per year with nausea
- A person-day less per year with colds or influenza

Irritations

- A person less complaining about noise from road traffic
- A person less complaining about unpleasant smells
- A person less complaining about soot or smudge

Injuries

- A non-fatal traffic injury less per year
- A fatal traffic injury less per year

Costs

- Costs per year of public actions

Our choice is based on the belief that environmental valuation will become an increasingly more important task in public planning, and that this requires focus on end-impacts. Dose response relations will consequently have to be developed even if that is not addressed in our project. This is in line with the policy adopted in the CEC/US Joint Study on External Costs of Fuel Cycles [1] where a major emphasis has been placed on specifying a set of well defined end impacts so that these can be valued in a consistent fashion in different countries. In our choice of effects, we have stayed close to the recommendations in the CEC/US study.

4. Cases

When carrying out an evaluation process with expert panels, it is helpful to refer to concrete cases. This makes the issues that arise more tangible, and the experts can use their own concrete knowledge of the cases to better perform value judgments. Two well known cases were selected, one national and one local. The national case was an evaluation of the effects of a graduate transition to electric vehicles, and the local one an evaluation of alternative traffic planning strategies in the city of Drammen over the next 10 years. Scenarios of the effects of different decision alternatives were made based on public planning documents.

5. Weight elicitation methods

Several weight elicitation methods have been computer implemented, and there are several reports comparing decision tools [4,6,8,10,12,17,18]. We selected pairwise ordinal comparisons of criteria as it is implemented in DEFINITE [7], and pairwise numerical trade off between criteria as it is implemented in Pro&ConW. The two approaches are different in two ways, as discussed below.

Criteria weights

Assume that there are n criteria, $x_1,..x_n$
The score of a criterion is restricted to its *working domain* where x^o_k is the least preferred value of the k'th criterion, and x^*_k the most preferred. The least and the most preferred values are usually inferred from the decision alternatives.
For each criterion, a utility function is specified so that $u_k(x^o_k)=0$ and $u_k(x^*_k)=1$
The weight of criterion k is w_k, $w_k<1$, $\Sigma w_k = 1$
The utility of a decision alternative with scores $x_1,..x_n$ is:
$U(x_1,..x_n) = w_1u_1(x_1)+w_2u_2(x_2)+..+w_nu_n(x_n)$

Pairwise ordinal comparison of criteria

The expert panel is first presented with a decision table describing a decision problem. Two criteria, viz. A and B, are selected and the expert panel is asked

whether they judge A to be more important than B, whether the opposite is the case or whether they are equally important. Assume that A is judged to be more important than B. They are then asked *how much more* important they judge A to be. The answer is to be given verbally on a nine point scale: 1: just as important; 3: moderately more important; 5: much more important; 7: very much more important; 9: extremely more important

Next, a criterion C is selected and A is compared to C in the same way. This continues until all $n(n\text{-}1)/2$ pairs have been evaluated.

With this method, questions about importance depend on the decision frame. In this case, the frame is the *decision table* as described by the cases in tables 1 and 2. This means that the method is *decision situation specific* and that decision makers who *know the decision problem well* should be able to give better answers than those who do not.

Numerical trade off between criteria

This elicitation process was developed in connection with the project and is implemented as an interactive spreadsheet file for Lotus 1-2-3 for windows (Pro&ConW). It is a stepwise procedure:

The expert panel is presented with a decision table describing the case in terms of criteria scores for each alternative

The panel is asked to specify working domains for the criteria, the default values being suggested by the decision table.

The panel is asked to rank the criteria according to their importance. "Importance" is to be understood in the same sense as in the verbal comparison method, either related to the decision table or to the adjusted working domains. This induces the panel to think in terms of trade-offs

The panel discussed the applicability of linear utility functions for the criteria. The concept of a Willingness-to-pay (WTP) to avoid one unit of an impact assumes linear utility functions which the panels accepted as reasonable approximations.

The panel is asked to select one criterion as a reference criterion against which all the other criteria are going to be compared. This criterion should be easily to comprehend and it should not be monetary.

The panel is presented with a two-by-two matrix and asked to adjust the numbers so that the "before" and "after" situations are equally preferable.

The relative weights are computed.

The weights are scaled to make their sum equal to one.

With *pairwise numerical comparison of criteria,* the judgments about the relative importance of the criteria are not related to the decision table. This may be a disadvantage when the experts have good knowledge about the decision problem, and an advantage if the opposite is true.

6. The elicitation process

The elicitation process spanned eight months. There were three interactive sessions with each panel. There was no cross-information between panels. Each session used one case and one elicitation method. We started with all panels using numerical trade off applied to the El-car case, then ordinal comparisons applied to the same case, and finally numerical trade off applied to the Drammen case. The selected criteria were reviewed and modified somewhat to fit the cases.
Each session lasted approximately three hours. A session would start by going through the case and elicitation method off line. This would always lead to much discussion which served to clarify many issues. After this, we used the interactive software which was installed on a portable PC connected to a color flatscreen on an overhead projector.
During the elicitation process, the panels were supposed to discuss all matters of judgments openly and with little involvement by the session leaders. There were at least two session leaders present at each session. Questions of clarification could always be asked to the session leaders. The panels were expected to reach unanimous judgments, if possible. If the situation required it, however, they could make more than one sets of weights.
When the weights had been produced, we continued the process by calculating the implied WTPs. This was possible since the criterion "annual costs of actions" was also weighted as a part of the process. A WTP associated with a criterion expresses how much the panel thinks the society should pay to avoid one unit of the corresponding impact. The panels were asked to review their weights in the light of the ensuing WTPs until they felt confident with the final result.

7. Results and analysis

Observations from the process

Higher management persons seem to perform judgments more confidently than middle level management who appear less inclined to assume the independent role of judges.
Intra-person uncertainties tend to overshadow inter-personal disagreements within a panel. Intra-person uncertainties were due rather to ambiguities regarding the decision criteria, than to uncertainties about value judgments. Typical questions were "what does it really mean when a person complains about chest irritation and coughing?" "What does it really mean when a person complains about traffic noise?" This points to a need for the identification of decision criteria that are clear and unambiguous.
The sessions were more effective in smaller groups than in larger ones. We think three is the optimal number.
Since weights in themselves are inherently difficult to appreciate while monetary terms are more readily understandable, to compute WTPs after a preliminary set

of weights have been established was an instrumental feedback for reappraisal of the weights.

Sources of variation in valuation results

Table 1 shows the sources of variation in valuation results according to analysis of variance. Natural differences between the criteria are the major source of variation as was to be expected. The different combinations of cases and evaluation methods contribute significantly to the variation as well. The raw data show that this is attributable to the cases and not the elicitation methods. There is also significant differences between the panels. However, inspection of the data reveals that this stems mainly from the three emission variables.

Table 1: Three-way Analysis of Variance of ln(WTP) - all criteria

Factors:	The eight criteria				
	The three expert panels				
	Three combinations of elicitation method and case				
Source	**DF**	**SS**	**MS**	**F**	**P**
Criteria	6	425	71	25	0
Panel	2	23	12	4,17	0,02
Case	2	63	32	11,3	0
Error	52	146	2,8		
Total	62	658			

The emission variables concerning SO_2, NO_x and CO_2 represent presumed geopolitical impacts of making a good performance at home, thus excluding real end impacts. For these criteria, the panels expressed conscious and articulated opinions during the valuation sessions which manifested themselves in diverging valuations.

Table 2: Three-way Analysis of Variance of ln(WTP) - health impact criteria

Factors:	Criteria - Cough, Headache, Flu, Noise, Smudge				
	The three expert panels				
	Three combinations of elicitation method and case				
Source	**DF**	**SS**	**MS**	**F**	**P**
Criteria	4	7,5	1,9	3,41	0,02
Panel	2	0,61	0,31	0,56	0,58
Case	2	50	25	46	0
Error	36	20	0,55		
Total	44	77			

If we restrict our analysis to those criteria having a strictly local impact, an even clearer pattern emerges as shown in table 2. Here, there is limited contribution to variation from between panel differences. The choice of case, on the other hand, has considerable impact, indicating *framing effects*. Several studies [15] report that panels are not always able to adjust the weights consistently according to the ranges of the criteria. We see an instance of this here. It seems that the much lower number of people in the Drammen case compared to the national case have lead the panels to place a higher emphasis on the *per person* impact evaluation. One should be careful not to draw a too strong conclusion, however, since we know that the WTPs from one session are dependent. Therefore, the F-values in the variance analyses must be interpreted with care. We do, however, observe exactly the same phenomenon in a parallel study with student panel data.

Statistical summary of the expert panel valuation results

Our results are presented in table 3 and figure 1.

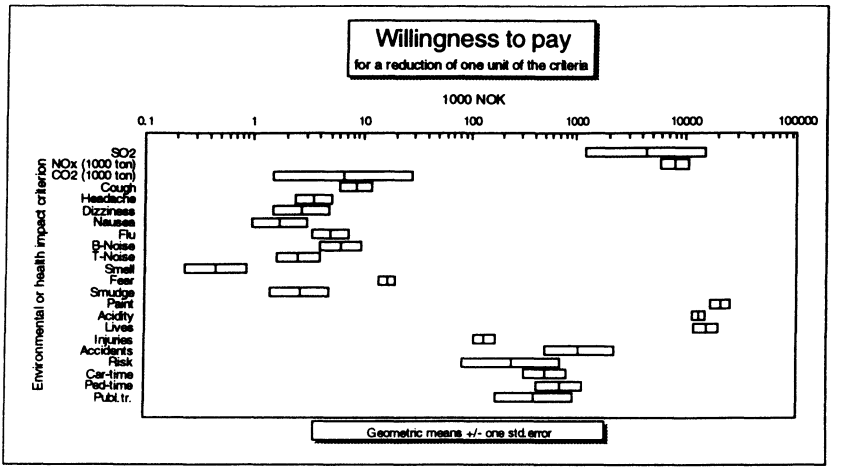
Table 3: Statistical summary of the willingness-to-pay data from all panels and methods.
Willingness to pay p.a. for one unit of the criterion (NOK 1000)

Criterion		Geometric mean	Lower limit	Upper limit
SO_2	% reduction from the 1991 level	4400	1200	15000
NO_x	ton reduction p.a.	7.8	5.8	10
CO_2	1000 ton reduction p.a.	6	1.5	28
Cough	one person less complaining	8.4	5.9	12
Headache	one person less complaining	3.5	2.4	5.1
Dizziness	one person less complaining	2.7	1.5	4.8
Nausea	one person less complaining	1.7	1	3
Flu	one person less complaining	4.9	3.4	7.2
Noise	one person less complaining	6.1	3.9	9.6
Smell	one person less complaining	0.4	0.2	0.9
Fear	one person less complaining	17	14	20
Smudge	one person less complaining	2.6	1.4	4.9
Lives1	lives lost p.a.	15000	12000	20000
Injuries	p.a.	130	110	170
Accidents	p.a.	1100	500	2200

1 The numbers are based on the judgments with the numerical evaluation method.

The raw data were highly skewed to the right. One reason is that the individual WTPs were calculated as the quotients of two weights, both of them subject to statistical errors. This means that the *logarithms* of the WTPs are approximately normally distributed. Therefore we have used *geometric means* of WTPs as our estimate of central tendency. The upper and lower limits in table 3 are calculated as the sample mean +/- one standard error of the sample mean. The large interval for CO_2 reflects articulated differences in judgments between the panels.

Figure 1: Graphical illustration of table 3.



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A Case-study in Military Decision-making With Mission-oriented Multiple Objectives*)

Diederik J.D. Wijnmalen
TNO Physics and Electronics Laboratory
P.O. Box 96864, 2509 JG The Hague, The Netherlands
Fax (+)31 70 3280961, Tel. (+)31 70 3264221

Abstract: In this paper we report on a problem of selecting military equipment. We have applied an evaluation methodology based on hierarchical multi-criteria analysis, which relates the contribution of alternative options (regarding equipment, organisational structure, etc.) to military objectives by explicitly defining military missions, tasks and functions. The analysis consists of two approaches, an aggregate ("holistic") and a detailed approach, which should complement each other and offer an opportunity for comparison and reflection. We begin with an overview of the evaluation methodology and then focus on the application.

1 Introduction

When deciding on the allocation of resources, the acquisition of systems, the organisational structure, the order of battle, etc. within the context of defence planning, it is necessary to relate means to ends. The ultimate end is the security of the country and of the alliance in which it is taking part, and the safeguarding of their interests. In one way or another the means should contribute efficiently and effectively to this end. This rather abstract objective is made more concrete by defining missions and tasks to be accomplished. While systems acquisition, for example, was traditionally a matter of assessing alternative options on costs and technical/tactical hardware requirements, there has been a growing need to look explicitly at the multitude of possible tasks and missions of the forces using those systems when deciding which one to acquire. Some recent developments have increased that need. With a changing political constellation and dynamic military developments taking place in Europe and elsewhere, the traditional East-West threat with well-defined battlefields and modes of operation has changed, triggering a process of reconsideration and restructuring of military forces and their tasks in the Netherlands as elsewhere.

We offer military planners a general approach that should stimulate structured thinking about the matter at issue, make all assessments and priorities explicit, and enable planners to combine tangible and intangible aspects, investigate sensitivities, and detect and then focus on a few essential aspects. The approach uses (hierarchical) multi-criteria analysis (MCA) and aims at a comparison of alternative options (whatever they may be) through a consistent and comprehensive evaluation of their contribution to the objectives of the defence

*) This paper does not necessarily represent the official view of the Netherlands Ministry of Defence or the Armed Forces

organisation. Military planners do occasionally use a general MCA format when they draw up tables of plusses and minuses to summarize their findings and recommendations. They also use the weighted sums method at a very aggregate level. The approach we have proposed, however, goes rather beyond that.

The essential idea of the approach is not novel in the relevant literature. Coyle [3], Anderson [1], and recently Buede & Bresnick [2] have proposed similar approaches based on (multi-criteria) decision analysis to military planning/acquisition problems. In their paper, Lee & Ahn [5] determine relative values of weapon systems in relation to a military mission using Saaty's Analytic Hierarchy Process [7]. As the comprehensiveness of the approach, however, was indeed new to the Netherlands military, we decided to restrict ourselves in proposing possibly sensitive or intellectually and mathematically difficult elements (like the value functions in [2] or the definition of different performance levels in absolute terms in [3]) and in emphasizing the detection of deficiencies (like in [1]). Moreover, we did not want to push one particular MCA technique (as in [5]), but rather to put forward a methodology, to incorporate military expertise as opposed to analytical studies (as suggested in Hartley & Haysman [4]), and to adopt familiar NATO military terminology and concepts.

In section 2 we give an overview of the approach, which is called MT(F)A: Multiple Tasks (and Functions) Analysis. In section 3 we focus on a particular problem: how to select an air-mobile special vehicle for the Royal Netherlands Army from several different candidate vehicle types. This selection problem had been considered previously by the military without, however, arriving at final conclusions. In order to see how the MT(F)A approach would work, the selection problem was studied again, using the MT(F)A approach and focusing on its military operational aspects. We describe how this study was carried out and discuss its results. The paper ends with some final comments and conclusions.

2 The "Multiple Tasks (and Functions) Analysis" approach

At the highest level of evaluation of alternative options (regarding matériel and/or manpower and/or infrastructure etc.), consideration may be given to criteria such as operational effectiveness, cost, political acceptance, organisational feasibility, etc. Notwithstanding the difficulties of getting hold of the other aspects, we shall focus here on operational effectiveness: the degree to which military operations are (potentially) able to achieve military objectives and, in particular, how each option performs in contributing to that.

The evaluation boils down to setting up a hierarchy of criteria, viewing them in different contexts (called scenarios), linking the options with the hierarchy, and, finally, assessing the relative importance of the criteria and the options on the elements on the lowest hierarchical level. This is no different from Saaty's Analytic Hierarchy framework [7]. To actually build the hierarchy we use standard military (NATO) concepts.

At the top level of the hierarchy the overall strategic mission is stated. This strategic mission depends on the scope of the problem being addressed. It is divided into key mission components: major task areas, each of which is essential

to the overall mission. At lower levels, tasks and sub-tasks are defined: mission elements that must be performed to execute the military operations concerned. Relations between missions and tasks are also defined. The totality of missions, tasks, and relations is called the *task model*.

Descending to the next levels of the hierarchy, formalized activities and capabilities to perform tasks are defined. As they correspond to functional elements of the organisation, these are called functions. Functions may be grouped into functional areas (called super-functions) and broken down further into sub-functions. The alternative resource options are then connected to the lowest functional level. This connection can be visualized as a *functional relation matrix*, which defines for each function the resources it needs, or for each resource option the functions it contributes to. We may either have a situation where one generic set of functions is related to different instances of the same resource (e.g. different manufacturing types of transport vehicle), or a situation where there are various sets of functions, each pertaining to a different configuration of resources (e.g. different organisational structures). The totality of functions and functional relations with resources is called the *function model*.

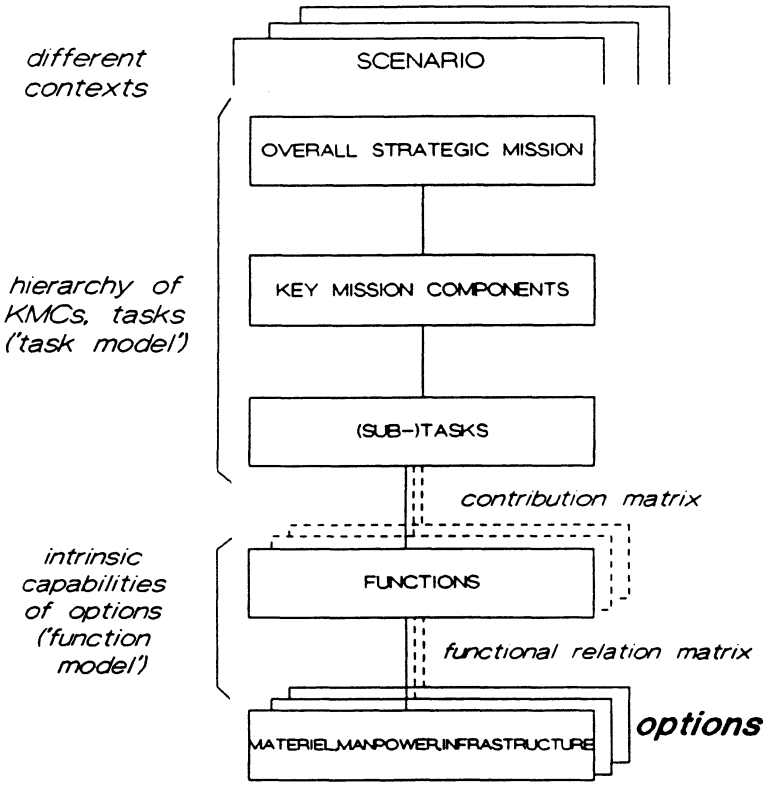

Figure 1: Hierarchical evaluation framework

The task model part of the hierarchy and its function model part must also be connected. This is done by defining a zero-one contribution matrix. This matrix indicates which functions are used/needed to perform each task. There is a clear distinction between a task and a function. A *function* is a formalized organisational element and represents an *ability to perform when needed.* A *task* is an activity which one *wishes or expects to carry out in order to achieve an objective.* A system may perform certain functions perfectly, but is of no importance if those functions are not relevant to the tasks.

The *evaluation model* consists of alternative options and scenarios, assessments of the hierarchical elements and the degree in which they contribute to each other (e.g. a differentiation of the values 1 in the zero-one contribution matrix). Figure 1 shows the elements mentioned so far.

The evaluation methodology proposes two procedures: a holistic procedure which evaluates the alternative options in the task model only, and a detailed procedure which evaluates the alternative options in the function model and relates the function model to the task model. In the detailed procedure, performance indicators are defined for each function. This may be interpreted as a refinement of the functional relation matrix introduced above. Results from simulation studies or data from manufacturers or other sources can be used to provide values for the performance indicators. In the holistic procedure, military expertise, experience and insight are used to assess the alternative options. This kind of analysis is used to express general feelings about task performance as opposed to the multitude of aspects that must be considered in the detailed analysis. Both procedures require weights to express the relative importance of tasks. In addition, the detailed procedure requires weighing functions, indicators, and the relative contributions of indicators to functions, and of functions to tasks. The reason for the brackets around the F in MT(F)A should by now be clear: it is only the detailed procedure that incorporates the function model.

Figure 2 shows a diagram of the analysis, clearly indicating the two parallel ways of proceeding, and summarizing per procedure the information to be provided by experts or analysts, and the results an MCA aggregation technique can produce. Figure 3 offers a concise layout in matrix format using mathematical symbols.

The results of both procedures can and indeed should be compared, and may trigger further analysis in case of considerable differences. The task scores based on the functional analysis in the detailed model can be compared with those provided by experts in the holistic model. Experts ought to be expert but they need not be the same for each procedure or for each part of the hierarchy. If the same experts are involved in both procedures, then it is a matter for discussion and depending on circumstances which should come first.

For aggregation purposes we have not adopted any specific MCA methods, although some might be more suitable for use in hierarchical MT(F)A than others; the software package that supports the approach contains a select set of methods. In section 3 we shall be more specific on this subject.

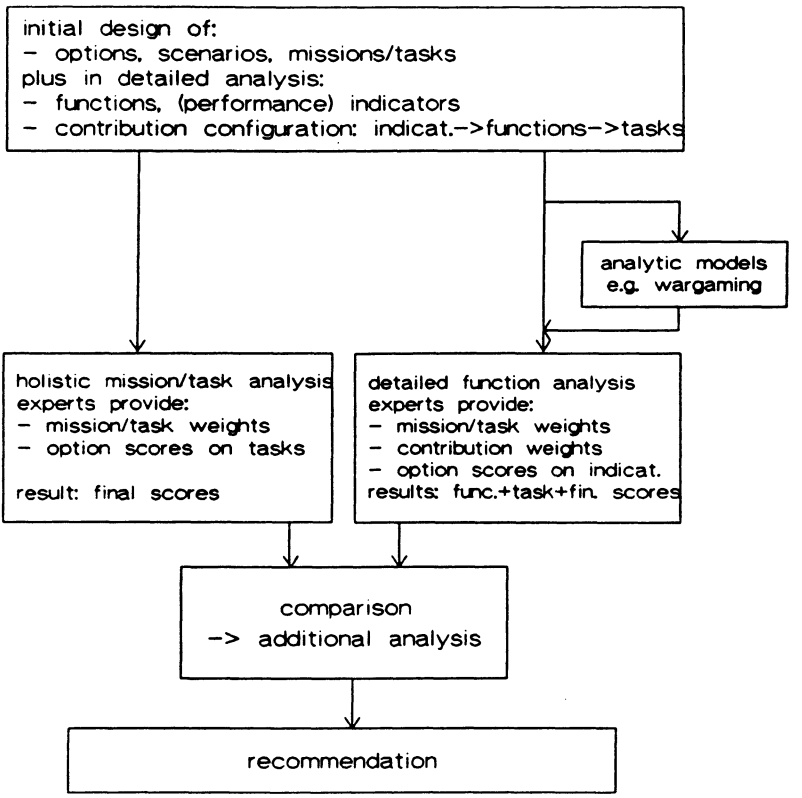

Figure 2: The parallel approach to Multiple Tasks (and Functions) Analysis; two ways to proceed

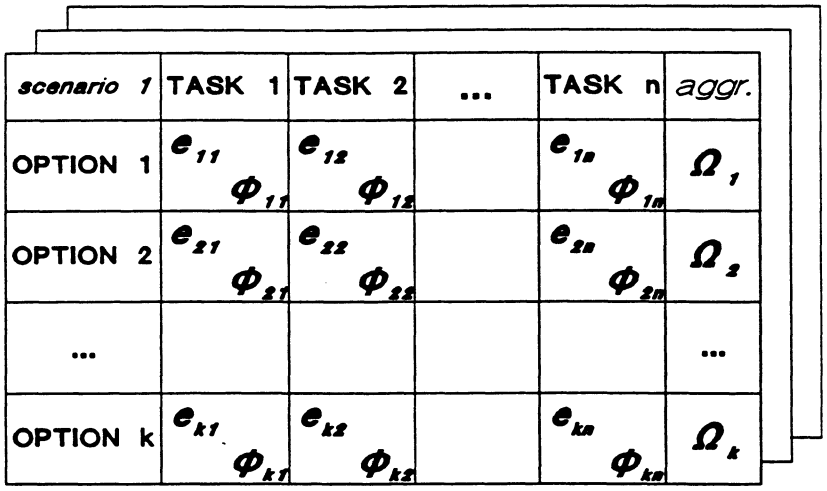

Figure 3a: Matrix layout of the *holistic* evaluation (not showing any clustering of tasks); e_{ij}=score of option i on task j; φ_{ij}=weight of task j (possibly depending on option i); Ω_i=final score of option i after aggregation

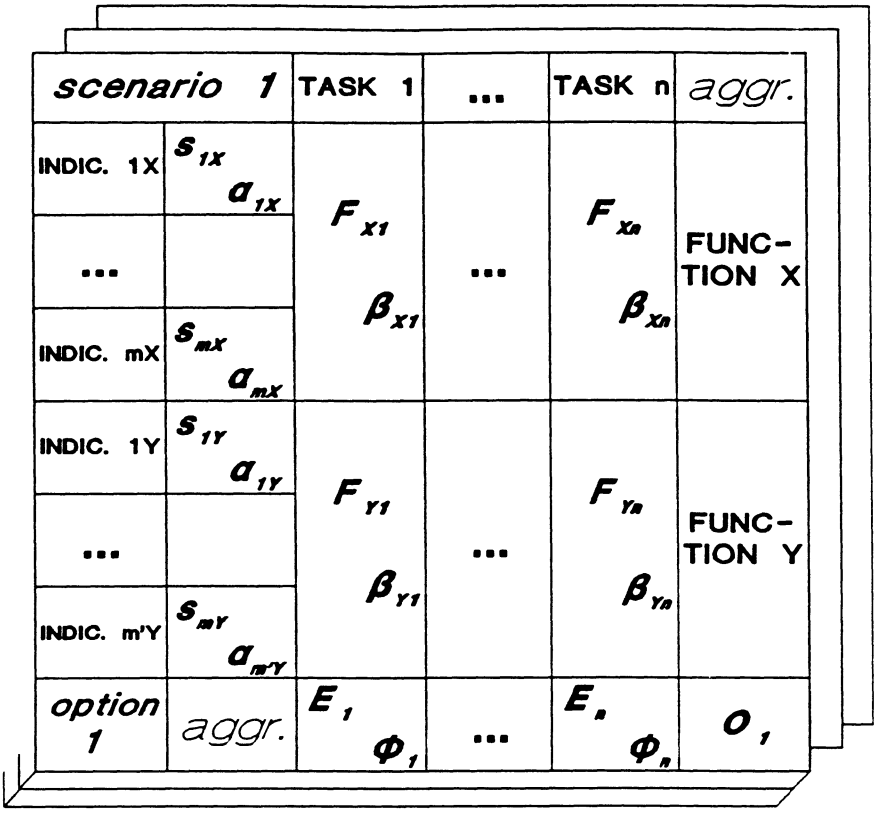

Figure 3b: Matrix layout of the *detailed* evaluation of a particular option (not showing any clustering of tasks or functions). An indicator iX, on which option 1 is scoring s_{iX}, is contributing with weight α_{iX} to function X. After weighted aggregation over the indicator scores, the computed function score F_{Xj} is contributing with weight β_{Xj} to task j. After weighted aggregation over the computed function scores, a task score E_j is calculated which has the weight φ_j; O_1 is the final score of option 1; compare Ω_1 (Figure 3a) with O_1 (Figure 3b), and e_{1j} (Figure 3a) with E_j (Figure 3b)

3 Selecting an air-mobile special vehicle, a case-study

3.1 Background

In the introduction we mentioned the process of reconsideration and restructuring of military forces in the Netherlands. The establishment of the 11 (NL) Air-mobile Brigade (AMB) has been one of the outcomes of this process, but its strategies, mode of out of usual area operation, etc., are as yet not entirely clear. These will largely depend on the particular circumstances, time and location the AMB, or parts of it, are brought into action. The AMB needs a special vehicle for its operations. Several candidate vehicle types had already been selected on the

basis of preliminary requirements. We shall not go into the Netherlands Defence Matériel Planning Process as this would exceed the scope of this paper. While the remaining problem of choosing the best candidate may seem a straightforward one (like the famous car selection problem in MCA), it is in fact a complex issue if expected task performance and mission achievement in the context of the AMB's future military operations are also to be taken into account. In order to do precisely this and to investigate the merits of the MT(F)A approach, it was decided to undertake a case-study.

The first objective of the study was to investigate whether or not multi-criteria analysis embedded in the evaluation methodology described in section 2 was a satisfactory way of dealing with the military mission-oriented aspects of the selection problem or not. The second study objective was to investigate the possible appeal of this explicit and systematic way of thinking about criteria for assessment of military equipment within the context of different scenarios and in relation to tasks at a higher level than the usual technical performance criteria.

There were five candidate vehicles of different types, e.g. wheel or track-driven, which we shall refer to as A, B, C, D and E. Some of them were re-included, though they had even previously been rejected. With a group of military experts from the Matériel and Operations Directorates we went through the whole process of defining objectives, tasks, functions and indicators together with their hierarchical relations, and of assessing their relative importance. This was done in small groups of experts who reported back in plenary sessions. The alternative vehicles were judged within the framework of each particular way of proceeding (the "parallel analysis"). We did not have quantitative results from simulation studies, but we did use information from manufacturers and results from the previous analysis performed as part of the Defence Matériel Planning Process and some information on the vehicles' performance in other countries. The outcomes of both ways of proceeding were then placed side by side and discussed. We shall go on first to describe the actual evaluation hierarchy and then to focus on the analysis.

3.2 The evaluation hierarchy

As stated earlier, the analysis took account not only of military operational, but also of technical and logistical aspects. The overall mission of the AMB is to carry out three types of operations: actions in low intensity conflict (low level of violence), actions in high intensity conflict, and humanitarian or other support actions in peacetime. Tasks were then defined for each type, in so far as they were considered relevant to the vehicle selection problem. Part of the high intensity conflict task cluster is shown in table 1. Examples of tasks which are typically relevant to low intensity conflicts are: protection and self-defence, operating observation posts and road blocks, and carrying out mounted patrols. Tasks relevant to peacetime actions include: distribution of medical supplies, evacuation of civilians, first aid, and providing support to UN monitoring missions. The definition of tasks was not straightforward. For example, if the AMB as a whole is engaged in a defence operation, then a particular battalion may

well be engaged in delaying and a particular company within that battalion in attacking. Tasks will depend therefore on the organisational level under consideration. Whenever relevant, we have looked at the lowest level, since this is the level that is closest to the vehicle, and at one task at a time. This task model suffices for the holistic procedure. In order to carry out the detailed procedure, however, we have also to define the function model and relate the two.

The number of vehicles to be purchased and the allocation of vehicles to sub-units of the AMB are the same for all five vehicle options. Therefore, we may define and classify the functions according to vehicle functionality (that is all functions to be performed by the vehicle) instead of according to the AMB's organisational structure. As a consequence, we do not have explicitly to define the functional relation matrix (see section 2). We would also like to point out that the decision to purchase an air-mobile special vehicle had already been taken; therefore, the option of carrying out operations without a special vehicle no longer existed. This would otherwise have forced us to look at all possible functions of the AMB. Table 2 shows part of the actual function hierarchy.

key task: attack	
task	**sub-task**
command&control	movement of command post passive protection of comm.post active protection of comm. post communications with comm. post working in command post
logistics	medical evacuation transportation of supplies maintenance salvage
reconnaissance	movement of reconn. platoon observation by reconn. platoon self-defence by reconn. platoon
anti-armour	movement of anti-armour platoon disposition of anti-armour plat. engagement by anti-armour platoon

Table 1: Part of task model

super-function	function	sub-function
protection	against weather against detection against light ammun. against heavy ammun. against NBC	 while stationary while moving
weapon carrier	TOW-system MRAT-system grenade launcher machine gun Stinger system	
equipment carrier	communication eq. observation eq. detection eq. inform. techn. eq. logistic opns. eq.	
vehicle logistics	vehicle salvage vehicle transport vehicle maintenance vehicle refuelling	

Table 2: Part of function model; a particular (sub)-function contributes to one or more sub-tasks (not shown here)

The next step was to connect each elementary (sub-)task (the ones not subdivided; 113 in total, including all 3 modes of operation) with the elementary (sub-)functions (the ones not subdivided; 46 in total) by setting up a zero-one contribution matrix. Notice that non-elementary functions were only used as an aid to defining the elementary ones. In order to evaluate the vehicles in relation to the functions, performance indicators were defined such as: loading capacity, maximum speed, cross-country capacities, ease of handling and range of action. For each elementary (sub-)function a set of indicators was defined. These sets were overlapping, since one particular indicator may well provide information on performance in relation to several functions (e.g. loading capacity says something about such functions as the ability to transport people, weapon systems, jerry cans containing fuel, etc.).

The result was a very large hierarchy with three major parts, each representing a type of operation and each with an impressive number of elements representing tasks, functions and indicators. To save space and to protect information, we do not show this hierarchy. Scenarios describing terrain and weather conditions, unit deployment, order of battle, enemy threat etc. were not defined, mainly due to time restrictions. The three generic types of operation were considered as "scenarios".

3.3 The evaluation

The proposed methodology does not prescribe any particular MCA technique for obtaining weights, scores, or aggregation results, merely stipulating the requirement that the aggregation should be able to deal with the hierarchical structure. We proceeded as follows. The weighing of relative importances and contributions, and judging of alternative options were always carried out in two steps. The first was to obtain an ordering within a particular group of elements (e.g. all functions contributing to a particular task), the second to assign the value 10 to element number 1 and further values on the real scale [1..10] to the remaining elements of the group, taking the ordering into account. The usual normalising procedure was applied to the weights.

The final results were calculated as weighted sums. The comparison method used is a relative method whose results require careful interpretation: only the preference ordering and *differences* in final scores have a meaning, not the scores themselves. The advantage of this method is that we were not obliged to define the exact absolute meaning of each scale value, and avoided both the cumbersome effort of doing many paired comparisons as in Saaty's approach [7] and the problem of rank reversal (as may occur when using Saaty's original priority method). As there were several experts involved, we tried to reach consensus by comparing outlying assignments with a "group value". This appeared to work out well. Some outlying values were used in a sensitivity analysis.

In the holistic procedure, the alternative vehicles were assessed in relation to the elementary (sub-)tasks. Final scores were calculated by weighing these assessments with the weights of the elementary (sub-)tasks. In the detailed procedure, by contrast, the vehicles were assessed in relation to the performance

indicators. Task scores were calculated in this case by ascending through the hierarchy and applying the weighted sums method to each cluster of elements at each level. Final scores were derived in the same way as in the holistic procedure, but now using computed task scores. The final scores were not aggregated over the three types of operation. Some results are summarized in table 3. Only (partial) rank orders are shown in this table; if the difference between final weighted sum scores of alternatives is 0.1 or less, then those alternatives have the same rank order.

3.4 Discussion

It can be concluded that the results of the weighted sums in the two procedures do not differ very greatly, except for a rather striking change in the position of option E in the low intensity conflict operation. Discussion revealed that the importance of a few aspects had been dealt with intellectually differently in the two procedures. Options A and C emerged as the best; analysis revealed that this was mainly due to the multi-functionality of these vehicles, an important asset for an AMB. A sensitivity analysis suggested that the results were rather robust; one had to change weights considerably in order to make option D end up higher than A and C, for example.

scenario:	low intensity	high intensity	peace-time
weighted sums method			
holistic	A>E>C>B>D	A>C>D>B>E	C>A>B>E>D
detailed	A>C>B>(DE)	A>C>(BD)>E	A>C>B>E>D
ELECTRE-3 I=0%, P=0%, V=100%			
holistic	A>(CE)>(BD)	A>(CBD)>E	C>A>B>D>E
detailed	A>(BC)>D>E	A>(CB)>D>E	A>B>C>D>E
ELECTRE-3 I=0%, P=15%, V=100%			
holistic	(ACE)>(BD)	A>(BCD)>E	C>A>B>D>E
detailed	B>A>(CD)>E	B>A>(CD)>E	B>A>C>D>E

Table 3: Evaluation results shown as rank orders (each ELECTRE-3 threshold value is a percentage of the scale range: I=indifference, P=preference, V=veto); > means 'more preferred than'.
If weighted sums of X and Y differ 0.1 or less, they will have an equal ordering (XY).

Once the whole approach was explained to them, the military experts felt fairly comfortable with it. It appeared to be the first time that they had been forced to define and think explicitly about the totality of missions, tasks, functions and indicators in the context of this particular selection problem on behalf of the AMB. In doing so, the evaluation hierarchy was adapted several times. They felt that the approach (including the multi-criteria analysis) stimulated a reasoned and comparatively objective evaluation, and would enhance the credibility (and defendability) of the selection recommendations. The price to be paid was, of

course, the considerable effort involved. There is a danger of overdoing things. The decision on when to stop adding or subdividing evaluation aspects is an important one. We lacked time to investigate this topic further.

We obtained no absolute statements about vehicle performance, since we did not rate the vehicles in an absolute way and did not define scale values in absolute military terms (as Coyle [3] proposes to do). With few relevant vehicles left, we were interested in identifying the best one rather than finding out exactly how good it was. Moreover, to do so would have involved lengthy and difficult discussions on how to define absolute performance (or "capability") levels and, after that, how to rate the alternatives.

4 Final remarks and conclusions

We have presented an evaluation approach to military decision-making involving a multitude of evaluation aspects. This approach is based on several ideas which have already been published. We have, however, adapted them and combined elements from various approaches together to produce a parallel procedure. One ("holistic") procedure consists of assessing alternative options in relation to high-level criteria (viz. military tasks) based on general expertise. The other ("detailed") procedure involves setting up a hierarchy of organisational functions and performance indicators and connecting them with the task hierarchy. We have applied the approach to a vehicle selection problem faced by the 11 Netherlands Air-mobile Brigade. We used the weighted sums and the ELECTRE-3 method to compute final scores within the context of each procedure. The experts involved concluded that the approach was useful: it offered a systematic and structured way of dealing with a multitude of objectives and requirements; it forced planners to state their assessments explicitly, and the use of a software package enabled planners to compute aggregated final scores in an mathematically "objective" way (albeit using partly subjective information). Comparison of the results of the holistic and detailed procedure revealed no significant differences but triggered additional discussions and more differentiated conclusions, as indeed it was supposed to do. We have reasons to believe that the approach would work equally well for civilian organizations, when one has to cope with evaluations as multi-faceted as the one of this case-study.

An important topic for further investigation is how to deal with dependency between different elements of the hierarchy. With problems of the type we have tackled, with its large and detailed evaluation hierarchy, it is to be expected that dependencies between tasks and between functions will exist. It seems to be preferable to deal explicitly with this phenomenon rather than ignore it.

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XI

Future Trends on Multicriteria Analysis

Several researchers present at the XIth International Conference on Multiple Criteria Decision Making were invited to present their views on the future trends in this field. This chapter contains nine papers expressing the opinions of the academics that accepted our challenge. These are presented in alphabetical order of their author's last name. Exceptionally, one of these invited authors is accompanied by a co-author, who is nevertheless welcome.

Thoughts on the Future of the Multicriteria Field: Basic Convictions and Outline for a General Methodology

Carlos A. Bana e Costa[1] and Marc Pirlot[2]

[1] Technical University of Lisbon, IST–Dpt. Civil/CESUR, Av. Rovisco Pais, 1000 Lisbon, PORTUGAL

[2] Faculté Polytechnique de Mons, 9, rue de Houdain, 7000 Mons, BELGIUM

Abstract. In this paper we develop some thoughts on the future of the multicriteria field. We defend the need of theoretical robustness and practical validation of the multicriteria tools, and we state two methodological convictions as pillars for the re-orientation of the future of MCDA: the conviction of the interconnection and inseparability of the objective and subjective elements of a decision context, and the conviction of constructivism and learning. Finally, we call for a collective effort to build a general methodology that can embrace the different practices and plurality of valid multicriteria methods.

1. Multi-Criteria Decision Making (Aid, Analysis), a new Babel ?

Since the famous First International Conference on Multicriteria Decision Making (MCDM) at the University of South Carolina in 1972, an international scientific community interested in the multi-criteria domain emerged in an organised form. Despite the fact that, ten years later, MCDM was already viewed as "an independent body of knowledge" (Zeleny, 1984, p.x), several important research currents in this field have developed on both sides of the Atlantic with little direct interaction and have since remained relatively independent.

This observation can be partly explained through the influence of the different cultural environments, as well as by the diversity of scientific origins that were brought to bear for the development of multicriteria approaches. In a scientific environment dominated at the time by the paradigms of the rational decision-maker, of the optimal solution, and of the quantitative / objective information, one could expect that a need for scientific synergy among the proponents of the new multicriteria paradigm would naturally emerge. Surprisingly, however, a conceptual misunderstanding occurred between different multicriteria streams of thought, primarily between the normative "decision-making" (DM) perspective that still dominates the International Society of MCDM (formally established in 1979) and the constructive "decision-aid" (DA) attitude that characterises the European Working Group on Multicriteria Decision Aid (MCDA) (created in 1975).

More recently, an integration vision has been progressively, though timidly, taking form. It recognises that the ideas emerging from the coexistent multicriteria schools – previously seen as supporting competing approaches – are valuable streams of complementary knowledge. Indeed, it is fundamental for the progress of our domain to not perceive the different multicriteria streams of thought as conflicting, but rather recognise them as mutually supporting approaches and sources of new and rich ideas. In 1985, this vision led to the formation of a new discussion group, the Euro Special Interest Group on Multicriteria Analysis (ESIGMA).

2. Theoretical foundations and operational validation

This integration vision will likely establish the multicriteria field as a coherent body of decision-aiding tools that are both practically operational and theoretically well founded (hence the importance of investigating their properties, and the need for theoretical demonstration of weaknesses in particular multicriteria procedures).

Before achieving this "desideratum", both multicriteria theoreticians and practitioners must face a long "rocky road" in exploring the links between the desires for theoretical robustness and practical validation of the multicriteria tools. In this sense, we fully agree with the objectives for the future stated in "A Manifesto for the New MCDA Era" (Bouyssou *et al*, 1993, p.126):

"We believe that two main subjects should be explored: (1) theoretical and axiomatic foundations of MCDA at all levels (approach, methodology, methods); (2) conceptual and operational validation of the use of MCDA in real-world problems. ... The development of our theories and methods should be guided by the desire to represent a larger number of problem situations. ... The theory of MCDA is thus an open theoretical field and not a closed mathematical theory solving a specific class of problems. Key theoretical and methodological issues have to be addressed. New potential areas of applications have to be explored. 'True' applications are needed as well as ex-post analysis. Connections between MCDA and other areas of research have to be explored ..."

3. Two basic convictions

This Manifesto can perhaps be criticised for its pessimistic assessment of the current state of our field. However, an overview of the contents of the great majority of the communications presented at the most recent international MCDM conferences begs the question: isn't our research domain risking to become simply a closed branch of pure mathematics solving a specific class of (abstract) problems?

In our view, the following two methodological convictions are very important pillars for changing this state of affairs and re-orient the future of MCDA:

The conviction of the interconnection and inseparability of the objective and subjective elements of a decision context: a decision process is a system of relations among elements of objective nature (e.g., the characteristics of the actions) and elements of subjective nature (stemming both from the value systems of the actors and the ill-defined decision context). These types of elements are intricately linked and neither can be neglected in decision support. Although the search for objectivity must be of great concern, one must not forget that decision-making is above all a human activity subtended by the notion of value. Subjectivity is always omnipresent in decision-making, and therefore one must recognise the limits of a pure objective approach.

In the future, greater attention needs to be paid to structuring (and framing) as a fundamental phase in a decision-aid process. Because structuring is a mixture of art and science, it is easy to understand why significant differences, as well as many common ideas, arise in the approaches proposed by prominent "multicriteria artists" – see, for example, (Saaty, 1980), (Roy, 1985), (Keeney, 1992b). Inspired by von Winterfeldt (1980), we assert that structuring a decision situation is that learning process which seeks to build a more-or-less formal representation, by integrating the objective components of the decision context, and the subjective points of view (concerns or objectives), in such a way that the value-systems of the actors are made explicit.

The conviction of constructivism and learning: a decision situation is, in general, an ill-defined entity, unclear even to the actors involved in the decision process. A constructivist methodological approach is therefore most appropriate for decision-aiding, in light of a new paradigm of learning, that should replace the old paradigm of normative optimisation (prevalent in Operational Research and Systems Analysis). A direct implication of the adoption of a constructivist attitude in decision-aid is that simplicity and interaction are fundamental tools for effective participation.

Bell, Raiffa, Tversky (1988, p.17–18) – see also (Keeney, 1992a, p.57) – tried to clarify the differences among descriptive, normative, and prescriptive views in decision making: "Descriptive models are evaluated by their *empirical validity*, that is, the extent to which they correspond to observed choices. Normative models are evaluated by their *theoretical adequacy*, that is, the degree to which they provide acceptable idealizations or rational choices. Prescriptive models are evaluated by their *pragmatic value*, that is, by their ability to help people make better decisions."

However, none of the above mentioned authors refers to the "constructivist approach" in decision aiding (not decision making).

Some difficulties still remain in distinguishing between the constructive attitude we share and the prescriptive conception of Raiffa and Keeney. For instance, one can find in the literature references to "constructive" as a synonym for "prescriptive": "... I believe that we are witnessing the emergence of a useful technical concept which can be labeled interchangeably by the terms constructive or prescriptive."

"Raiffa's distinction between normative and prescriptive was neatly captured by Jevons (1877), specifically in relation to the theory of probability: 'Nothing is more requisite than to distinguish carefully between the truth of a theory and the truthful application of the theory to actual circumstances" (Dempster, 1988, p.286–287).

Still according to this article, Raiffa's preference for the term "prescriptive", over the term "constructive", is only justified by reasons of etymological nature: "... Shafer did write that prescriptive suggests actively helping real people, as when a doctor prescribes. Raiffa agreed that 'constructive captures an essence of what we mean by prescriptive' but worried that constructive 'may seem to some to be too systematic, too linear in conception, too structured.' Raiffa indicated that no uniqueness conception is implied by prescriptive."

Moreover, like us, Keeney (1992b) also places a great importance to structuring and problem framing: "Keeney notes that a decision analyst must be concerned with structuring the problem and with communicating the results of the analysis effectively, topics about which normative theory is silent" (Edwards, 1992, p.257).

Nevertheless, there is a methodological distinction between a "Multicriteria Decision Making" prescriptive approach designed to help people make *better* decisions, and a "Multicriteria Decision Aid" constructivist approach designed to facilitate decision-making by constructing (in interaction with the actors involved in the process) "a set of keys which might enable the actors to go forward, progressing according to their own objectives and their value-systems" (see Roy, 1993). In a prescriptive approach, an analyst begins with a description and draws up *prescriptions* based on normative hypotheses (axioms) validated by the reality thus modelled (see Keeney, 1992a). Conversely, in the constructivist approach, a facilitator helps to build a model of value-judgements seeking working hypotheses for elaborating *recommendations* (Bana e Costa, 1993).

As stated by Phillips (1990, p.150): "The model is only a guide to action, not a normative prescription, and it is at best conditionally prescriptive. In addition, the model differs from purely descriptive models in that it participates in the reality it is supposed to model. As the results of the modelling become available to participants, they compare these results to their holistic judgements. It is the inevitable discrepancies that arise, especially early in the modelling process, that drive the dialectic."

4. Towards a general methodology

The question arises: What are the future needs for the progress of the multicriteria field? We need a general methodology which would unify the field.

Not in the sense that one school of thought would impose its concepts on all and eliminate alternative ways of thinking; in fact, we believe that there are various pertinent manners of modelling a given situation and also that not all situations can be pertinently modelled within a given approach.

What we mean by "general methodology" is rather a common framework for the various specific methodologies.

What is the state of the field today? The landscape is a very contrasted one. Lots of small towns and little villages spread all over, each with its own ethnic and architectural peculiarities; their names are MAUT, AHP, Multiobjective Optimisation, Outranking methods, Problem Structuring, Negotiation, DSS, Expert Systems, etc. Some of their inhabitants are mostly concerned with technical issues, algorithms, formal procedures, while for others the process of interaction is essential. For the latter group, computations and formal procedures are of subsidiary importance if not absent. A third group are those who migrate among villages, who are not concerned with theories or general methodologies; they develop methods and software on the basis of heuristic thinking, sometimes motivated by a specific application.

What are the basic principles which would improve fruitful communication between the different villages? In our view it is mainly the recognition by everyone that different methods can in general be pertinent in a given decisional situation, and that the concept of an "optimal decision" is much weaker here than in a single objective context. This is another way of talking about the two "pillars" of a methodology stated above (see § 3). Both represent the basic ingredients of a "Decision Aid" conception of our field. People from villages like Problem Structuring, Negotiation, DSS and Outranking are closer to these ideas than people from other villages. On the other hand, the plurality of pertinent methods does not mean that any method can be applied in any situation; the people of the other villages perhaps better know that.

What we call for is therefore a collective effort to build a general methodology that can embrace the different practices and plurality of valid methods. Such a methodology can only be elaborated on the basis of a variety of practical cases of application of various methods in concrete decision processes. We thus need an extensive and well-described collection of real-world case studies.

5. Formal validity

Besides the need for a general methodology, each method must be accompanied by a particular methodology which specifies its field and modalities of application. Therefore, it is necessary to be knowledgeable of the formal properties of the methods. For those methods involving formal procedures (e.g. computational procedures), it is of great importance to manipulate (constructed) data and exploit outputs in a manner consistent with their meaning in the decisional context (e.g. summing up data which obviously have an ordinal character should be avoided.)

The literature is abound with lots of methods and procedures for choosing among, or ranking between alternatives. Most of them implement some reasonable or even appealing heuristic idea; most of them however appear as all-purpose and omnipotent methods.

In a constructive methodological perspective which embraces a plurality of pertinent approaches, we need guidelines for choosing among them. In order to provide such guidelines one has to be able to identify the types of decisional situations in which each of them is most adequate, and what are the minimal requirements for their use. In particular, both the nature of the "inputs" to the formal procedures involved in the methods and the type of conclusions which can be meaningfully derived from the "outputs" of the procedures should be known. This is necessary for guaranteeing proper practical use of the methods, and postulates that the properties of the procedures should be investigated and understood. Note that a theoretical investigation of the properties also enables improvement to the procedures which are involved in a particular methodology; indeed, one may discover procedures which dominate those used with respect to a set of desirable properties; conversely it may demonstrate that those procedures used are the best possible in the specific context. Therefore, it is important that research be carried out in studying, improving and classifying procedures that can be used as technical tools in particular methodologies. This is essential from the perspective of building a unified field of multicriteria-aid for decision-making.

The problem we are facing here is a very general one; it is the control of the correct articulation of a real-world situation with a model (in a very general sense). While this is quite a difficult philosophical problem in the physical sciences, it is still much more difficult when the modelled situations involve human judgement and interaction. The difficulty we face can be found also in statistics, classification, classical OR, etc. For example, an ongoing debate on the compared virtues of probability and fuzzy sets (see e.g. the discussion in *Technometrics* 37, 3, 1995, 249–292) can be compared to our dilemma. Beyond the technical differences, the key issue is that reasoning in natural language leads to the formulation either of a probabilistic model or of a fuzzy model; the reasonings are semantically very different and the natural arithmetic operations performed on the objects are specific to each theory; essentially probabilities add up while fuzzy numbers are naturally combined through logical operations.

6. Conclusion

In conclusion, we assert that constructivism naturally follows the process of all the activities concerning decision-aid. Indeed, by adopting a constructive approach, a facilitator does not take axioms as "norms to prescribe" once they are "accepted"; he or she will use them as "working hypotheses to recommend", through the adoption of a permanent attitude of critical discussion of the tools that he or she will select during the evolution of the interactive process of learning. Therefore, the existence of a theoretical base is a desirable but not sufficient condition for legitimating a decision-aid theory; it must be also subjected to practical validation. Thus, the application of formal or computational procedures should be done in harmony with the entire decision process, i.e. the operational treatment of the information should not violate the semantics of the interaction with the actors. Both mathematical and substantive meaningfulness are needed.

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MCDM Trends in Natural Resources and Environmental Management

Jared L. Cohon Yale University
School of Forestry and Environmental Studies

Recent events and future trends in natural resources and environmental management suggest that multiple criteria decision making (MCDM) will play an ever-more important and central role in these sectors. The emergence of ecosystem management and sustainable development, the first signs of movement away from "command and control" approaches for environmental protection, more emphasis on risk-benefit analysis, and the trend toward decentralized, consensus-based decision making form the basis for the increased need for MCDM approaches. I discuss each of these elements in turn after first defining some terms.

"Green vs. Brown ": Natural Resources and Environmental Management

There is a distinction between natural resources ("green") management and environmental, i.e., pollution ("brown") management. The distinction is not a crisp one, and the line that separates the two areas is becoming increasingly and thankfully more blurred as we move toward sustainable development. Nevertheless, we see the distinction in most of our laws and bureaucracies — the US Environmental Protection Agency has jurisdiction over water quality issues while the US Corps of Engineers and Department of Interior handle water quantity issues — and in the professionals that we train, which is especially so in developing countries where, for example, there are irrigation engineers and water quality problems are generally left to public health professionals. Natural resources management refers to the use of land and water resources to meet society's goals and objectives. Forest management — choosing what to plant and cut (and not) and when — wildlife management, protected areas management, ecotourism and river basin planning are examples of focussed problem areas and subfields within natural resources management. In contrast to these "green" activities, environmental management deals with cleaning up pollution, preventing it in the first place and restoring damaged environments. The trends discussed below generally affect one of these areas, either natural resources management or environmental management. However, the first trend that we will discuss — and the overall search for "sustainable development" — is moving us toward an integration of the green and the brown.

Ecosystem Management

Ecosystem management is the new paradigm in natural resources policy and decision-making. We see this especially in forestry and water resources planning,

but it is rapidly becoming the basis for the management of all natural resource problems, including land (e.g., for grazing or real estate development) and wildlife. Forestry is a particularly interesting example for the relative speed and drama of the changes that it is undergoing. A profession based on the utilitarian ethic of the conservation movement of the late 19th and early 20th century — using forests to provide the greatest good to the greatest number of people — forestry is now struggling to incorporate a fundamentally different view of forests, their purposes and their management. Here, as Gordon (1994) so cogently explains, ecosystem management means treating forests as integrated physical-chemicalbiological systems and managing them in a way that is responsive to the full range of society's objectives: preservation of habitat, especially for endangered species; recreation; aesthetics; ecological health; as well as the traditional and enduring desire for forest products. These objectives unavoidably conflict — consider habitat and forest products — and many of them are difficult, if not impossible, to quantify, e.g., ecological health and aesthetics. The development and implementation of ecosystem management in forestry will require the use of MCDM techniques. Academic forestry researchers have started to apply some methods (Howard, 1991), but much remains to be done to adapt and extend MCDM for forest management. Forest management is first and foremost a difficult mathematical problem. It is dynamic (trees grow over the decades-long planning horizon which foresters must consider), stochastic (there is great variation in the growth among individual trees, and prices and other parameters of objective functions are highly uncertain) and large (a forest may consist of several thousands of blocks for which interrelationships in management practices must be enforced, resulting in very large integer programs.) Added to this mathematical complexity is the challenge of quantifying criteria. Ecological health is a widely accepted goal of forest management, but measuring it is still beyond our understanding of forests as ecosystems. Recognizing that ecological health is itself a multicriterion concept may assist ecologists and forest managers in developing measures to be used in practice. The successful use of MCDM in natural resources management requires a planning and organizational context in which it can be used. Forestry, however, is still in search of mechanisms to engage and involve the public in decision making. Until it does, MCDM will be primarily an academic exercise.

The story is similar in water resources planning, but I think this sector is a few years ahead of forestry. The idea of ecosystem management — "watershed planning" is the term most often used — is not new in water resources. It has been around in one form or another for almost forty years, when the Harvard Water Program (Maass et al., 1963) articulated the need for a systems analytic and multiobjective approach to water planning. Many elements of the Harvard approach have been implemented, after decades of further development and refinement. Indeed, water resources systems analysis has been the birthplace and proving ground for many MCDM methods.

Watershed planning as it is pursued today represents a significant shift from the past, both in the problems that we now confront and the methodologies that they require. Driving this is the same shift in society's expectations that we see in forestry, which has led to a broader and more emphatically pursued set of objectives. This has forced integration of water quality and quantity issues — the green and the brown — to an unprecedented extent. Concomitant with these trends, and, in part, as a result of them, decision-making processes for water resources have become more

inclusive and open. Interest groups ("stakeholders") are increasingly formally included in planning and decision-making. All of these shifts require methodologies that can deal explicitly with multiple, conflicting objectives in a way that gives voice to stakeholders' concerns. MCDM has been used to meet these challenges in the Netherlands (Ridgley and Rijsberman, 1992) and in South Africa (Stewart and Scott, 1995.) Replicating theses fine studies and extending them to other physical and institutional settings is the next challenge.

Beyond Command and Control

Since the passage in the early 1970's of the major environmental laws in the US, environmental protection has been guided by the philosophy of "command and control," which assumes that sources of pollution can be identified and their abatement prescribed by law and regulation. This approach largely worked, leading in the United States and most industrialized countries to significant improvements in air and water quality. It worked, however, because the major point sources — smokestacks and discharge pipes at large facilities — could be identified and their facilities forced into compliance with environmental standards and penalties for non-compliance. As expensive and difficult as the last twenty-five years of environmental improvement have been, this phase was relatively easy compared to what the future holds.

While water bodies and airsheds are significantly cleaner in many places of the industrialized world, environmental problems persist. The continuing problems are a result of non-point sources of pollution — runoff from urban streets, suburban lawns and farms, and air emissions from millions of motor vehicles and wood-burning fireplaces — and they include a host of pollutants that are now getting attention — a bewildering variety of organic compounds, metals and other toxic contaminants. In addition to the technical challenge of understanding the origins, transport, fate and impacts of all of these compounds, the current problems present two large regulatory problems: cleaning them up is relatively very expensive and they are a result of the daily decisions of hundreds of million of individuals, rather than a few hundred or thousand large corporations and cities. MCDM has a role to play in responding to both of these problems.

The high costs associated with future pollution abatement is due, in part, to the fact that so much treatment is already in place. In the case of water resources, most point sources in industrialized countries already remove 90% or more of the pollution in their wastewater. These facilities are far out on cost curves that show severe diseconomies as the percent treated approaches 100%. Thus, going beyond where we are today will be very expensive, which gives pause to dischargers and politicians alike.

The future costs of environmental protection will also be driven by the addition of large numbers of new pollutants — new both in the sense of being recently considered as pollutants and actually being new compounds often in very small, but long-lived concentrations in the environment and always with uncertain impacts on human and ecological health. Removing each of these compounds from the environment or avoiding their introduction in the first place is likely to be very expensive, though with great variation from one compound to another.

Facing such high costs of pollution control, industry and many politicians are questioning whether environmental protection is worth it. This leads in short order to the idea of using benefit/cost analysis or "risk/benefit" analysis to determine environmental policy and whether to pursue specific abatement actions. This is inherently a problem in MCDM, in which decision-makers search for acceptable tradeoffs between the benefits of environmental protection, measured in terms of the reduction of health and ecological risks (which are themselves multidimensional), and the costs of providing their protection. There are many interesting issues here related to defining and measuring criteria in a way that is meaningful to the public and nontechnical decision-makers.

The other large regulatory challenge is the diffuse nature of today's environmental problems. The relative administrative ease of controlling large point sources has been replaced by the daunting task of influencing the behavior and consumption patterns of whole populations. This has led to an interesting search for new regulatory frameworks that are market-oriented, based on incentives rather than penalties and aim to prevent pollution rather than clean it up. It's an MCDM problem on a grand scale, and it has added to a movement toward decentralized decision-making for environmental problems.

In Search of Sustainability

"Sustainable development," a term now more then ten years old, has spurred a wave of reassessment of natural environmental policy and focussed attention on global environmental problems. The challenges and changes that have resulted will create new demands for MCDM. Sustainable development came to the fore at the "Earth Summit" at Rio de Janeiro in 1992. Since that time, national governments have struggled to define sustainability and to translate the idea into policy. Although each country is unique and the search for definitions and policies continues, there are some common directions that seem to be emerging. First, achieving sustainability will require an unprecedented level of integration in analysis, planning and decision-making. The interplay between economics, culture, politics and environment is central to sustainability. This not only requires ecosystem management and a fusing of the "green" and the "brown", it requires a deeper understanding of how human society relates to and is part of the natural environment, which raises fundamental and inescapable value questions. Thus, planning and designing for sustainability will present very complex analytical problems that will benefit from MCDM approaches. A second theme which is starting to emerge from national and multilateral sustainable development studies is the need for more local determination in environmental policy. Sustainability may, in fact, be most meaningful at the level of relatively small regions which can be treated as ecosystems or landscapes (interconnected ecosystems) and in which there are strong economic and social ties to the natural resources. This focus on "sustainable communities" naturally creates the need for planning and decision-making methodologies consistent with MCDM approaches. Consider this passage from a recent speech by former U.S.E.P.A. Administrator William Ruckelshaus, in which he calls for "a new sort of consensus process, in which all the significant stakeholders are brought together to hammer out a solution to a set of environmental problems." He goes on to note that such a process "is particularly

applicable to problems confined to specific geographic regions." He cites as an example the use of such an approach for forest management in the state of Washington in the U.S. Another, cited earlier in this paper, is Ridgley's and Rijsberman's (1992) application of MCDM to the Rhine River in the Netherlands, where the use of objective hierarchies was effective in bringing together the "significant stakeholders. . .to hammer out a solution. "

At the other end of the spectrum is a set of problems that are inherently global, i.e., where the relevant ecosystem is, in fact, the earth's biosphere. Global climate change and depletion of the earth's ozone layer are two excellent examples. Dealing with these problems will require cooperation by most of the countries in the world, and, as we know, multi-party problems are MCDM problems.

Recent attempts to develop plans to deal with global climate change illustrate an important point: despite the strong, almost overwhelming multiobjective nature of environmental problems, most of those involved in their analysis do not use MCDM. Neoclassical economics is still the basis for analysis of environmental problems. A news article in Nature (Vol. 376, 3 August 1995, p. 374) reported a controversy over the draft report on the social costs of climate change, then under production by a working group of the Intergovernmental Panel on Climate Change. Among the issues in dispute was the value put on human deaths due to climate change. Nature reported, "The value put on a death in a developed country, for example, was calculated to be 15 times higher than in a less industrialized nation. Such disparities result partly from the conversion of all estimates of loss from national currencies into US dollars. '$1 in, say, Cambodia is not the same as $1 in the United States' one delegate remarked. " Indeed. It's enough to make an MCDMer blush. We have our work cut out for us.

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Research Agenda in MCDM

Oleg Larichev
Institute for Systems Analysis
Moscow
Russia

Looking in the past one can trace the first origin of MCDM in economical research related to the field of the utility theory [1].That research gave a foundation to the notion of a "rational man" making the best economical decisions. The second origin of MCDM is the field of operation research [2]. The operation research gives tools for a choice of optimal solutions in complex systems:methods of mathematical programming[3]. The tools are very efficient in solving well structured problems. It is possible also to say that MCDM has an origin in engineering research on military technical systems[4]. Well known results of the research are the systems analysis and the method "cost-efficiency"[5].

A multicriteria statement of a problem is a way to a solution of ill-structured and non-structured problems widespread in the reality. But multicriteria approach is not directed to a better approximation of the reality. The central figure in multicriteria presentation of a problem is a human being with a variety of the wishes. Multicriteria models represent a human perception of the reality in the form of decision maker's preferences.

Multicriteria approach is quite different from all previous approaches. According to the economic theory of "rationality", a person selects the best decision based on one criterion: a maximization of the utility. It means a very narrow view of a person as a "maximizer" upon one criterion. Human beings described by models of the operations research (e.g. transportation problems) have no freedom of choice.Their behaviour is described in the same way as a movement of physical objects. In "cost-efficiency" approach one is trying to convert one criterion (cost or efficiency) into a constraint to receive the usual engineering task.

In the contrary to all previous approaches multicriteria approach is focused on the attention to a human being (decision maker) making a free choice with many conflicting criteria. This approach is much more promising but at the same time much more difficult. One can look at a decision maker (DM) from different points of view: psychological (how a person is making a decision); mathematical (how to develop a rational procedure for a decision making); neuropsychological (how the brain is operating in a decision making process); behavioural (how decisions are made in an organisation) and so on. That is why multicriteria problems need various tools for an investigation, research and solution.

There are several problems which are crucial for a future development of MCDM.

1.Extended understanding of the Decision Process

H.Simon defined three principal phases in a decision process: search of an information ("finding occasion for making a decision"), design of alternatives ("finding possible courses of action"), choice of the best alternative ("choosing among courses of action") [6].

Up to now the majority of research is directed to the third phase of a decision process: the comparison of alternatives. But in the reality the decision process usually starts much earlier. Sometimes it is necessary to spend big efforts to find or to construct alternatives. Sometimes the search for the criteria of alternatives evaluation is a difficult task.

The extended understanding of decision processes was demonstrated in the recent book by R.Keeney [7].

Of course two first phases are poorly formalized. The skill of an analyst plays a critical role in those phases. But it is possible to develop some tools aiding the consultant in a structuring of a problem. A first DSS of such kind was MAUD [8].This system allows one to gather in a systematic way the information at the first phase of a problem's investigation.

The second phase — finding or creation of alternatives, is especially difficult. Only human beings (DM, expert, analyst, consultant) can invent new alternatives, but DSS should help them in determining particular unsatisfactory features in existing alternatives and to form the dezirable combination of them [9]. For example, in the case of gas pipeline route selection in north region of Russia [10] there are several active groups participating in the decision process. Each active party has its own perception of the best alternative and different subsets of criteria. The phases of structuring, design of best alternative and selection are interconnected. For complex problems of such kind auxiliary computer systems are especially needed. They can play different roles: a data base; a tool for analysis of active parties policies; DSS for negotiation and so on. Let us note here Multiple Party Multiple Criteria descriptive model developed first by H.Kunreuter [11].

An item in the agenda for future research in MCDM is needed: development of tools aiding analysts in: a) structuring of a problem: b) design of new alternatives; c) finding a solution for MCMAP-multiple Criteria Many Active Parties problems.

2.Measurements of uncertainty

The problem how to measure a quality of objects (alternatives, variants of a decision and so on) in terms of many criteria deserves a special attention. Very often decisions are being taken in a condition of uncertainty. The common approach for such cases is a utilisation of subjective probabilities as a degree of belief expressed by DM. It is taken for granted that a human being can express his belief in numbers located on a segment from 0 to 1. Moreover, a possibility to receive a distribution of probabilities for an uncertain factor is accepted. The procedure has been elaborated where DM is to give an answer to a question like the following one: "Give me the value of an uncertain factor such that the probability of this event above and below that value are equal". It is supposed that DM after some hesitations can give this number. After several questions of this type a nice probability curb can be

constructed. This procedure can be valid in case when DM has in his/her head some information about the uncertain factor connected with his/her previous experience but in a vague form.

There is evidence that in many cases such procedure is an unreliable one. Many studies of "heuristics and biases" demonstrate that people are poor experts in nomination of quantitative probability. Really, without a pressure of an analyst DM can only honestly say: "I don't know a numerical probability of such an uncertain event" or: "I can only say that this uncertain event is more probable than that one". But those answers are not suitable for quantitative models like ,for example, one of subjective expected utility. We can observe a domination of quantitative models with numerical probabilities in the majority of textbooks on decision analysis.

But there is a more reliable way to measure a probability of uncertain events: verbal, qualitative measurements. There are experiments demonstrating that from the very childhood people can consistently use verbal expressions of probabilities for uncertain events [12]. So, in some cases, a measurement in natural language can be very reliable. It is a way to overcome some kind of uncertainty in decision processes: adaptation to a way of measurement typical for human beings [13,14].

Different type of adaptation we can observe in the case of very uncertain events when people have no previous experience. For such situations even qualitative evaluation of uncertainty in an absolute form is unreliable. One can use only comparative judgements in this case.

Let us suppose that one wants to measure the weight of an object but a balance is absent. If there are two objects one can define in an approximate way which object is heavier. A measurement could be done by using a crude scale like this one: 1. heavier; 2. approximately the same weight; 3. lighter. An output of such measurements can not be more exact. Exactness of an output of measurements must correspond to an accuracy of data.

The second item in an agenda of future MCDM research consists of development of models appropriate for verbal and comparative measurements of uncertainty.

3. Combining descriptive and prescriptive

Nowadays we witness a big gap between descriptive (psychological) research on one hand and multicriteria methods and decision support systems (DSS) on the other hand. The psychologists are not interested in practical consequences of their findings. The mathematicians consider an axiomatic foundation of decision methods as the only scientific one. They look traditionally at a human being as a measuring device giving a necessary information without errors.

But the are some changes. Some books combine results of descriptive and normative studies [15]. The research on possibilities and limitations of human information processing system takes a substantial place in the psychology [16]. During MCDM conference held in Coimbra for the first time the discussion of "Behavioural Decision Making " was organised. One can stress also attempts to construct new prescriptive theories taking in account behavioural aspects like the Prospect theory[17]. Unfortunately, there is no general prescriptive theory for construction MCDM methods. We have now many interesting results demonstrating "what a human being can not do"[18].Much less known about

"admissible"[14] information processing operations for which human performance is quite satisfactory. Only operations of such kind are used in verbal decision analysis [14,19].

The third item in an agenda for future research in MCDM is a development of psychologically valid normative methods and DSS oriented to a wide class of problems.

4.A "Guide" for MCDM world

MCDM has some features of mature research field: many societies exist and conferences are carried out; special journals have appeared; good textbooks have been published [20,21,22]. Along with that one can see an abundance of multicriteria methods: MAUT [7], AHP [23], ELECTRE [24], different methods of Multicriteria Mathematical Programming [20] and so on. Nowadays frontiers between domains of the most suitable applications for different MCDM methods are not defined.

On the other hand, there is big variety of MCDM problems in diverse practical fields such as economy, engineering, medicine, policy making and so on. The diversity of problems requires a diversity of multicriteria methods and DSS. I believe that for majority of MCDM methods it is possible to find the type of problems or a field of application in which a method has an advantage against other one.

An item in an agenda for future research is the elaboration of a "guidebook" in MCDM world.

MCDM is an exciting field of research having big potential for future development. The basis for this development is an interesting MCDM community — a family having members from different branches of science and from many countries. Exchange of ideas during conferences and in journals is vital for a future successful life of this family.

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MCDM at the Crossroads

Wojtek Michalowski
School of Business
Carleton University
Ottawa, Ontario K1S 5B6
Canada
email: wojtek@business.carleton.ca

Abstract

This paper discusses challenges and opportunities facing the multiple criteria decision making (MCDM) researchers. It reviews some of them and presents the author's view regarding possible courses of action.

1 Introduction

MCDM is a field concerned with solving decision problems involving multiple, conflicting objectives. For the purpose of this paper, the developments within multi attribute utility theory (MAUT) will be excluded from considerations. MAUT as a field is usually served by different journals and conferences, and it has much closer ties to the classical decision-theoretic paradigm than to the MCDM.

An overview of papers published in the MCDM conference proceedings and journals allows to establish the following taxonomy of problems comprising the MCDM:
- single decision maker continuous choice problems (treated as continuous mathematical programs);
- single decision maker discrete choice problems (treated as either discrete programs or ranking problems);
- multiple decision makers with single or multiple criteria (treated as group decision making or negotiations problems).

A significant number of publications covering different aspects of the above areas indicates sufficient maturity of the field. Achieving this level of maturity prompts discussion on future directions and perspectives of the MCDM. Zionts (1992, 1994) was one of the first to initiate such a discussion, advocating that we should:
- increase the number of applied studies in the MCDM;
- develop eclectic approaches combining different decision paradigms;
- emphasise theoretical and applied research on negotiations.

Similar views were presented by Dyer *et al* (1992), where in addition the future of the MCDM was considered as depending on:
- relaxation of assumptions regarding precision in problem formulation;
- incorporation of behavioural considerations into normative models;
- development of heuristics for generation of solutions to MCDM problems.

An interesting and inspiring document analysing state-of-the-art and prospects of the MCDM was prepared by Bouyssou *et al.* (1993). The authors of this document view lack of *ex post* analysis of the MCDM applications and lack of positive feedback between theoretical and applied studies as one of major shortcomings of the field. They advocate that the MCDM be treated as "an open field and not closed mathematical theory solving a specific class of problems". They also ground their justification for the MCDM development in the utilisation of the MCDM tools in real life situations.

The issues outlined in the discussion papers point towards the significant consensus of the opinions regarding desired trends of the MCDM development and factors affecting it. Hence, it is valid to ask what impact these opinions have had on mainstream research in the field of MCDM? Review of the past 15 years of the MCDM research suggests that the field is almost immune to these discussions. Apart from the somewhat superficial impact of desktop computing on algorithmic developments, mainstream MCDM research follows quite closely patterns established in the 1970s and mid 1980s. This paper is an attempt to analyse the causes of this situation and to provide some conclusions.

2 Challenging Established Wisdom

One of the manifestations of a healthy and vigorous scientific discipline is associated with challenging the "collective wisdom" forming its axiomatic foundations. Without such challenges scientific discovery would suffer, thus impending the ability to analyse and undertake more complex problems.

Zionts (1994) in a paper discussing the state of the MCDM, presents "ten myths about MCDM" as anchors for stimulation of future research. The analysis presented here uses these ten myths as a framework to present the author's view on the nature of challenges* presently pursued and the implications of these challenges to the future of the MCDM.

The myth of solving a problem

Most methodological research in the MCDM is concerned with development of axioms or procedures which allow for the generation of a "better" solution to a problem. At the same time, few of us tackle the issue of purposefulness of the very process of solving a problem. In my opinion, this lack of interest results from the "algorithmic roots" of the MCDM research. Natural to these roots is a view that "a model is a decision problem", with the collorary that "a model has to be solved". Challenging this myth should give us a better understanding of the *raison d'etre* of the MCDM modelling.

The myth of a decision maker

This myth is a straightforward consequence of "solving a problem" attitude. A decision maker is equated with some utility function which is mechanistically

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* This paper focuses on "mainstream MCDM research" represented by most of the conference presentations and publications. It does not preclude studies which go beyond the scope of the challenges discussed here..

applied to evaluate decision alternatives. Whilst in reality, the decision maker being the driving force of a decision problem, is subject to psychological considerations and cognitive limitations. His/her role in a decision process changes depending on the type and scale of choices facing him/her. The fact that we do not challenge common perceptions about the decision maker's role in the MCDM processes limits our abilities to understand the intertwined character of the provision of decision support and the decision maker's need for it.

The myth of fixed set of alternatives

The MCDM is frequently perceived as a mathematical theory developed to solve some abstract models. This perception does not stimulate studies on the relaxation of the fixed set assumption, because the core foundations of the theory would then be challenged. Unfortunately, there are very few of us who are taking up this challenge.

The myth of an optimal solution

In the situation where the myth of solving a problem is not being sufficiently challenged, it is normal that we are content with searching for "optimal" solutions. We concentrate our efforts on developing "measures of optimality" leaving aside reasons for searching for "a solution". This attitude is further enforced by our acceptance of a definition of "a solution" as being given and undisputable.

The myth of considering nondominated solutions

Early research on economic behaviour had a significant impact on the MCDM by providing it with one of the conceptual pillars, namely a concept of efficiency (Pareto-optimality, dominance). Despite existing evidence that it is possible for a decision maker to wilfully select nonefficient decisions, we consistently adhere to the notion of dominance as a cornerstone for our research. If we expect wider acceptance of our approaches, especially in situations involving multiple participants, we need to challenge assumptions of economic behaviour and develop new measures for assessing the effectiveness of the decisions.

The myth of the existence of an utility function

While developing new approaches to the MCDM problems we try to demonstrate how they relate to different utility axioms. We are comfortable in stating that a given approach is superior to others because its underlying utility function is more sophisticated. We do that, despite the fact that a cognitive value of using utility functions as decision models, is being challenged by behavioural decision making. We should benefit from this challenge by studying decision situations with insufficient information for making utility assumptions, or when this function is temporal and nonstationary.

The myth of decisions and outcomes being static

This myth can be attributed to limited knowledge we have about studies in the cognitive science and the strong impact of applied mathematics on the MCDM. A univocal mapping F: X->Y (where X denotes a set of decisions and Y represents a set of the outcomes) is one of the unquestionable assumptions of our work. A cognitive scientist would challenge that, saying that choice is a multistage

cognitive process subjected to several external stimuli which frequently distort the crisp relation between decision and its outcome.

The myth that sophistication/complexity is good

The conventional wisdom amongst us is that complex decision problems require complex methodology. This wisdom combined with the perceived non-importance of applied studies contribute to increased methodological complexity and decreased applicability of our work. If we emphasise practical applications (not necessarily confined to case studies) and the resolution of practical problems, then we should be able to discover that complex problems can be also approached in a simple and intuitive manner.

The myth of mathematical convergence

Applied mathematics is a rigorous discipline which requires the exact specification of termination conditions of a solution process. If this requirement is transplanted into the MCDM problem, it results in a definition of convergence conditions for an interactive decision process. In our studies we attempt to demonstrate that certain approaches converge faster than the others without challenging the correctness of such comparisons. Decision processes are convergent in a psychological sense, and mathematics do not have a lot to do with that.

The myth of satisfying all model assumptions

This myth implies that a model *is* a decision problem, as opposed to being its abstract representation. Real life decision problems are sufficiently rich, so adherence to modelling assumptions limits our cognitive capabilities. Challenging this myth should allow for the appreciation of a process of creating problem representation which at some later stage of a choice process may be transformed into a normative model. Analysis of such a model should account for some of its assumptions not being satisfied and should provide us with a feedback regarding problem representation and understanding. These are the issues which deserve answers.

Challenging the above myths might help us to realise where we are and in what direction we are going. A fear of asking difficult questions and questioning established truth, combined with a fear of failure are detrimental for the development of our field. In comparison with some "sister" disciplines, the MCDM resembles French society 100 years prior to French revolution. A society characterised by a well established hierarchy of truth and strongly guarded against any dissent.

There are two important catalysts which might change this situation. They are:
- establishment of closer contacts with business/government organisations;
- introduction of changes to the undergraduate management science curriculum taught at the universities.

Importance of real life applications for the development of the MCDM should not be underestimated. Establishment of the "Wiley prize in applied decision analysis" is the first and very important step in that direction. Increased interaction

with business and government people should provide us with a better comprehension of the reasons for an organisation to apply our tools. This understanding, stimulated by challenging the established wisdom should propel the MCDM into a qualitatively different sphere of thought. The depth and enduring character of this sphere will greatly depend on the understanding and perception of the MCDM by the decision makers at large. That is where another catalyst comes into play - the importance of educating students about the tools of the MCDM. Currently, most of the management science textbooks confines our field to goal programming models and the analytical hierarchy process analysis. This narrow view of the MCDM does not allow future decision makers to appreciate the opportunities and breath offered by the MCDM methods. Hence, it is very important that our community develops a broader treatment of the MCDM in the management science curriculum. Such an effort should be combined with the increased exposition of the MCDM topics in our lectures. This exposition should focus on the benefits associated with the MCDM modelling, and not on the algorithmic aspects of the MCDM problems. We should pay attention to the fact, that the management science is increasingly focused around modelling and problem structuring, as opposed to the classical algorithmic side (Powell, 1995). Encouraging students' interest in the MCDM at the undergraduate level will bring us significant benefits in the future.

3 Next Twenty Five Years

The state of any scientific discipline is determined by the people who work in it, and by the ideas they pursue. The next twenty five years of the MCDM will also depend on these two factors.

An informal survey conducted by F. Lootsma during one of the recent IIASA meetings demonstrated that most of us are academics (similar conclusions can be drawn after reading the address list published in the *WorldScan* newsletter). Very few of us are practitioners who would actually benefit from the tools of the MCDM analysis. It should be our goal to increase non-academic representation amongst us. Following the example of INFORMS annual meetings, we should try to organise "practitioners workshops" which facilitate the exchange of ideas between us and potential users of our work. Such workshops would not only make our conferences more interesting, but will most likely be the source of new ideas for our research.

Most of us agree that the MCDM is a mature field of study. Any "mature organism" requires more oxygen to keep up with the demands of the environment. In our case, this "need for additional oxygen" can be translated into a need for the increased presence of Ph.D. students in our activities. A "Ph.D. forum" organised at one of the MCDM meetings is first and important step in that direction. This effort should be combined with some provision of support for Ph.D. students willing to participate in our activities and facilitation for publication of their work.

The MCDM should be an open field allowing for interaction of ideas from the classical decision analysis, artificial intelligence, mathematics, cognitive science, and psychology. This openness will produce new axioms, new approaches, and new methodologies which will deal with such issues as:

- creation of decision problem representations as opposed to solution of decision models;

- creation of integrative representations encompassing often incompatible decision paradigms;
- fostering modelling activities as insight providers as opposed to problem solvers;
- development of axiomatic foundations for decision support and decision structuring.

Obviously, the above list is non-exclusive and highly subjective. Nevertheless, it may give some direction for discussion concerning challenges facing the MCDM.

4 Conclusions

The MCDM is a very interesting field of study with great potential. It is up to us who work in this field, to live up to this potential and to contribute to the prosperity of society. I hope that this paper, together with other contributions published in this volume will add to the ongoing discussion on a redefinition of our goals, our positions, and our prospects. The existence of such a discussion manifests the strength of the MCDM and encourages an optimistic outlook for the future of our field.

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Non-Fully Resolved Questions about the Efficient/Nondominated Set

Ralph E. Steuer
Management Science
Brooks Hall
University of Georgia
Athens, Georgia
30602-6255 USA

Introduction

In this paper is discussed the state-of-the-art in the following topics pertaining to efficient and nondominated sets in multiple objective programming:

(a) number of efficient extreme points
(b) software implications
(c) distribution of maximally efficient facets as to their dimensionality
(d) criterion cone decomposition
(e) proportion of unsupported nondominated criterion vectors
(f) quad-trees for storage and retrieval
(g) distributions of U-valuesNotation and Terminology Employed

A multiple objective program is of the form

$$\max \{f_1(x) = z_1\}$$
$$\vdots$$
$$\max \{f_k(x) = z_k\}$$
$$\text{s.t. } x \in S$$

where S is the feasible region in decision space. Let $Z \subset R^k$ be the feasible region in criterion space where $z \in Z$ iff there exists an $x \in S$ such that $z = (f_1(x),...,f_k(x))$. Let $K = \{1,...,k\}$. In R^k, criterion vector $\underline{z} \in Z$ is nondominated iff there does not exist another $z \in Z$ such that $z_i \geq \underline{z}_i$ for all $i \in K$ and $z_i > \underline{z}_i$ for at least one i. In decision space, point $\underline{x} \in S$ is efficient iff $\underline{z} = (f_1(\underline{x}),...,f_k(\underline{x}))$ is nondominated. Let the set of all efficient points be termed the efficient set E and the set of all nondominated criterion vectors be termed the nondominated set N.

Let $U: R^k \rightarrow R$ be the decision maker's utility function. A $z^0 \in Z$ that maximizes U over Z is an optimal criterion vector, and any $x^0 \in S$ such that $(f_1(x^0),...,f_k(x^0)) = z^0$ is an optimal point. Our interest in E and N results from the fact that if z^0 is optimal, $z^0 \in N$ and $x^0 \in E$.

(a) Number of Efficient Extreme Points

In the all linear case, we have the multiple objective linear program (MOLP)

$$\max \{c^1x = z_1\}$$
$$\vdots$$
$$\max \{c^kx = z_k\}$$
$$\text{s.t. } x \in S = \{x \in R^n \mid Ax = b, b \in R^m, x \geq 0\}.$$

For up to k = 5 objectives, m = 50 constraints, and n = 100 decision variables, the numbers of extreme points of S that are efficient have been carefully studied, for instance, in Steuer (1994). However, above these sizes, the numbers of efficient extreme points can easily climb into the tens of thousands, but the rate at which this happens is not fully known. What might be interesting would be to conduct additional experiments on larger problems and then develop regression models that relate the number of efficient extreme points and CPU time to k, m, n, the size of the criterion cone (convex cone generated by the c^i), and perhaps, the A-matrix zero density. It might also be interesting to determine whether special structure plays a role (e.g., multiple objective transporation versus regular MOLP).

(b) Software Implications

Although the efficient set has been studied by many, few durable computer codes have emerged. Two MOLP codes still in distribution by their authors are TRIMAP (see Climaco and Antunes, 1989) and ADBASE (see Steuer, 1995).

As its name suggests, TRIMAP is only for 3-objective MOLPs, and on such problems, TRIMAP provides a nice graphical rendition of all efficient extreme points and all efficient 1-dimensional and 2-dimensional facets. Unfortunately, TRIMAP only runs on MacIntosh computers.

ADBASE, on the other hand, can compute all efficient extreme points of an MOLP with any number of objectives, but beyond being able to compute all unbounded efficient edges, it is unable to compute higher dimensional efficient facets. While ADBASE runs on PCs, workstations, mainframes, and supercomputers, it has not yet been programmed for use the MacIntosh.

What is needed is a code for computing all efficient extreme points and all maximally efficient facets that runs on <u>all</u> computers.

(c) Distribution of Maximally Efficient Facets as to their Dimensionality

An efficient facet of the feasible region is maximally efficient iff that facet is not a subset of an efficient facet of higher dimensionality. The convenience of maximally efficient facets is that in an MOLP, the efficient set E cannot be expressed more parsimoniously than by the union of all maximally efficient facets.

There are three main approaches to computing all maximally efficient facets. One is the <u>inside</u> approach which involves adding logic to a computer code that

computes all efficient extreme points as was implemented in EFFACET (Isermann and Naujoks, 1984), a code that was available for a period during the 1980s. Another is a post-processing approach involving an analysis of the active constraints (slack = 0) at each efficient extreme point (Zeleny, 1974). A third is the generalized reachable sets approach outlined in Bushenkov, Chernykh, Kamenev and Lotov (1995) which involves convolution techniques on systems of linear inequalities. Because of the computation involved, all approaches have been restricted to small MOLPs.

However, with the speed of today's computers, it is believed that with these approaches, further progress can be achieved. Also, since all maximally efficient facets need not be of the same dimensionality, it might be interesting to do a study on how an MOLP's maximally efficient facets are distributed by dimensionality from 0 to n-1.

(d) Criterion Cone Decomposition

Because of the time required to compute all efficient extreme points in large MOLPs, it may be beneficial to subdivide a large MOLP (say, with more than 10,000 efficient extreme points) into parts. This can be accomplished by decomposing the criterion cone into subset criterion cones, and then solving separately for those efficient extreme points pertaining to each subset cone. Savings can be realized in several respects.

One is that the time required to consult a list to determine whether a newly detected efficient basis has been visited before should be less because the lengths of the lists in the subset problems should be roughly proportional to the sizes of the subset criterion cones. Also, there should be memory savings because of the expected reduced lengths of the lists. A third is that the subset-cone problems can be run in parallel on a computer thus enabling reduced elapsed times. Computational testing appears to be about the only way to confirm the magnitude of these projected savings.

(e) Proportion of Unsupported Nondominated Criterion Vectors

Let $\Lambda = \{\lambda \in R^k \mid \lambda_i > 0, \Sigma_{i \in K} \lambda_i = 1\}$. A nondominated criterion vector $\underline{z} \in Z$ is unsupported iff there does not exist a $\lambda \in \Lambda$ such that $\underline{z}$ solves the weighted-sums program

$$\max \{\lambda^T z \mid z_i = f_i(x), x \in S\}.$$

While unsupported nondominated criterion vectors cannot occur in an MOLP, they can easily occur in multiple objective integer and nonlinear programs. The significance of an unsupported nondominated criterion vector $z^u \in Z$ is that there exists no $\lambda \in \Lambda$ such that z^u is computable via the weighted-sums program. Instead we must use a Tchebycheff type of program along the lines of those discussed in Wierzbicki (1986).

Beyond knowing that unsupported nondomiated criterion vectors can occur when the feasible region Z in criterion space is nonconvex, little is known about their degrees of prevalence. This would be another deserving area for a computational investigation.

(f) Quad-Trees for Storage and Retrieval

A quad-tree, originated by Finkel and Bentley (1974) and adapted to multiple criteria use by Habenicht (1982, 1986) and Sun and Steuer (1996), is a datastructure that is believed to have great potential for detecting, storing, and retrieving nondominated criterion vectors.

In two dimensions (with criterion vectors from R^2), a quad-tree is formed as follows. The first criterion vector is used to divide R^2 into "quadrants". The next criterion vector divides the (empty) quadrant into which it falls into four "subquadrants". If a third criterion vector falls into an empty quadrant, it divides that quadrant into four subquadrants. Otherwise, the criterion vector divides the empty subquadrant into which it falls into four subsubquadrants. A fourth criterion vector will divide the empty quadrant, subquadrant, or subsubquadrant into which it falls into four subquadrants, subsubquadrants, or subsubsubquadrants, respectively, and so forth. An advantage of the datastructure is that by examining only a small subset of the vectors in a quad-tree, it can be determined whether a new vector (a) is dominated by a criterion vector in a quad-tree, or (b) dominates a criterion vector already in a quad-tree.

(g) Distribution of U-values

Suppose, using a utility function, the U-values of all efficient points are scaled 0 to 100 over the efficient set. Assuming an MOLP with 1,000 efficient extreme points, one might be tempted to expect 10 of the efficient extreme points to be within 1% of the optimum. However, the number is usually far fewer as the efficient extreme points tend not to be uniformly distributed by U-value. Moreover, with nonlinear utility functions, the best efficient extreme point might not even have a U-value of 100! From the preliminary results reported in Steuer (1994), one might hypothesize that the frequency of efficient extreme points by U-value roughly follows a gamma distribution, with the order number of the gamma distribution increasing, for instance, as the p-value of an L_p shaped utility function ranges from $p=1$ to $p=\infty$.

Conclusions

There is still a large amount of computational-based research to be conducted on the efficient/nondominated sets in multiple objective programming that should still be welcomed by the literature.

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Future Trends in MCDM

Theodor J. Stewart
Department of Statistical Sciences
University of Cape Town
Rondebosch 7700
South Africa

Abstract
We comment firstly on the wide diversity of MCDM techniques which exist, and suggest that at least partly, the diversity arises from three different decision contexts which can be recognized. Recognition of these different contexts may assist in developing a "meta-approach" to MCDM, which is the first challenge to MCDM which is identified. Other lines of development are then suggested, including the need for alternative methods of treating risk in MCDM, for methods of visualization of the decision space, and for standards for validation of MCDM procedures. Finally, we emphasize the need for MCDM to exploit methods of soft systems and problem structuring, and to contribute to the development of groupware for group decision support.

1. Current State

Before starting to make projections of where the field of multiple criteria decision making is heading, we need to take stock of where we are now. Superficially, there seems to be a Babel-like confusion of tongues as we look at the widely divergent classes of techniques for MCDM (cf. Stewart [6] and Bana e Costa et al. [1]). It may be useful, however, to look firstly at the types of context within which multicriteria decision aid may be required, with a view to categorizing MCDM techniques according to context. At least three contexts can be identified:

(1) Direct aid to the ultimate "decision maker" (an individual or small homogeneous group): The primary aim of MCDM here is to facilitate exploration of the decision space and the available trade-offs, and to provide information and guidance as to the extent to which certain choices may violate apparently desirable normative axioms. In this context there is little or no need for an "audit trail" to document the rationale for the decision; it suffices that the decision maker be comfortable with the final decision.

(2) Aid to a representative group operating on behalf of a larger body, and required to produce a final recommendation (a single course of action or a short-list): In this case, the aim of MCDM is to facilitate the incorporation of the values of different interest groups in a relatively formal manner, and in such a way that "justice can be seen to be done" in properly accounting for all group interests. The availability of an "audit trail" to document how recommendations were arrived at is essential, while conformity to nominally "rational" norms may further enhance credibility.

(3) Aid to a staff group in preparing alternatives for decision by a higher body: The primary aim is to screen out clearly less desirable options and/or to provide some partial preference orderings on the basis of expressed decision maker values. The staff group would of itself have no decision making authority, and would need to minimize the extent to which value judgements or axiomatic assumptions are imposed on the process.

The first context is typical of private sector (particularly in small corporations) or personal decisions, while the last is typical of public sector decisions in some countries at least. The second context can arise in private (particularly larger corporations) or public sectors. To some extent, the three contexts can be associated repectively with "progressive articulation of preferences", "prior articulation of preferences" and minimal or no articulation of preferences, to use a classification which has been used elsewhere (e.g. Goicoechea et al. [3]) to differentiate between processes (methodologies), as opposed to application context. Different workers in MCDM may tend to focus on approaches that are relevant to the contexts with which they are familiar (through experience and/or cultural background). There is no problem with this, but emphasis on differences in methodological approach may obscure the different aims and generate sterile conflict rather than constructive synthesis of ideas in MCDM.

2. Synthesis of Existing MCDM Techniques

We have asserted elsewhere (Bana e Costa et al. [1]) that the confusion of approaches to the MCDM problem should not be seen as a monster, or as destructive competition, but rather as a number of resources to be harnessed to work together. This is one of the major challenges to research in MCDM over the next few years. The classification by problem context given in the preceding section may be helpful in giving direction to such research. Initially, it may be useful to make an inventory of existing MCDM techniques, and to evaluate the extent to which each technique appears to be able to contribute to decision aid in each of the three contexts. Another possibility is a critical analysis of published MCDM case studies, in order to identify both the context and the methodologies which were successfully employed. A better understanding of the context in which different methodologies are appropriate will assist in identifying the manner these methodologies can be linked to provide synergistic gains. Different contexts demand different types of insight, and this is mirrored in the methodologies. But all insights are ultimately of value in all contexts, and the scope thus exists for evolving a meta-approach to MCDM that utilizes all methodologies to generate multiple perspectives. As illustration of potential synergies, consider the following:

Interactive Goal Programmimg or Reference Point Approaches
appear to be particularly relevant to the first problem context as they involve the decision maker directly in the search from the beginning. On the other hand, some tentative exploration of the decision space in the other two contexts, even if this is of a rather subjective nature, may assist task groups or teams in understanding where the critical choices lie (to be referred back to principals) and/or in conveying results.

Value (Utility) Function Approaches appear to be particulaly
relevant to the second context, and require, in principle, a priori value judgements (such as trade-offs). Coupled with sensitivity analysis, however, they can be used to reveal precisely where value judgements are most necessary (valuable in the third context), and to indicate what are realistic goals in other contexts (or where more creative thinking is necessary to extend the decision space to make "unrealistic" goals realizable).

Outranking methods (ELECTRE) and Data Envelopment Analysis (DEA)
appear to be particulaly relevant to the third context, as they emphasize what can be concluded with a minimum of assumptions. It is perhaps valuable to utilize approaches such as these as a pre-screening of alternatives in the other contexts, so as not to waste effort and time in establishing values and trade-offs which are entirely unnecessary. Some readers may find the inclusion of DEA in this context unusual, but a recent interchange in the journal Omega ([2],[7]) illustrates the above points well.

In summary, one major new area of development in MCDM should be the development of a meta-approach which effectively synthesizes the existing methodologies to achieve greatest synergy.

3. Scope for New MCDM Techniques

The search for synthesis of existing approaches does not imply that there is no longer a need for developing new MCDM techniques. Undoubtedly, many gaps in our existing toolkits still exist, but we shall comment here only on two:

The treatment of risk and uncertainty: All decision problems
contain a substantial element of risk and uncertainty, but much of the MCDM literature largely ignores this fact! It is true that multiattribute utility theory provides a formal decision analytic manner of treating this problem, but this is solidly entrenched in the prior articulation of preferences school, and requires complex value judgements and full elicitation of probability distributions. We have argued elsewhere [8], that there is considerable scope for development of alternative procedures, based for example on scenario planning concepts.

Descriptive/exploratory techniques: MCDM has developed largely
in a prescriptive, and sometimes normative, framework, and the aim has tended in consequence to be to produce at very least a shortlist or partial ordering of alternatives. There is scope for using modern three-dimensional graphics and multivariate statistical techniques to provide the user with a visualization of the decision space, without normative characteristics. Some start has been made in this direction (cf. section 7 of [6]), but much more could be done.

4. Validation of MCDM Techniques

This appears to be one of the major challenges facing MCDM over the next decade. It is well recognized that many new MCDM tools and techniques are published each year with very little being done to provide a generally acceptable standard of validation of these tools. Some of the problems are lack of recognition of the different contexts which we have described above, and lack of consensus concerning what validation entails.

The first step would be to establish standards of validation for MCDM techniques. Such standards would need to recognize, amongst others, the following:

(1) Are the models used (e.g. for aggregation, elicitation of weights) internally and mathematically consistent, and how robust are they? For example, the use of additive value functions methods implies certain properties of the decision maker's preference structures, and it may need to be asked whether the methods of elicitation are consistent with these same properties. Sensitivity of the methodology to small changes in inputs, or to violations of assumptions of specific classes of preference structures, would also be a matter of concern.

(2) What norms are imposed by the methodology itself? Restriction to non-dominated solutions may be considered desirable in many contexts, whereas restriction to non-convex-dominated solutions may be much more arguable. Imposition of transitivity is viewed by some as desirable, but not by others (possibly as a result of the contexts with which they are familiar). There seems to be a need to classify different normative impositions which can arise, and to reach some form of consensus as to which of these are strongly desirable, or strongly undesirable, in each context.

(3) How efficient is the procedure? How long does the decision maker need to spend in responding to questions, and how demanding or stressful are the judgements which are needed?

(4) How satisfied are users with the results? In one sense, this is truly the "bottom line", but there are many problems in making this concept operational. What constitutes satisfaction? What may be satisfying in the short-term may turn out to be far less satisfying when reflecting back on the decisions months or years later. Clearly any measure of satisfaction will be highly context-dependent. What, for example, are the pros and cons of basing measures of user-satisfaction on groups of MBA students (which are repeatable measures of satisfaction in artificial problems, but with questions about real committment), or on real decision makers (where the problem and committment are real, but there is no chance of repeatibility or control for factors such as the effect of the facilitator)? These are hard questions which need to be resolved.

Comparative studies of methodologies should in principle be valuable, but tend to be problematical. In some cases, actual case studies have been used, but then conjectures have typically had to be made about how the decision makers would have answered the questions imposed by methods other than the one actually used. These conjectures might well be influenced by the researcher's own

background and experience. In artificial problem settings, decision makers may be asked to analyze the same problem setting using a variety of methods in randomized order, but the generalization from artificial to real problems remains debateable. In recent work [9] we have suggested a simulation approach, but this only addresses the first three points above. Clearly, this issue remains an important challenge to MCDM.

5. Problem Structuring

The use of "soft" systems and problem structuring approaches is becoming widely accepted in management science in general, but little of this has yet filtered into the MCDM field. This, too, seems to be an important area for development in MCDM over the next 5-10 years. These "soft" methodologies have at least two potential roles to play which are specific to MCDM (i.e. apart from the general aid given to systems modelling, which is relevant to assessing impacts of alternatives in terms of specific criteria), viz:

(1) Support for the identification of criteria: Although
desirable features of a set of criteria have been suggested in a number of places (e.g. Keeney and Raiffa [4] and von Winterfeldt and Edwards [10]), and although some anecdotal experience has been reported regarding how analysts have elicited sets of criteria, there has been little research into the overall facilitation process for identifying criteria. The soft systems and problem structuring methods appear to have much to offer here, and what is required is for the MCDM community to integrate these concepts into the MCDM "meta-approach".

(2) Support for generation of alternatives: MCDM has
traditionally focussed on the analytical process of evaluating a given set of decision alternatives. As indicated by Keeney [5], the MCDM field needs to broaden to include a "value-focused thinking", i.e. the identification and design of alternatives which best satisfy decision goals. The use of the softer methodologies offers scope for facilitating this constructive process.

As a final challenge to the MCDM world, I wish to draw attention to the use of "decision rooms" to facilitate group decision making. Much groupware is already available, linked to impressive hardware set ups. These certainly provide powerful tools for the generation and synthesis of ideas in strategic decision making contexts. But this development has been driven by computer scientists and those of related disciplines, who seem largely ignorant of developments in MCDM and decision analysis in general. Certainly the groupware which this writer has seen includes extremely naive scoring systems for the all-important phase of bringing together the ideas which are generated, in order to facilitate consensus-forming on final conclusions and recommendations for course of action. If MCDM does not get into this process, we shall be left behind having no influence on where it is really happening. This is perhaps the greatest challenge of all.

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Towards the Tradeoffs-Free Optimality in MCDM

Milan Zeleny
Graduate School of Business Administration
Fordham University at Lincoln Center
New York, New York 10023, U.S.A.

Multiple Criteria Decision Making (MCDM) has for decades relied on the traditional optimality concept of OR/MS, based on a single scalar function maximization with respect to a priori given constraints. Whenever it encountered a proper vector function, MCDM has promptly resorted to its aggregation and scalarization.

This fundamental limitation is at the core of the continued lack of MCDM success in replacing or even complementing the OR/MS philosophy in decision-making practice. It thus appears to be destinned to follow the same if not worse course of decline into irrelevancy.

The MCDM problem is *not* a problem of scalar maximization subject to constraints. It is a *vector optimization* problem and it has to be treated as such.

John von Neumann and Oskar Morgenstern had the following to say about the vector optimization problem [1]:

> *This is certainly no maximum problem, but a peculiar and disconcerting mixture of several conflicting maximum problems ... This kind of problem is nowhere dealt with in classical mathematics. We emphasize at the risk of being pedantic that this is no conditional maximum problem, no problem of the calculus of variation, of functional analysis, etc. It arises in full clarity, even in the most "elementary" situations, e. g., when all variables can assume only a finite number of values.*

A vector optimization problem [2] arises when two or more (K) scalar-valued objective functions (or criteria) are to be maximized over a set of feasible solutions:

$$opt\,\{f(x) \mid x \in X\}, \tag{1}$$

where $X \in \Re N$ and $f(x) = [f_1(x), ..., f_K(x)]$.

A feasible solution x" is nondominated with respect to X and f(x) if and only if there exists no $x' \in X$ such that $f(x') \geq f(x")$ and $f(x') \neq f(x")$. Solving a vector optimum problem means identifying all x", i. e. the set $\mathcal{N}(X)$, which are nondominated with respect to X and f(x).

Each component scalar-optimum problem,

$$opt^{+}\,\{f_k(x) \mid x \in X\},\ k = 1, ..., K \tag{2}$$

has an optimal solution x_{k+} with $f_{k+} = f_k(x_{k+})$. The vector $f^+ = (f_{1+}, ..., f_{K+})$ represents an ideal value of f(x) with respect to X. If $x_{k+} = x^+$, k = 1. ..., K, the vector optimum problem has a *perfect solution* $f^+ = f(x^+)$. This excepted, no feasible x yields vector f^+.

It is then possible and often necessary to redefine (or "reshape") X so that a perfect solution is brought forth and the vector optimization problem effectively resolved [3].

The perfect solution f^+ is mirrored by the lowest, separately acceptable values achieved by "anti-optimizing" over the nonodominated set $\mathcal{N}(X)$ of X. Each component scalar anti-optimum problem,

$$opt^- \{f_k(x) \mid x \in \mathcal{N}(X)\}, k = 1, ..., K \qquad (3)$$

has anti-optimal solution x_{k-} with $f_{k-} = f_k(x_{k-})$. The vector $f^- = (f_{1-}, ..., f_{K-})$ represents anti-ideal value of f(x) with respect to $\mathcal{N}(X)$. If $x_{k-} = x^-$, k = 1. ..., K, the vector optimum problem has the imperfect solution $f^- = f(x^-)$.

Imperfect solution is not acceptable as a whole (because it is necessarily dominated) although its individual component values are acceptable separately (as they could become part of $\mathcal{N}(X)$).

Performance values of perfect and imperfect solutions identify the ranges of achievable values $[f_{k-}, f_{k+}]$, k = 1. ..., K, defined over $\mathcal{N}(X)$. These ranges provide the potentials for change for each criterion, as in IDEA [6].

Barring several algorithmic results in identifying and interactively reducing (or filtering) the set of nondominated solutions $\mathcal{N}(X)$, the above summary covers most of the advances made by MCDM in the area of vector optimization.

It is therefore time to face the truth. Even though MCDM formally originated from *vector optimization*, it has, so far, not passed beyond the above initiatory definitions and calculations. Instead, the vector optimization problem has been directly and without sufficient justification transformed into an aggregate *scalar* optimization problem and most research has concentrated on determining the multiplier-like "weights of importance" for component criteria functions entering the aggregate scalar function.

Vector problem (1) has thus been conveniently replaced by a scalar problem:

$$max \{\mathcal{U}[f(x)] \mid x \in X\}, \qquad (4)$$

or by

$$max \{\mathcal{U}[w, f(x)] = w_1 f_1(x) + ... + w_K f_K(x) \mid x \in X\}, \qquad (5)$$

or by some other of the countless utility-function variations of $\mathcal{U}[w, f(x)]$.

Multiple criteria problem has thus been transposed into a single criterion problem. Only the name, *Multiple* Criteria Decision Making (MCDM),

still persists - dating from the first MCDM conference in South Carolina in 1973.

The scalarization of MCDM has had its obvious effects on the treatment of criteria weights of importance. The $\mathcal{U}$[w, f(x)] is a single-criterion aggregation structure which overrules the individual decision criteria and renders their weights of importance, wk, k=1, ..., K, as either meaningless or as mechanical normalizing multipliers. The larger the weight wk, the more valued is criterion's *contribution* to the overall aggregate performance of $\mathcal{U}$[w, f(x)].

Why should the notion of criterion importance be related to or derived from the performance of an aggregate superfunction $\mathcal{U}$[w, f(x)] is not easy to answer.

Clearly, a decision maker does not seek to maximize any aggregate, superfunction or collective performance, but searches for a specific mix of actual criteria performances, ideal if possible, closest to the ideal if necessary. Search for the close-to-ideal equilibrium is therefore his primary concern. Modern cognitive sciences clearly show that humans do not maximize any collective superfunction but search for a balanced or equilibrium performance, close to perfect solution in accordance with the expressed weights of criteria importance. Individual performances are more important than the overall performance.

The scalarization and single-objective aggregation also disposes of the problem *of tradeoffs.*

Criteria tradeoffs exist *only* among different multiple criteria. Whenever there is only a single criterion, like $\mathcal{U}$[w, f(x)], there can be no tradeoffs. The original criteria tradeoffs have been measured, evaluated and subsumed within the aggregate. Subsequent maximization of $\mathcal{U}$[w, f(x)] does not take any tradeoffs into account. Without evaluating and comparing tradeoffs there can be no decision making, just measurement and search.

There is another, more effective and practically more desirable way of disposing of tradeoffs: through the redefining (reshaping) the set of alternatives X and thus of $\mathcal{N}$(X). In fact, the world-class management practice has shown that pursuing and achieving lower cost, higher quality and improved flexibility, all at the same time, is not only possible and competitively desirable, but also necessary.

Tradeoffs can only be postulated among different, conflicting and multiple objectives or criteria. Conventional wisdom recommends dealing with such conflicts through "tough choices and a careful analysis" of the trade-offs. Yet, many Japanese factories have achieved lower cost, higher quality, faster product introductions, and greater flexibility, all at the same time. We can show that companies can have it all if only they would embrace the tradeoffs-free thinking and tradeoffs-free methodology of optimal systems design via De novo programming [4, 5].

John C. Anderson at al., reviewed "The Need to Make Trade-offs" [7] in 1989 and concluded:

> *Recently, trade-offs have been called into question as operations are being designed which have better quality, lower cost and faster delivery than the competitors. These operations*

> *have moved to a new level of performance rather than making trade-offs on an existing level. Because of these new insights, the exact nature of trade-offs is no longer clearly understood* [the emphasis M. Z.]

How should the traditional tradeoffs be "eliminated" or "overcome"? Are not tradeoffs generic to multiple-criteria conflicts? Can we have it both ways? Can one decrease cost and increase quality, all at the same time - and continue doing so? The answer is: Yes. Tradeoffs are properties of inadequately designed systems and thus can be eliminated through designing better, preferably optimal, systems.

There are no conflicting objectives per se. No human objectives are in conflict by definition, that is, inherently conflicting. Everything depends on the given situation, the historical state of affairs, the reigning paradigm, or the lack of imagination.

We often hear that one cannot minimize unemployment and inflation at the same time. We are used to the notion that maximizing quality precludes minimizing costs, that safety conflicts with profits, Arabs with Jews, and industry with the environment. Although these generalizations may be true, they are only conditionally true. Usually inadequate means or technology, insufficient exploration of new alternatives, lack of innovation - not the objectives or criteria themselves - are the causes of apparent conflict.[1]

If we are to deal with the problems of reality in terms of non-scalarized vector optimization and via "reshaping" of X in order to eliminate or reduce the magnitude of tradeoffs, one has to significantly extend the traditional concept of optimization. In Table 1 we summarize all major optimality concepts according to a specific dual classification: single versus multiple criteria and the extent of the a priori "given", ranging from the "all-but" to "none-except". Traditional concept of optimality, characterized by so many "givens" and a single criterion, now appears to be the farthest removed from the useful optimal conditions or circumstances for human problem solving and decision making, as indicated in the Cognitive Equilibrium (Matching) concept in the lower right-hand corner.

The *eight optimality concepts* of Table 1 are all distinct and fundamentally irreducible to each other. The field of MCDM should derive its contributory charge and challenge from pursuing and implementing all eight optimality concepts in all their practical combinations. MCDM should not limit itself to the top left-hand corner of Traditional "Optimality" and thus cut itself off the significant part of management and business reality. Otherwise, MCDM shall remain just a philosophical copy of the traditional OR/MS, sharing in its fate, reputation and practical relevance in the modern space of world-class management and business.

Optimization remains and shall remain at the core of MCDM. But optimization itself is multidimensional and contextually variable. In reality, it

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[1] These two paragraphs are reprinted from the conclusion of author's text on *Multiple Criteria Decision Making*, McGraw-Hill, New York, 1982, p. 402.

has at least eight corner manifestations (or dimensions) which should not be aggregated or reduced. Their scientific and practical pursuit is the challenge of MCDM.

Number of Criteria Given	Single	Multiple
Criteria & Alternatives	Traditional "Optimality"	MCDM
Criteria Only	Optimal Design (De Novo Programming)	Optimal Design (De Novo Programming)
Alternatives Only	Optimal Valuation (Limited Equilibrium)	Optimal Valuation (Limited Equilibrium)
"Value Complex" Only	Cognitive Equilibrium (Matching)	Cognitive Equilibrium (Matching)

Table 1. Eight Concepts of Optimality

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Some Thoughts on MCDM: Myths and Ideas

Stanley Zionts

School of Management, State University of New York, Buffalo, NY 14260-4000, USA

1 Introduction

Decision making is a most important function and process of management. It is one of the most pervasive problems of management and a pervasive problem in general. Multiple criteria decision making is an approach to decision making. It is the making of decisions taking into consideration multiple conflicting objectives. How do we make decisions? What is involved? There are many simple paradigms, and until recently about all that could be said was that you should list the alternatives, score them on certain dimensions, and then somehow aggregate the scores, and choose an alternative. Unfortunately, many excellent decision makers simply choose an alternative, based on instinct, intuition, or feel. Then they go on. Though that is fine in many cases; in some cases it is not. In this paper I consider some myths of decision making. I then propose some steps that we, as decision analysts and scientists, should consider taking to try to sensitize decision makers to thinking more about their decisions and making better decisions.

Since the development of management science, there has been great interest in making decisions based on mathematical models. The first such models appeared around 70 years ago. However, there were earlier developments. To get a flavor of a really early decision making "technique", perhaps the first in recorded history, consider what American statesman and inventor Benjamin Franklin did, approximately two hundred years ago (Parton, 1864). When faced with a proposal that he had to act upon, he folded a sheet of paper and kept it handy, and wrote arguments in support of the proposal on one side of the sheet, and wrote arguments against the proposal on the other side of the sheet. After a period of time, when he had written down all the arguments he could come up with, he then identified one or more arguments on one side of the issue roughly of the same importance and effect as one or more arguments on the other side. He then crossed off those arguments. He continued the process until all the arguments on one side of the sheet were eliminated in that fashion. Because he had struck off arguments of equal importance and effect at each stage, the remaining arguments made the difference and helped him to determine his decision.

Franklin's approach may be considered a decision making method, an early one to say the least!

The field of multiple criteria decision making has now matured. Numerous models have been developed and applied. Let's now consider some myths of MCDM and explore ways to overcome the problems of the myths.

2 Ten Myths of MCDM and Decision Making

When I was invited to this meeting, I was asked to talk about myths of MCDM/decision making that I presented earlier. Some time ago I came up with ten reasonable assumptions of decision making, which I presented and then argued were myths. I now briefly review the myths, with some brief comments on each. The following are the myths:

1. The myth of a decision. Though there is sometimes a well-defined decision and a decision point or time, often there is not. Many decisions simply evolve.

2. The myth of a decision maker. We think of a decision maker making a decision, like a wise man sitting on a throne. (We use man and male pronouns in this paper generically, and not otherwise.) What is a more accurate scenario is that often a group rather than an individual makes a decision. Even where there is not a group, many people influence the decision maker, and the decision is a result of a complex interactive process.

3. The myth of a fixed set of alternatives. We tend to think as the set of alternatives as being fixed. More likely the set is dynamic, and changes over time.

4. The myth of an optimal solution. Though in principle decision makers would like to make optimal decisions and have optimal solutions, they are probably not able to distinguish between optimal solutions and good solutions. The emphasis is therefore on finding good solutions and sometimes only on finding solutions.

5. The myth of seeking nondominated[1] solutions. Unfortunately, most decision makers do not concern themselves with nondominated solutions. They are happy to have a solution.

6. The myth of a utility or value function. Though a utility or value function may be a useful construct for decision makers, it seems more important to analysts than to decision makers.

7. The myth of decisions being static. In decision analysis, we think about good and bad decisions and good and bad outcomes. Of course decision makers want to make good decisions. But when a good decision is made, and a bad outcome occurs, a good manager doesn't shrug his shoulders and say "That's just too bad." He proceeds to adjust the decision in some way, to improve the bad outcome.

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[1]One solution dominates a second if it is at least as good as the second in all respects, and is strictly better than the second in at least one respect. A solution that is not dominated by any other is said to be nondominated.

8. The myth that sophistication/complexity is good. From an analyst's point of view, we sometimes get the feeling that sophistication/complexity is good. However, as we all know, that is not true. Sophistication/complexity is not good. Simplicity where attainable is much preferred.

9. The myth that mathematical convergence is good. Here too, from an analyst's or a mathematician's point of view, mathematical convergence may be desirable. However, if we can help people think through problems, then even though a process may not converge mathematically, it may still be fine from a practical perspective.

10. The myth that all technical model assumptions must be satisfied for a decision-making model to be useful in practice. Like the last two myths, we might expect that all technical model assumptions must hold in order for a model to be useful. However, several researchers, including Howard Raiffa, have stated that this need not be the case.

3 Overcoming the Problems of the Myths

I have found in working with managers and entrepreneurs is consistent with the myths. Though the myths may be appropriate in theory, they are not necessarily true in practice; they may be erroneous in practice. This is also consistent with work that I have done together with Eero Kasanen, Hannele Wallenius, and Jyrki Wallenius (1994). In that study we examined six real-world major strategic decisions. We examined the myths and found only weak support for them. Many of the points we raised in our brief discussion of the myths were valid.

The two myths that were most strongly supported were :

1. We were able to identify a nominal decision maker in every instant. However, as was pointed out above, even though it was clear who was the nominal decision maker, there were others involved either in influencing the decision maker or in making the decision.

2. We were also able to show that the decision point or time could be isolated by the people making the decision.

The other myths were only weakly supported. There are several messages that we learned from the study, and I incorporate them into the discussion below.

What have we learned? First, our decision-making framework and our decision-making myths are only rough approximations to the way decisions are made. There is no standard framework for decision making, and there are many different ways in which decisions are made. We should study actual decision making further to learn more about decision making. That is what we have attempted to do in Kasanen et al. (1994). Of course, there is much more to do. However, we can do certain things to

improve decision making and the role of MCDM methods in decision making. We shall consider some things we may do below.

First, we have to make our models more desirable from the perspective of decision makers, and more usable to decision makers. Models are intended to help decision makers.

Second, we have to abandon our algorithm first approach, whereby we develop algorithms and then try to peddle them, rather than work with decision makers and understand their decision-making process.

Finally, we have to sensitize decision makers and have them think more about decisions, rather than having them simply make a decision and move on. How can we do this? Here are a few ideas that may be useful:

1. We should try to sensitize decision makers to think about their decisions. When confronting a decision, they should think about the magnitude of the decision, and some idea of what an optimal decision is worth compared to a less-than-optimal decision. If the difference does not justify the time for analysis, then they should make the decision and go on, as many do now.

2. Where analysis is worthwhile, decision makers should first do what I like to call a preliminary utility analysis. Rather than use formal decision analysis, decision makers should decide whether any alternatives have an unacceptably large chance or probability of a truly unacceptable or disastrous outcome. Such alternatives may be eliminated. Only alternatives with a sufficiently small chance of disastrous outcomes should be considered as active alternatives for possible selection. In case none of the alternatives have sufficiently small probabilities of unacceptable or disastrous outcomes, or for other reasons, decision makers want to take decisions that risk unacceptable outcomes, that is of course their choice. They should at least be aware of such risks! There are examples of companies that have made decisions where they ignored the chance of unacceptable outcomes, and disaster followed.

3. Decision makers should think about tradeoffs among criteria, that improving one criterion should worsen another.

4. Where a utility function would be appropriate, one may be calibrated and used. However I believe that utility functions will be used only in certain very important situations.

5. Alternatives should be scanned, with the concept of dominance in mind. It makes sense to use a computer program for doing this, because scanning for dominance may be quickly and easily done using a computer. A spreadsheet or a decision-making program is appropriate. Dominated

alternatives should generally be eliminated, except for certain special instances.[2]

6. Too often when decision makers determine a set of alternatives, they limit their choices to the original set. Some judgment must be made as to whether additional alternatives should be generated or not. A simple cost-benefit approach is appropriate, whereby the search is justified if the expected gain from the search is greater than the expected cost of the search. In our latest computer version of AIM (Lotfi et al., 1992), we have a built-in procedure to help generate new alternatives. Sometimes radical thinking can be useful in generating new alternatives. Involving others, and encouraging and asking dumb questions are ways of generating alternatives that are creative and perhaps valuable.

4 Conclusion

A lot of fine work has been done in the MCDM area. Our field has addressed many problems and come up with many solution approaches. Much of that work has been programmed for computer and used in various studies. What we need to do is to learn from what we have done, and use and peddle what is useful. There is a need for something simple, something perhaps that can be thought of a modern version of Ben Franklin's approach. An idea would be to use a spreadsheet with weights, or perhaps something a bit more involved. Just as the electronic spreadsheet has revolutionized the use of the computer for many tasks, a simple spreadsheet decision method could also be of great use to decision makers in making all sorts of decisions. This is the idea that Lotfi, Stewart, and I used in developing our Aspiration-level Interactive Method (AIM) for MCDM (Lotfi et al., 1992). It involves specifying aspiration levels for each objective, and internally using those aspiration levels to generate weights for a proxy utility function to rank alternatives. We also use many ideas of others in trying to help the user choose an alternative. For example, we use the ideas of Wierzbicki (1979) in creating our metric for ranking alternatives, and we use ideas of Roy et al. (1968) in identifying similar solutions to ones under consideration. A different approach with a similar philosophy is V*I*S*A for Windows (1995).

We have had some success with our AIM approach, and are currently reconfiguring it to have a graphical user interface. We hope to have the new version ready in about six months. We encourage others to consider some of the issues we have been thinking about.

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[2]Dominated alternatives should be excluded from further consideration, except where some of the criteria are proxies for unknown underlying criteria (such as asking price in buying a house is a proxy for the unknown underlying criterion of price), and where there are some unformulated, unstated, or hidden-agenda criteria.

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The European School of MCDA : Some Recent Trends

Constantin Zopounidis
DSS Laboratory, Dept. of Production Engineering and Management,
Technical University of Crete,
University Campus, 73100, Chania,
Greece

Abstract
This paper shortly describes the main characteristics of the European School of MCDA. Then, it provides some recent research directions showing the future development of this School. The paper also includes a recent bibliography on the subject of MCDA.

1. Introduction

The great success of multicriteria analysis today is manifested by the increasing number of theoretical and application papers published in several scientific journals of operations research and decision science and by the great number of communications presented in scientific meetings. Furthermore, several international working groups have been established to analyze this topic on a regular basis. According to the evolution of the multicriteria analysis, several schools of thought have been developed, each one insisting on a different idea of supporting or aiding decisions. Moreover, several overviews proposed by the specialists in the field show clearly the different points of view of the multicriteria analysis (cf. Dyer et al., 1992; Korhonen et al., 1992; MacCrimmon, 1973; Starr and Zeleny, 1977; Stewart, 1992). Among these schools, we can distinguish the European School of multicriteria analysis called, also, multicriteria decision aid (MCDA). The members of this school nearly all come from the European Working Group known as "Multicriteria Aid for Decisions".

Section 2 briefly describes the basic characteristics of the European School which distinguish it from the others. Its recent trends and main new ideas are presented in section 3. Finally, some discussion elements of the European School are given in section 4.

2. Basic characteristics

The European School of MCDA is very well presented in the excellent paper of Roy and Vanderpooten (1995). In this paper entitled "The European School of MCDA : emergence, basic features and current works", the authors lay the foundations of this school of thought. The present paper constitutes, according to our modest opinion, a continuation of the above paper by presenting some new developments of the European School of MCDA.

According to Roy (1994a), the European School is different from the other schools of thought, mainly the American School, on the following points :

- a general methodological framework in order to structure a decision problem and a consistent family of criteria able to facilitate the communication in the bosom of the decision process ;
- a decision aid by the gradual development of a conviction rather than researching of an optimum or a pre-existent preference ;
- the notions of imprecision, uncertainty and ill-determination are included in the decision process, modelling by use of the concepts such as thresholds (preference thresholds, indifference thresholds, veto thresholds) rather than the probability distributions;
- the relative importance of criteria or pseudo-criteria by reconsidering the usual notions of weighting, and the development of new approaches in order to apprehend and to quantify this idea of importance, according to the kind of the proposed model;
- the notion of the incomparability in the preference modelling by using, mainly, the concept of outranking relation.

In the above, features one can add two others which concern the use of appropriate interactive procedures and the systematic use of a robustness analysis (cf. section 3, Roy and Vanderpooten, 1995; Mousseau and Roy, 1994).

Bana e Costa (1993) states that one of the methodological aspects of disagreement between the American and European multicriteria schools lies in the procedure (prescriptive/descriptive versus constructive) used in order to construct a decision aid model. Using the prescriptive way of modelling (American School), a decision analyst makes the prescriptions on the basis of normative assumptions which are validated by reality. In the constructive way (European School), the decision analyst's objective is to help the decision maker to construct a decision aid model on the basis of work assumptions which are not taken as norms. These work assumptions must be considered as recommendations rather than as norms.

The analysis of the above characteristics leads to the concept of a general decision aid methodology such as has been proposed by Roy (1985). This European decision aid methodology can be enlarged to the other multicriteria schools (e.g., American) and includes a way of identifying and defining alternatives, the techniques for constructing criteria and a way of stating a decision problem by choosing an appropriate problematic.

3. Some recent trends

In the European School of MCDA one can distinguish three main categories of methods, although the boundaries between these categories are rather fuzzy :

- The outranking synthesis approach,
- The unique synthesis criteria approach,
- The interactive approach.

A large family of methods can be found in the outranking approach such as the ELECTRE methods (cf. Roy and Bouyssou, 1993), the PROMETHEE methods (cf. Brans et al., 1984), the ORESTE method (cf. Roubens, 1982), the ELECCALC method (cf. Kiss et al., 1994), etc. Within the single criterion approach one can find the Preference Disaggregation Approach proposed initially by

Jacquet-Lagräze and Siskos (1982) with the UTA method (see also : Siskos and Yannacopoulos, 1985; Despotis et al., 1990). Other techniques in the same category are the REMBRANDT method (cf. Lootsma, 1993) and the MACBETH method recently developed by Bana e Costa and Vansnick (1994).

Several methods have been proposed in the interactive approach starting with the pioneering work of Benayoun et al. (1971). However, according to Roy (1987) and Vanderpooten (1989), the main contribution of the European School is the development of an alternative concept of interactive procedures, which is the rejection of any assumption about the decision maker's preferences (e.g., the assumption of a preexisting value function).

After the great popularity of MCDA in the last twenty years, the question is how to forecast the future developments of this science. Many academic researchers have considered this problem. Our objective, in this paper, is not to describe all the recent developments of the MCDA in Europe. Obviously, this is a very ambitious project. However, our aim is more modest, concerning only the presentation of the most characteristic research works of the European School of MCDA. Another difficulty, in order to present all the recent trends of the MCDA, was the space restrictions posed by the editorial policy.

First, we can begin the analysis of the future development on MCDA with the work of five specialists in the field who express theirs thoughts for the future. In fact, Bouyssou et al. (1993) in their work "The manifesto of the new MCDA era" present the MCDA as :

- an attitude toward providing decision-aid to actors involved in a decision process;
- a methodology for providing such a decision-aid;
- a collection of methods, and
- a corpus of experience obtained after many real world applications.

For these authors, MCDA is an open theoretical field rather than a closed mathematical one, and they propose the following areas for future research :

- Key theoretical and methodological issues have to be addressed;
- New potential topics of applications have to be explored;
- "True" applications are needed as well as ex-post analysis, and
- Connections between MCDA and other areas of research have to be explored (ie. fuzzy sets, cognitive science, artificial intelligence, value theory, organisational theory, control theory, logistics, etc.).

The need to have more applications and tests of the available multicriteria methods is clearly stated in the works of Pomerol and Barba-Romero (1993), Vansnick (1995), and, also, by authors from the American School (cf. Dyer et al., 1992; Stewart, 1992).

Roy (1994b) investigates the following important questions about the future of MCDA, in order to complete the propositions made by Bouyssou et al., (1993) in their manifesto.

- What is the scientific basis of a decision aid science ?
- What is the relationship between the decision aid science and the science of decision behavior ?
- Does one have a habit of confusing the result of a procedure with the recommendation ?

Another important future direction of the MCDA is the development of multicriteria decision support systems (MCDSSs). A MCDSS is simply a DSS

that helps to implement multicriteria methods. Characteristics of MCDSSs include analyses of multiple criteria, involvement of multicriteria methods, and the integration of user input (decision maker's preferences) in the modelling processes (cf. Jelassi et al., 1985). The development of some MCDSSs can be found in Siskos and Zopounidis (1993) and Jelassi and Brans (1994).

Studying the scientific reports, the information on MCDA research groups in Europe (cf. Newsletter of the EWG "Multicriteria Aid for Decisions" edited by Joao Climaco) and through personal contacts with colleagues in the field, it is possible to make an overview of the main future axis in the field of MCDA.

In the LAMSADE Laboratory, the research team directed by Bernard Roy works on seven topics related to the decision aid theory (cf. Scientific Report 1993-94, Document no 88, 1995). More specifically, among the seven topics, topic No 1 concerns research works on MCDA. This topic is divided in the following four subjects :

- conceptual and axiomatical foundations;
- procedures and algorithms;
- imprecision, uncertainty and inaccurate determination, and
- methodologies, approaches and transversal point of view.

On the first subject one can cite the following research works.

- In the field of preferences modelling, the study of the structure of non-conventional preferences is made by Tsoukias. With collaboration of Vincke, he proposes a new axiomatic foundation for the partial comparability theory by using a four valued paraconsistent logic. Tsoukias proposes, also, a generalisation of the concordance-discordance concept by considering them as "positives-negatives" reasons for and/or versus the establishment of an outranking relation.
- Bana e Costa provides some new ideas about the concept of problematic. In fact, he proceeds to the research and the enlargement of the trilogy : selection, sorting, ranking.
- The problem of the relative importance of criteria is studied by Mousseau and Roy. The future work in this field is to determine a theoretical framework in order to analyse the notion of the relative importance of criteria under general conditions.
- Roy and Slowinski have proposed an original way to construct a criterion of distance in order to compare two partial preorders coming from two quite different points of view (socio-economic priority and technical programming).
- The contribution of the rough set approach in the decision aid theory is jointly studied by the LAMSADE laboratory (Vanderpooten) and the Institute of Computing Science of the Poznan University of Technology (Slowinski and Stefanowski). At the beginning, the rough set has been used for the analysis of multiattribute preference systems in the sorting decision problematic, while, very recently, it has been extended to the choice and ranking decision problematics (cf. Greco et al., 1995).

In the second subject, it is important to note the following research works :

- Development of a new multiattribute sorting method, in order to assign – in a fuzzy way – each alternative into one category among a set of categories ordered

or non-ordered, defined in advance (Vanderpooten). This sorting method is used by Lenard in his Ph.D. thesis on stock management.

- In the field of the multicriteria interactive procedures, several methods have been proposed (interior methods and interactive procedures in multicriteria mathematical programming using outranking relations as preference models, the light beam search over a non-dominated set interactive method, the software DISCC : Désirs Individuels à Satisfaire sous Contraintes Collectives, etc., (cf. Scientific Report 1993-1994).
- Development of the MACBETH method which is an interactive approach for cardinal measurement of judgements and preferences of a decision maker (Bana e Costa and Vansnick).

Concerning the third subject, the following research works are analysed.

- The pseudo-criterion model used as a tool to make comparisons under the conditions of imprecision, uncertainty and inaccurate determination (Roy).
- The robustness analysis in multicriteria decision aid. The robustness analysis is a new concept in MCDA and is different from the classic concept of the sensitivity analysis, because it searchs for conclusions which remain valid for a great number of data set. This field needs more future work on two topics : conceptual work (cf. Roy and Mousseau) and development of tools (e.g. rough set theory, cf. Slowinski; data base systems).
- The imprecision of the substitution rates between criteria in an additive model of aggregation preferences is studied by Bana e Costa and Vincke.

The fourth subject includes the following research works :

- Investigation of the links between the decision aid and the operations research (Roy).
- Conception of a French-English thesaurus of MCDA index terms (Van Asbroeck and Champ-Brunet).
- The manifesto of the new MCDA era presented above. Other topics include also research works on MCDA as follows :
- multicriteria optimisation (topic no 2);
- improvement of the organizations' management by using multicriteria analysis (topic no 3);
- introduction and application of the MCDA methods in other fields of research (environment, production, transport, finance, personnel, marketing, etc.), (topic no 4)
- research into the complementarity and the interdependence between the multicriteria approaches and the artificial intelligence ones (topic no 5).

The activities of the other European laboratories about the MCDA research are generally similar with little differences. According to the informations on MCDA research groups (see Newsletter of the EWG on MCDA), one can mention the main features of the MCDA research made in each laboratory.

The research on MCDA of the Decision Support Systems laboratory (Technical University of Crete) concerns the following subjects (cf. Zopounidis, 1992) :

- Multicriteria decision aid methods;

- Application of MCDA methods in the fields of the financial management and marketing management;
- Multicriteria decision support systems.

For the Laboratory of Mathematics and Operations Research of the "Faculté Polytechnique de Mons", the MCDA research concerns four specific themes (cf. Teghem, 1993) :

- Building vigorous foundations for outranking methods;
- Fuzzy preference modelling and aggregation;
- MOLP under uncertainty;
- MOLP with discrete variables.

The team of the Multiple Criteria Decision Support of the Helsinki School of Economics and Business Administration poses the following questions about the research on MCDA (cf. Korhonen and Wallenius, 1993) :

- The importance of criteria;
- The impact of starting point on the final outcome in a multiple criteria decision problem;
- The progressive group decision making.

The MCDA research of the Laboratory of Operations Research of the "Université Libre de Bruxelles" refers to the following subjects (cf. Vincke, 1994) :

- Multicriteria decision-aid : theory, methods, software;
- Preference modelling;
- Group decision support systems ;
- Multicriteria industrial evaluation (finance company)
- Multicriteria evaluation of suppliers (chemistry industry). For the team of the Laboratory of Intelligent Decision Support Systems (Institute of Computing Science, Poznan University of Technology), the research on MCDA concerns the following subjects (cf. Slowinski, 1995) :
- Preference modelling ;
- Interactive procedures for multi-criteria choice, sorting and ranking;
- Multi-objective mathematical programming in deterministic or fuzzy environment ;
- Applications of MCDA.

Also, it is important to note for the same Laboratory, the research work which is made on knowledge-based systems (e.g. rough set approach, fuzzy set approach, etc.) and its relation to MCDA work.

A last point to mention is the research about the development of effective group decision support systems. As Roy (1994b) remarks, the MCDA can provide a substantial support in a group decision-making situation where there are considerable value conflicts between group members. The importance of the development of group DSS is also stressed in other school of thought (cf. Stewart, 1992, Zionts, 1992).

4. Conclusion

The aim of this paper is to describe some new developments of the European Multicriteria School. In relation to this objective, recently a book entitled

"Advances in Multicriteria Analysis" edited by Pardalos, Siskos and Zopounidis (1995) has been published, including several new ideas about the development of the MCDA. On the other side, we are aware that many other new ideas on the development of MCDA are not included in this paper, due to its limited length of this paper. In my opinion, the variety of those mentioned in this paper provides a representative view of the future research activity of the European School of MCDA.

The research activity on MCDA has supported by the operation of two European working groups "Multicriteria Aid for Decisions" (300 members from over 30 countries, formed in 1975) and "European Summer Institute Group on Multicriteria Analysis" (200 members from over 30 countries, started in 1985). These two working groups keep an excellent harmony and cooperation between the members, which lead to the exchange of reflections and the creation of new ideas. Moreover, these groups remain open to methods, approaches, and results coming from the other schools of thought and that, being this in our opinion, an important element of progress for the European School.

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